ECE2025 SPRING2011 HW#4

problem 4.1(a)

for Linear-FM chirp, $x(t) = A\cos(2\pi u t^2 + 2\pi f_0 t + \phi$

$$\varphi(t) = 2\pi u t^2 + 2\pi f_0 t + \phi$$

$$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}\varphi(t) = 2ut + f_0$$

$$f_i(t=0) = f_0 = 4800\,Hz$$

$$\frac{f_i(t=2) - f_i(t=0)}{2-0} = \frac{800-4800}{2} = -2000\,Hz$$
$$= 2u$$

$\therefore$ $u = -1000\,Hz$

$$\Rightarrow x(t) = A\cos(-2\pi\cdot 1000\cdot t^2 + 2\pi\cdot 4800\cdot t + \phi) \quad \#$$

prob. 4.1(b) $y(t) = \cos(400\pi t^2 + 500\pi t - \frac{\pi}{4})$, $0 \le t \le 3$ sec

$$= \cos(2\pi\cdot 200\cdot t^2 + 2\pi\cdot 250\cdot t - \frac{\pi}{4})$$

from 4.1(a) $\Rightarrow$ $u = 200\,Hz$ $f_0 = 250\,Hz$

$$f_i(t) = 2ut + f_0$$

The starting freq $f_i(t=0) = f_0 = 250\,Hz$ #

The ending freq $f_i(t=3) = 2\cdot 200\cdot 3 + 250$

$$= 1450\,Hz \quad \#$$

Problem 4.2

$$g(t) = \cos(2\pi t) \sum_{l=-2}^{2} \left(\frac{1}{l+j1.5}\right) e^{j8\pi l t}$$

$$= \sum_{k} a_k e^{jk\omega_0 t}$$

(a) $$g(t) = \frac{1}{2}\left(e^{j2\pi t} + e^{-j2\pi t}\right) \sum_{l=-2}^{2} \left(\frac{1}{l+j1.5}\right) e^{j8\pi l t}$$

$$= \sum_{l=-2}^{2} \left(\frac{1}{2}\right)\left(\frac{1}{l+j1.5}\right) e^{j2\pi(4l+1)t} + \sum_{l=-2}^{2} \frac{1}{2} \frac{1}{l+j1.5} e^{j2\pi(4l-1)t}$$

$$= \sum_{k} a_k e^{jk\omega_0 t}$$

for the first summation let $4l+1=k$

l	k	a_k
-2	-7	$\frac{1}{2}\left(\frac{1}{-2+j1.5}\right)$
-1	-3	$\frac{1}{2}\left(\frac{1}{-1+j1.5}\right)$
0	1	$\frac{1}{2}\left(\frac{1}{0+j1.5}\right)$
1	5	$\frac{1}{2}\left(\frac{1}{1+j1.5}\right)$
2	9	$\frac{1}{2}\left(\frac{1}{2+j1.5}\right)$

for the second summation, let $4l-1=k$

l	k	a_k
-2	-9	$\frac{1}{2}\left(\frac{1}{-2+j1.5}\right)$
-1	-5	$\frac{1}{2}\left(\frac{1}{-1+j1.5}\right)$
0	-1	$\frac{1}{2}\left(\frac{1}{j1.5}\right)$
1	3	$\frac{1}{2}\left(\frac{1}{1+j1.5}\right)$
2	7	$\frac{1}{2}\left(\frac{1}{2+j1.5}\right)$

prob. 4.2(a) Continued

for $k \geq 0$ $\quad a_k = a_1, a_3, a_5, a_7, a_9$

a_1	$\frac{1}{2}\left(\frac{1}{0+j1.5}\right)$
a_3	$\frac{1}{2}\left(\frac{1}{1+j1.5}\right)$
a_5	$\frac{1}{2}\left(\frac{1}{1+j1.5}\right)$
a_7	$\frac{1}{2}\left(\frac{1}{2+j1.5}\right)$
a_9	$\frac{1}{2}\left(\frac{1}{2+j1.5}\right)$

prob. 4.2(b) fundamental period of $g(t)$ is $\frac{2\pi}{2\pi} = 1$ sec $\boxed{T_0 = 1 \text{ sec}}$

when $k=1$

$$g(t) = \cdots + a_1 e^{j2\pi t} + a_3 e^{j6\pi t} + a_5 e^{j10\pi t} + \cdots$$

fundamental freq $= GCD$ of $[2\pi, 6\pi, 10\pi, \cdots] = 2\pi$.

