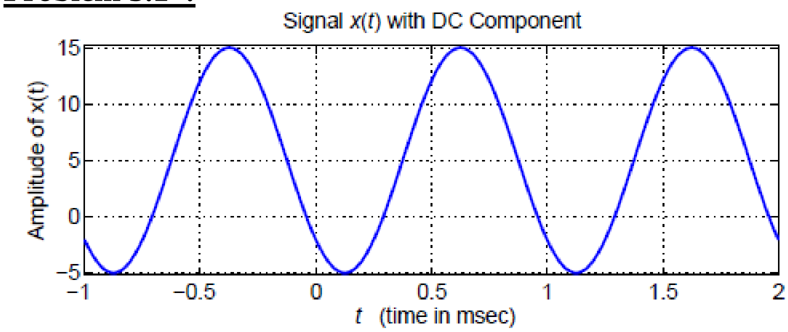
ECE-2025 Homework #3 Solutions **Spring 2011**

Problem 3.1*:



(a)

By applying the definition of a DC-component, it doesn't vary with time, so it must have a frequency of zero rad/sec or 0 Hz. Next, subtract the DC-component from *x(t)* to extract the cosine component. To find the frequency of the cosine component select a full cycle of the signal and measure the duration on the time axis to obtain the period, *T*, which is 1 msec. So the frequency of the cosine component is 1/(1msec) = 1 kHz.

(b)

We know that we are looking for an equation of the form:

$$x(t) = [DC\ component] + A\cos(2\pi f t + \varphi) = [DC\ component] + A\cos(2\pi f(t - t_m))$$

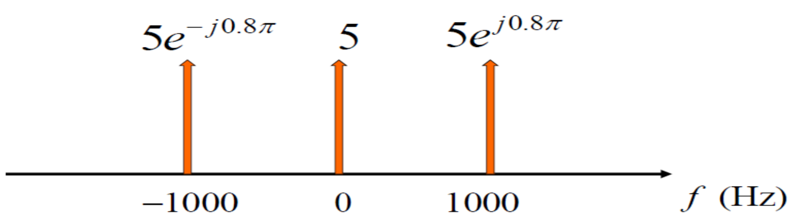
In part (a) we found the *DC-component* of amplitude 5. By inspecting the cosine component, we can determine its maxima and minima and therefore the amplitude, $A = ($maximum - minimum$)/2=10$. The phase, ϕ, can be determined by inspection and noting a cosine maximum occurs at t_m and therefore the formula $\phi = -2\pi f t_m = \frac{-2\pi t_m}{T}$ can be applied to compute the phase where t_m =-0.4msec is a time shift of the positive maximum near time 0. *W*e can use the formula: $\phi = \frac{-2\pi t_m}{T} = \frac{-2\pi(-0.4m\sec)}{1m\sec} = 0.8\pi$ radians. In summary, we have

$$x(t) = 5 + 10\cos(2\pi(1000)t + 0.8\pi)$$

(c)

The DC-component has a value of 5. For the cosine, the Inverse Euler's Formula yields:

$$10\cos(2\pi(1000)t + 0.8\pi) = (10)\tfrac{1}{2}\left(e^{j0.8\pi}e^{j2\pi(1000)t} + e^{-j0.8\pi}e^{-j2\pi(1000)t}\right)$$

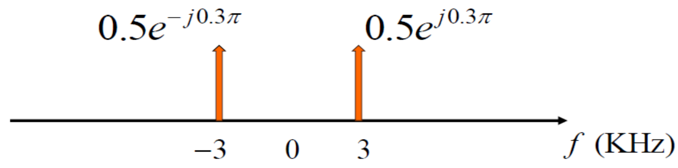


Problem 3.2:

(a)

$$v(t) = \cos(2\pi(3000)t + 0.3\pi) = \frac{1}{2}\left(e^{j0.3\pi}e^{j2\pi(3000)t} + e^{-j0.3\pi}e^{-j2\pi(3000)t}\right)$$

