GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2011

Problem Set #4

Assigned: 4-Feb-11

Due Date: Week of 14-Feb-11

Reading: In *SP First*, Chapter 3: *Spectrum Representation,* Sections 3-4 through 3-8.

The *SP First* Toolbox for MATLAB has been posted on **t-square** under the "Lab Assignments" link. You can install it to get some useful functions and GUIs for manipulating complex numbers. The direct link to the toolbox is: `http://users.ece.gatech.edu/mcclella/SPFirst/Updates/SPFirstMATLAB.html`

The web site for the course uses **t-square**: `https://t-square.gatech.edu`

⟹ Please check **t-square** daily. All official course announcements will be posted there.

ONLY the **STARRED** problems should be turned in for grading; a random subset of these will be graded.

Some of the problems have solutions that are similar to those found on the SP-First CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 4.1*:

A linear-FM "chirp" signal is one that sweeps in frequency from $\omega_1 = 2\pi f_1$ to $\omega_2 = 2\pi f_2$ as time goes from $t = 0$ to $t = T_2$ s.

(a) Determine the mathematical formula for a chirp whose instantaneous frequency sweeps from $f_1 = 4800\,\text{Hz}$ down to $f_2 = 800\,\text{Hz}$ as time goes from $t = 0$ to $t = 2\,\text{sec}$.

(b) In the following chirp

$$y(t) = \cos(400\pi t^2 + 500\pi t - \pi/4) \qquad 0 \le t \le 3\,s$$

determine the starting and ending instantaneous frequencies.

PROBLEM 4.2*:

Suppose that a periodic signal is defined via:

$$q(t) = \cos(2\pi t) \sum_{\ell=-2}^{2} \left(\frac{1}{\ell + j1.5}\right) e^{j8\pi\ell t}$$

$$= \sum_{k} a_k e^{jk\omega_0 t} \qquad \text{(Fourier Series of the periodic signal } q(t))$$

(a) Make a table listing all the nonzero Fourier coefficients, $\{a_k\}$, for $k \ge 0$.

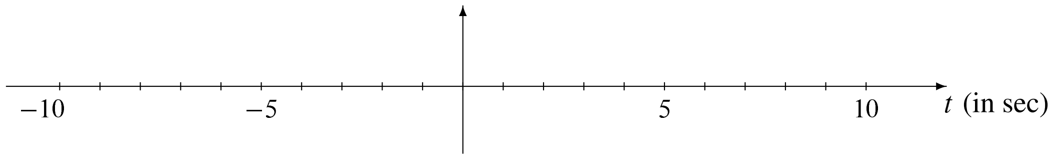
(b) Determine the fundamental period of $q(t)$.

(c) Determine the DC value of $q(t)$.

PROBLEM 4.3*:

Suppose that a periodic signal is defined (over one period) as: $x(t) = \begin{cases} 7 & \text{for } 0 \le t \le 5 \\ 3 & \text{for } 5 < t < 8 \end{cases}$

(a) Assume that the period of $x(t)$ is 8 s. Draw a plot of $x(t)$ over the range $-10 \le t \le 10$ s.



(b) Determine the DC value of $x(t)$ from areas, or from the Fourier series integral.

(c) Write the Fourier integral expression for the coefficient a_k in terms of the specific signal $x(t)$ defined above. Set up all the specifics of the integral, i.e., limits of integration, integrand.

(d) Determine a general expression for the Fourier series coefficients, $\{a_k\}$, by evaluating the Fourier integral expression. Simplify the expression into polar form, so that the magnitude and phase are explicit.

(e) It might be true that every fifth, or every eighth, coefficient is zero for $k > 0$. For example, $a_5 = 0$, $a_{10} = 0$, $a_{15} = 0$, etc.; or, $a_8 = 0$, $a_{16} = 0$, $a_{24} = 0$, etc. Determine which speculation is true.

PROBLEM 4.4:

Musical tones can be modeled mathematically by sinusoidal signals. If you read music or play the piano you are aware of the fact that the piano keyboard is divided into octaves, with the tones in each octave being twice the frequency of the corresponding tones in the next lower octave. To calibrate the frequency scale, the reference tone is the A above middle-C, which is usually called A440 since its frequency is 440 Hz. Each octave contains 12 tones, and the ratio between the frequencies of successive tones is constant. Since middle C is 9 tones below A440, its frequency is approximately $(440)2^{-9/12} \approx 262$ Hz. In musical notation the tones are called notes; the names of the notes in the octave starting with middle-C and ending with high-C are:

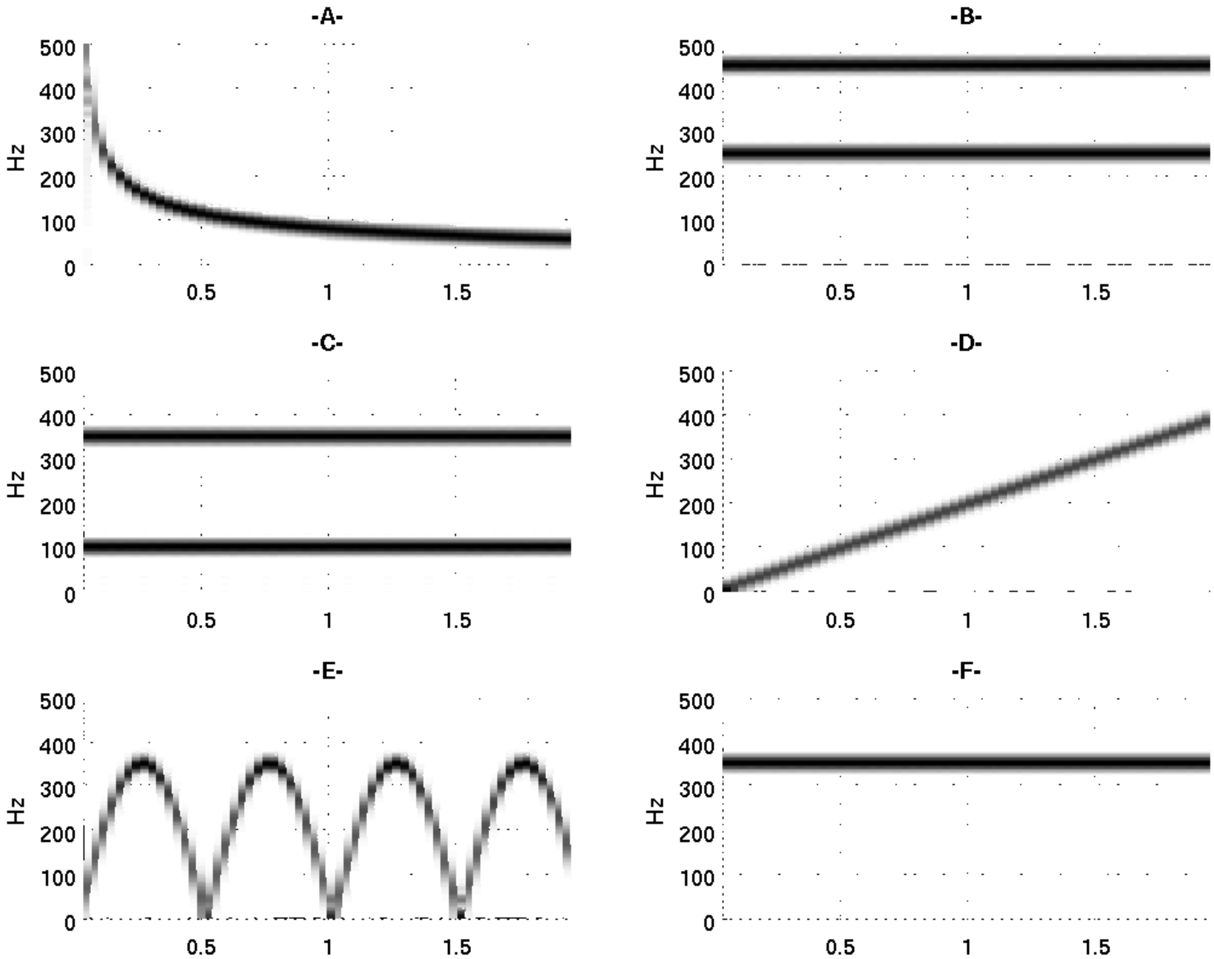
note name	C	$C^\#$	D	$E^\flat$	E	F	$F^\#$	G	$G^\#$	A	$B^\flat$	B	C
note number	40	41	42	43	44	45	46	47	48	49	50	51	52
frequency													

(a) Explain why the ratio of the frequencies of successive notes must be $2^{1/12}$.

(b) Make a table of the frequencies of the tones of the octave beginning with middle-C assuming that A above middle C is tuned to 440 Hz.

(c) The above notes on a piano are numbered 40 through 52. If n denotes the note number, and f denotes the frequency of the corresponding tone, give a formula for the frequency of the tone as a function of the note number.

PROBLEM 4.5:
Shown in the figure below are spectrograms (labeled with letters A-F) for six signals over the time period $0 \leq t \leq 2$. The frequency axis for each plot has units of Hz. For each signal description below, identify the corresponding spectrogram.



(a) $x(t) = \cos(1000\sqrt{t})$

(b) $x(t) = \cos(200\pi t - \pi/4) + \cos(700\pi t)$

(c) $x(t) = \cos(350\cos(2\pi t))$

(d) $x(t) = \cos(700\pi t) + \cos(700\pi t + \pi/3)$

(e) $x(t) = \cos(200\pi t^2)$

(f) $x(t) = \cos(200\pi t)\cos(700\pi t)$