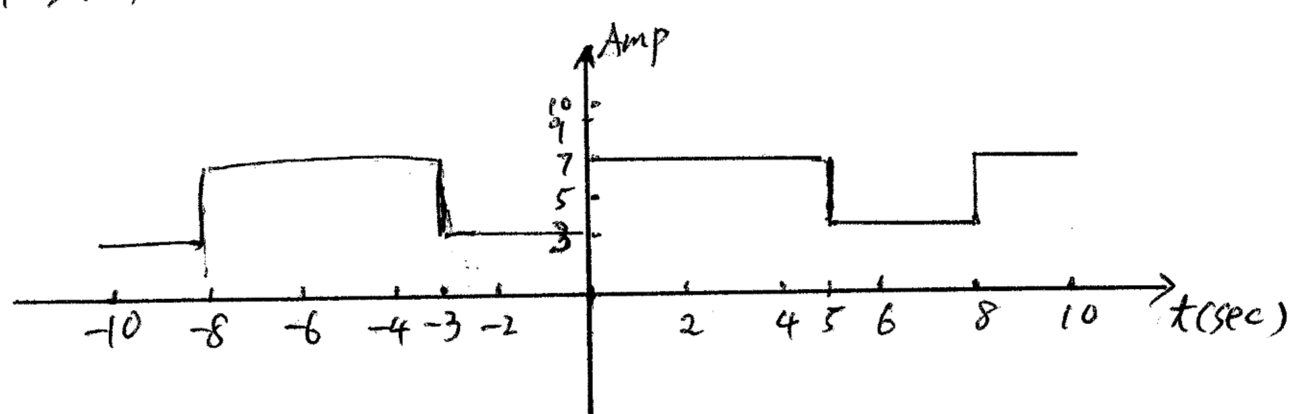
prob. 4.2(c) The DC value of $g(t)$ is 0

since $a_0 = 0$

problem 4.3

$$x(t) = \begin{cases} 7 & \text{for } 0 \leq t \leq 5 \\ 3 & \text{for } 5 < t < 8 \end{cases}$$

4.3(a) assume T_0 for $x(t)$ is 8 sec, plot $x(t)$



prob. 4.3 (b) Determin DC value of $x(t)$

$$a_0 = \frac{1}{8}\int_0^5 7 * e^{-j2\pi \cdot 0 \cdot t} dt + \frac{1}{8}\int_5^8 3 \times e^{-j2\pi \cdot 0 \cdot t} dt$$

$$= \frac{1}{8} \cdot 7 \cdot t \Big|_0^5 + \frac{1}{8} \cdot 3 \cdot t \Big|_5^8 = \frac{35-0}{8} + \frac{9}{8} = \frac{11}{2}$$ ※

DC value $= \frac{11}{2}$ ※

4.3(a) $$a_k = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-j\frac{2\pi}{T_0} k \cdot t} dt$$

$$= \frac{1}{8}\int_0^5 7 \cdot e^{-j\frac{2\pi}{T_0} kt} dt + \frac{1}{8}\int_5^8 3 \cdot e^{-j\frac{2\pi}{T_0} kt} dt$$ ※

4.3(d) $$a_k = \left(\frac{1}{8} \frac{7}{-j\frac{2\pi}{8}k}\right) e^{-j\frac{2\pi}{8}kt} \Big|_0^5 + \left(\frac{1}{8} \frac{3}{-j\frac{2\pi}{8}k}\right) e^{-j\frac{2\pi}{8}kt} \Big|_5^8$$

$$a_k = \left(\left(\frac{7}{8} \frac{1}{-j\frac{2\pi}{8}k}\right)\left(e^{-j\frac{2\pi}{8}k \cdot 5} - e^{-j\frac{2\pi}{8}k \cdot 0}\right)\right.$$
$$\left. + \left(\frac{3}{8} \frac{1}{-j\frac{2\pi}{8}k}\right)\left(e^{-j\frac{2\pi}{8}k \cdot 8} - e^{-j\frac{2\pi}{8}k \cdot 5}\right)\right)$$