The spectrum plot is: (note that the x-axis scale is in kHz):



(b)

$$x(t) = (v(t) + A)\cos(2\pi(680\times10^3)t) = (v(t)+1.5)\cos(2\pi(680\times10^3)t)$$
$$= \cos(2\pi(3000)t + 0.3\pi)\bullet\cos(2\pi(680\times10^3)t) + 1.5\cos(2\pi(680\times10^3)t)$$

For the cosine product term we can apply the trigonometric identity:

$$\cos a\ \cos b = \tfrac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)$$

$$\cos(2\pi(680\times10^3)t)\ \cos(2\pi(3000)t + 0.3\pi) =$$
$$\tfrac{1}{2}\cos(2\pi(683\times10^3)t + 0.3\pi) + \tfrac{1}{2}\cos(2\pi(677\times10^3)t - 0.3\pi)$$

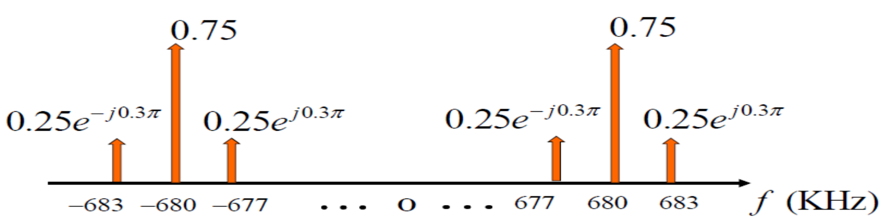
Therefore,

$$x(t) = \tfrac{1}{2}\cos(2\pi(683\times10^3)t + 0.3\pi) + \tfrac{1}{2}\cos(2\pi(677\times10^3)t - 0.3\pi) + 1.5\cos(2\pi(680\times10^3)t)$$

Expressing each cosine as a sum of two complex exponentials with the Inverse Euler's Formula, we have:

$$X(t) = 0.25(e^{j0.3\pi}e^{j2\pi(683\times10^3)t} + e^{-j0.3\pi}e^{-j2\pi(683\times10^3)t})$$
$$+0.25(e^{-j0.3\pi}e^{j2\pi(677\times10^3)t} + e^{j0.3\pi}e^{-j2\pi(677\times10^3)t}) + 0.75(e^{j2\pi(680\times10^3)} + e^{-j2\pi(680\times10^3)})$$

The spectrum plot is: (note that the x-axis scale is in kHz):



Problem 3.3*:

The two-sided spectrum of a signal *x(t)* is given in the following table:

Frequency (rad/sec)	Complex Amplitude
$-\omega_2$	$8e^{-j\pi/3}$
-80π	X_{-1}
0	B
ω_1	$\sqrt{2}+j\sqrt{2}$
100π	X_2

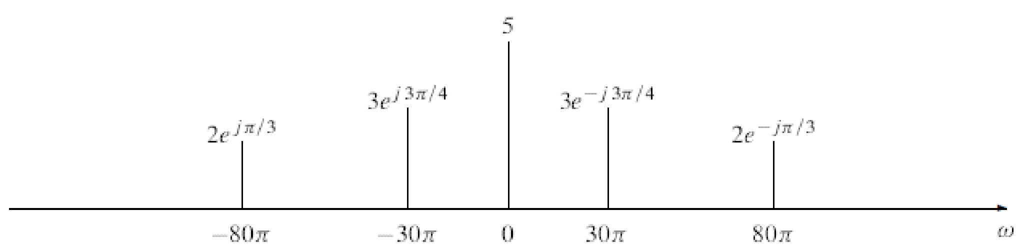
(a) If *x(t)* is a real signal, then based of the symmetry of positive and negative frequency components with complex amplitudes conjugate to each other we have $\omega_2 = 100\pi,$ and $\omega_1 = 80\pi$ and we also have $X_{-1} = (\sqrt{2} + j\sqrt{2})^* = \sqrt{2} - j\sqrt{2} = 2e^{-j\pi/4}$ and $X_2 = (8e^{-j\pi/3})^* = 8e^{j\pi/3}$

