GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2011
Problem Set #5

Assigned: 11-Feb-11
Due Date: Week of 21-Feb-11

Quiz #2 will be held in lecture on Friday 25-Feb-2011. It will cover material from Chapters 3 and 4, as represented in Problem Sets #3, #4 and #5, as well as Labs #3 and #5.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, both sides)

Reading: In *SP First*, Chapter 3: *Spectrum Representation,* Chapter 4: *Sampling and Aliasing*

The *SP First* Toolbox for MATLAB has been posted on **`t-square`** under the "Lab Assignments" link. You can install it to get some useful functions and GUIs for manipulating complex numbers. The direct link to the toolbox is: `http://users.ece.gatech.edu/mcclella/SPFirst/Updates/SPFirstMATLAB.html`

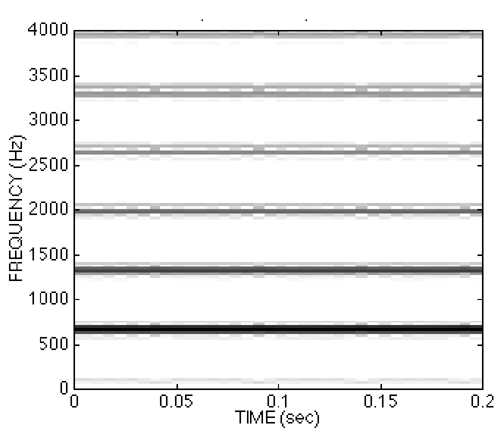
The web site for the course uses **`t-square`**: `https://t-square.gatech.edu`
$\Longrightarrow$ Please check **`t-square`** daily. All official course announcements will be posted there.

ONLY the **STARRED** problems should be turned in for grading; a random subset of these will be graded.

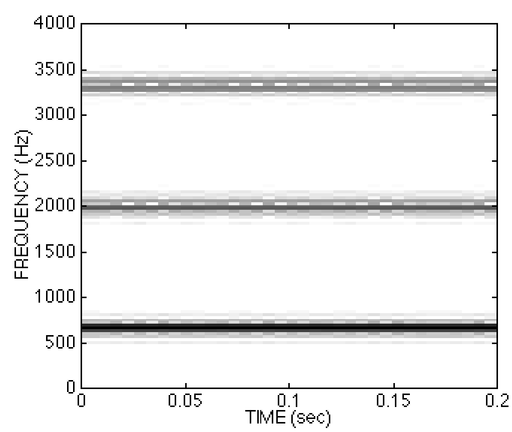
Some of the problems have solutions that are similar to those found on the SP-First CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

PROBLEM 5.1:
Shown in the figure below are spectrograms for two periodic signals, $x(t)$ and $y(t)$. *The frequency axis has units of Hz.*



Spectrogram of $x(t)$



Spectrogram of $y(t)$

(a) The spectrum line at $f = 2000$ Hz is the same N-th harmonic of both signals; determine N.

(b) Both signals have the same fundamental frequency. Determine the fundamental period T_0 for these signals using *accurate estimates* from the spectrograms.

(c) One of these signals is a 50% duty-cycle square wave; recall the harmonic structure of its Fourier Series. Determine which signal best fits this description and explain your answer.

PROBLEM 5.2*:

Sketch the spectrum for signals defined by the MATLAB code below. Be sure to label all of the frequencies and complex amplitudes.

(a) The following MATLAB code defines a vector zz, which corresponds to the signal $z(t)$.

```
tt = -10:0.00001:10;  %- in seconds
zz = 0*tt - 29;
for kk = -2:2
   zz = zz + 13i*kk*exp(55i*pi*kk*tt);
end
```

(b) The following MATLAB code defines a vector xx, which corresponds to the signal $x(t)$.

```
tt = -10:0.00001:10;  %- in seconds
xx = 0*tt;
for kk = -1:1
   xx = xx - 1i*kk*exp(55i*pi*kk*tt);
end
xx = 7.*cos(11*pi*tt).*xx;
```

PROBLEM 5.3:

A periodic signal $x(t)$ is represented as a Fourier series of the form

$$x(t) = 10 + \sum_{k=-\infty}^{\infty} k^2 e^{j32\pi kt}$$

(a) Determine the fundamental **period** of the signal $x(t)$, i.e., the minimum period. Explain.

(b) Define a new signal by adding a sinusoid to $x(t)$, $y(t) = 9\cos(96\pi t + \pi/2) + x(t)$

The new signal, $y(t)$ can be expressed in the following Fourier Series with new coefficients $\{b_k\}$:

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{j32\pi kt}$$

Fill in the following tables, giving *numerical values* for each $\{a_k\}$ and $\{b_k\}$ in polar form.
Note: A magnitude value must be nonnegative.

Signal: $x(t)$

a_k	Mag	Phase
a_3		
a_2		
a_1		
a_0		
a_{-1}		
a_{-2}		
a_{-3}		

Signal: $y(t)$

b_k	Mag	Phase
b_3		
b_2		
b_1		
b_0	***SAME***	
b_{-1}		
b_{-2}		
b_{-3}		

Note: Whenever a b_k coefficient is equal the corresponding a_k coefficient, just write ***SAME*** in the b_k table.