$$= \left(\frac{7j}{2\pi k}\right)\left(e^{-j\frac{5\pi}{4}k} - 1\right) + \left(\frac{3j}{2\pi k}\right)\left(1 - e^{-j\frac{5\pi}{4}k}\right)$$

$$= \left(\frac{j}{2\pi k}\right)(7-3) e^{-j\frac{5\pi}{4}k}) + \left(\frac{j}{2\pi k}\right)(-7+3)$$

$$= \frac{j}{2\pi k} \cdot 4 \cdot \left(e^{-j\frac{5\pi}{4}k} - 1\right) = \frac{2j}{\pi k}\left(e^{-j\frac{5\pi}{4}k} - 1\right)$$

$$a_k = \frac{2j}{\pi k}\left(e^{j(2\pi - \frac{5\pi}{4})k} - 1\right) = \frac{2j}{\pi k}\left(e^{j\frac{3\pi}{4}k} - 1\right)$$ ※

$$a_k = \frac{2j}{\pi k}\left(e^{j\frac{3\pi}{4}k} - 1\right)$$ ※

4.3(a) $\because a_k = \frac{2}{\pi k}\left(e^{j\frac{3\pi}{4}k} - e^{j\pi}\right)$

$a_k = \frac{2j}{\pi k}\left(e^{-j\frac{5\pi}{4}k} - 1\right)$

when $k = 8, 16, 24 \cdots$, $e^{-j\frac{4\pi}{4}k} = 1$

$\Rightarrow a_k = 0$, when $k = 8, 16, 24, \cdots$

prob 4.4(a) Let's define the freq of Tone 40 $= f_{40}$

then $f_{41} = r f_{40}$, $f_{42} = r^2 f_{40}, \cdots, f_{52} = r^{12} f_{40} = 2 f_{40}$

$r^{12} f_{40} = 2 f_{40} \Rightarrow r^{12} = 2$

$\therefore r = 2^{1/12}$ #

4.3(b)

Note Name	C	$C^\#$	D	E^b	E	F	$F^\#$	G	$G^\#$	A	B^b	B	C
Not Num.	40	41	42	43	44	45	46	47	48	49	50	51	51
freq(Hz)	262	277	294	311	330	349	370	392	415	440	466	494	523

4.3(c) consider the formula $f_{k+n} = f_k r^n$

when $k=49$, $f_{49} = 440\text{ Hz}$

The note number is $m = k+n = n+49$

$\Rightarrow n = m-49$

For note number m,

$f_m = (440)\, 2^{(m-49)/12}$ ##

prob. 4.5

(a) $x(t) = \cos(1000\sqrt{t}) = \cos(\psi(t))$

$f_i(t) = \frac{1}{2\pi}\frac{d\psi(t)}{dt} = \frac{1}{2\pi}\frac{1}{2}\times 1000 \times t^{-\frac{1}{2}}$

$= \frac{79.58}{\sqrt{t}}$ ※ which matches $\boxed{-A-}$ spectrogram diagram

(b) $x(t) = \cos(200\pi t - \frac{\pi}{4}) + \cos(700\pi t)$

$= \cos(\psi_1(t)) + \cos(\psi_2(t))$

$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}(\psi_1(t) + \psi_2(t)) = f_{i1}(t) + f_{i2}(t)$

$f_i(t) = f_{i1}(t) + f_{i2}(t) \Rightarrow$ 100 Hz, 350 Hz

which matches $\boxed{-C-}$ spectrogram.

(c) $x(t) = \cos(350\cos(2\pi t))$

$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}(350\cos(2\pi t)) = \frac{1}{2\pi}(-350\cdot 2\pi \cdot \sin(2\pi t))$ ※

which matches $\boxed{-E-}$ spectrogram.

(d) $x(t) = \cos(700\pi t) + \cos(700\pi t + \pi/3)$

Matches $\boxed{-F-}$ spectrogram

(e) $x(t) = \cos(200\pi t^2)$

$f_i(t) = \frac{1}{2\pi}\frac{d}{dt}(200\pi t^2) = \frac{1}{2\pi}200\pi\cdot 2t = 200t$

$\Rightarrow$ the slope $= 200$ Hz which matches $\boxed{-D-}$

(f) $x(t) = \cos(200\pi t)\cos(700\pi t) = \frac{1}{2}(\cos(900\pi t) + \cos(500\pi t))$

which matches $\boxed{-B-}$ spectrogram

※