(b) The signal can then be expressed as

$$x(t) = B + 4\cos(80\pi t + \frac{\pi}{4}) + 16\cos(100\pi t + \frac{\pi}{3})$$

Problem 3.4*:

(a) A real signal has the following two-sided spectrum:



then $x(t) = 5 + 6\cos(30\pi t - \frac{3\pi}{4}) + 4\cos(80\pi t - \frac{\pi}{3})$

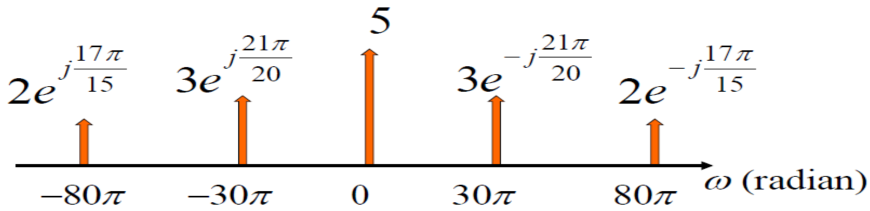
(b) $y(t) = x(t - 0.01) = 5 + 6\cos(30\pi[t - 0.01] - \frac{3\pi}{4}) + 4\cos(80\pi[t - 0.01] - \frac{\pi}{3})$

$$y(t) = 5 + 6\cos(30\pi t - [0.3\pi + \frac{3\pi}{4}]) + 4\cos(80\pi t - [0.8\pi + \frac{\pi}{3}])$$

Or

$$= 5 + 6\cos(30\pi t - \frac{21\pi}{20}) + 4\cos(80\pi t - \frac{17\pi}{15})$$

Now it is clear that the spectrum of *y(t)* is similar to that of *x(t)* with only phase changes for the same frequency components with the same complex amplitudes:



<u>Problem 3.5*:</u>

Signal Processing First, Chapter 3, Problem 19, Page 69-70

(a) The waveform looks like a cosine with a small negative phase shift because of a positive time shift and a positive DC-component. The best candidate is spectrum **(3).** From the spectrum, we can get the formula for $x(t)$

$$x(t) = 2 + 3\cos(2.4\pi t - 0.25\pi).$$

(b) The waveform is a cosine that is 180 degrees out of phase and has an amplitude of ~3, so the spectrum is **(5)** with a signal expressed as:

$$x(t) = 3\cos(3\pi t + \pi).$$

(c) Again, the waveform is a phase-shifted cosine with a positive DC-component. The difference from **(a)** is that the cosine is now with a larger positive phase because of a larger negative time shift. The spectrum is **(1)** with a signal denoted as:

$$x(t) = 2 + 3\cos(2.4\pi t + 0.5\pi)$$

(d) The signal doesn't seem to have a DC-component and its period is approximately 3.3sec. If the waveform is a sum of harmonically related cosines then the fundamental frequency would be $f_0 = 1/T \approx 3\,\text{Hz}$. Spectrum **(2)** seems like a good candidate since the two frequencies are 0.6Hz and 1.5Hz which gives the greatest common divisor (GCD) of 0.3 Hz. The other option is spectrum **(4)**, but the GCD there is 0.4 Hz which does not match with the waveform in **(d)**. From the spectrum, we get the sum of two sinusoids:

$$x(t) = 3\cos(1.2\pi t - 0.25\pi) + 3\cos(3\pi t + \pi)$$

(e) From the discussion for **(d)**, we can see that the spectrum for **(e)** is **(4).** Also the signal in **(e)** has a period of ~2.5 sec. Therefore, a fundamental $f_0 = 1/(2.5\text{sec}) = 0.4$ Hz, which is the GCD of the two frequencies, 1.2Hz and 2 Hz, and we get the sum of two sinusoids:

$$x(t) = 3\cos(2.4\pi t - 0.25\pi) + 3\cos(4\pi t + \pi)$$