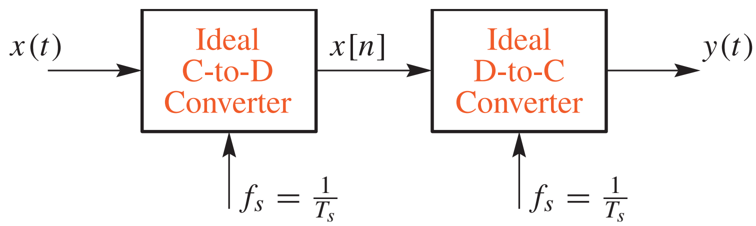


Figure 1: Ideal sampling and reconstruction. An ideal C-to-D converter samples $x(t)$ with a sampling period $T_s = 1/f_s$ to produce the discrete-time signal $x[n]$. The ideal D-to-C converter then forms a continuous-time signal $y(t)$ from the samples $x[n]$. The output $y(t)$ will have no spectrum lines outside of $\pm\frac{1}{2}f_s$.

PROBLEM 5.4*:

Consider the ideal sampling and reconstruction system shown in Fig. 1.

(a) Suppose that the discrete-time signal $x[n]$ in Fig. 1 is given by the formula

$$x[n] = -29 + \sum_{\ell=-2}^{2} j13\ell\, e^{j0.3\pi\ell n}$$

Plot the spectrum for the discrete-time signal $x[n]$ over the frequency interval $-\pi \le \hat{\omega} \le \pi$.

(b) Suppose that the discrete-time signal $x[n]$ in Fig. 1 is given by the formula

$$x[n] = 9.87\cos(0.3\pi(n-1))$$

If the sampling rate of the C-to-D converter is $f_s = 1500$ samples/sec, many *different* continuous-time signals $x(t) = x_i(t)$ could have been inputs to the above system. Determine two specific inputs whose frequencies are between 0 and 1500 Hz. In other words, find $x_1(t) = A_1\cos(\omega_1 t + \varphi_1)$ and $x_2(t) = A_2\cos(\omega_2 t + \varphi_2)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = 1/1500$ s and $\omega_1 \ne \omega_2$.

PROBLEM 5.5*:

Refer to Fig. 1 again for this problem. Now suppose that the input $x(t)$ to the system in Fig. 1 has the spectrum representation shown below,

$$x(t) = -29 + \sum_{\ell=-2}^{2} j13\ell\, e^{j550\pi\ell t}$$

(a) Determine the *minimum* sampling rate f_s such that the output $y(t)$ is equal to the input $x(t)$.

(b) Using the signal $x(t)$ from part (a), form the signal $x[n]$ by sampling at a rate $f_s = 1000$ samples/sec. Determine the spectrum for $x[n]$ and make a plot for your answer over the interval $-2\pi \le \hat{\omega} \le 2\pi$. This interval is more than the minimum, $[-\pi, \pi]$, so a few aliased components will have to be included.

Simplify your answer as much as possible, and then label the frequency and complex amplitude (magnitude and phase in polar form) of each spectral component.

PROBLEM 5.6*:

Here are some operations that are often done in MATLAB. Recall that `soundsc(xx,fs)` is effectively an ideal D-to-C converter; and `t1:(1/fs):t2` is sampling in time. In each case, the length of the `nn` or `tt` vector is huge, so the code cannot be run in MATLAB. Therefore, you should analyze the code and determine the answer via the theory of sampling and aliasing. For each part, you should draw a cascade of C-to-D and/or D-to-C blocks (with their sampling rates) that is equivalent to the MATLAB code. Sketching the spectrum of the discrete-time sinusoid might also be helpful in discovering whether aliasing has happened. *Note:* The sampling rates of the C-to-D and D-to-C blocks might be different.

(a) Suppose that a student enters the following MATLAB code:

```
nn = 0:2301832;
xx = cos(2*pi*0.75*nn + 5);
soundsc(xx,8000)
```

Determine the analog frequency (in hertz) that will be heard.

(b) Suppose that a student writes the following MATLAB code to generate a sine wave:

```
tt = 0:1/8000:10000;
xx = (28*pi) * cos(2*pi*800*tt - 7);
soundsc(xx,fsamp);
```

Although the sinusoid was not written to have a frequency of 400 Hz, it is possible to play out the vector `xx` so that it sounds like a 400 Hz tone. Determine the value of `fsamp` that should be used to play the vector `xx` as a 400 Hz tone.

(c) Consider the following piece of MATLAB code:

```
tt = 0:(1/8000):40;
xx = cos(2*pi*7000*tt);
soundsc(xx,20000);
```

Determine the frequency (in Hz) and the duration (in seconds) of the final played tone. (Assume that the computer has an infinite amount of memory so that we can store the entire signal in MATLAB.)