GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2011
Problem Set #6

Assigned: 17-Feb-11
Due Date: Week of 7-Mar-11

Reading: In *SP First*, Chapter 5: *FIR Filters*.

The *SP First* Toolbox for MATLAB has been posted on **`t-square`** under the "Lab Assignments" link. You can install it to get some useful functions and GUIs for manipulating complex numbers. The direct link to the toolbox is: `http://users.ece.gatech.edu/mcclella/SPFirst/Updates/SPFirstMATLAB.html`

The web site for the course uses **`t-square`**: `https://t-square.gatech.edu`
$\Longrightarrow$ Please check **`t-square`** daily. All official course announcements will be posted there.

ONLY the **STARRED** problems should be turned in for grading; a random subset of these will be graded.

Some of the problems have solutions that are similar to those found on the SP-First CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

Two-Parts in each HW Solution: Two distinct pieces of information are required for a complete solution:

(a) *Justification:* Write a clear explanation of ***how*** you are solving the problem. This can be with or without mathematical formulas, but should convey your understanding of the solution.

(b) *Details:* Carry out the solution of the particular problem. Details mean getting the algebra correct, making precise plots, and doing the numerical calculations.

PROBLEM 6.1*:

A linear time-invariant discrete-time system is described by the difference equation

$$y[n] = \tfrac{1}{2}x[n] - \pi x[n-1] + x[n-2] - 3x[n-4]$$

(a) Determine the impulse response $h[n]$ for this system.

(b) Make a stem plot of $h[n]$ versus n.

(c) Determine the filter coefficients b_k in the causal FIR representation: $y[n] = \sum_{k=0}^{M} b_k x[n-k]$.

(d) Determine the *order* of the filter (M), and the *length* of the filter (L).

PROBLEM 6.2*:

Suppose that $\mathcal{S}$ is a linear, time-invariant system whose exact form is unknown. It is tested by running some inputs into the system, and then observing the output signals. Suppose that the following input–output pairs are the result of the tests:

Input: $x[n]$	**Output:** $y[n]$
$\pi\cos(0.25\pi n + 0.25\pi)$	$\pi^2\cos(0.25\pi n + 0.5\pi)$
$\delta[n] - \delta[n-1] + \delta[n-2]$	$7\delta[n-1]$

(a) Make a plot of the signal:

$$s[n] = \delta[n] - \delta[n-1] + \delta[n-2]$$

(b) Use linearity and time invariance to find the output of the system when the input is

$$x_1[n] = \pi\delta[n-1] + \pi\delta[n-4]$$

PROBLEM 6.3*:

Consider a system implemented by the following MATLAB program:

```
% xx.mat is a binary file containing the vector of input samples
%     called "xx" which represents the input signal x[n]
load xx
yy1 = conv((-1).^(0:4),xx);      %- First subsystem
yy2 = conv([0,0,1],xx); %- Second subsystem
ww = yy1 + yy2;
yy = conv(ones(1,2),ww);   %- "yy" represents the output y[n]
```

The overall system from input `xx` to output `yy` is an LTI system composed of three LTI subsystems.

(a) From the MATLAB code, draw a block diagram showing how the three component subsystems are connected. There should be three blocks—one for each LTI subsystem.

(b) For the first subsystem, with output `yy1`, plot the impulse response.

(c) For the second subsystem, with output `yy2`, write the difference equation.

(d) The overall system is an LTI system. Determine its impulse response; give the answer as a stem plot.

(e) The overall system is an LTI system. Hence, a single MATLAB convolution will suffice to obtain the output vector `yy` from the input vector `xx`,

```
bb = ???;  yy = conv(bb,xx);
```

Give the correct MATLAB definition of the vector `bb`.

PROBLEM 6.4*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

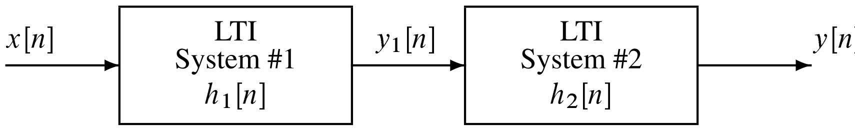

Figure 1: Cascade connection of two LTI systems.

(a) Suppose that System #1 is a "blurring" filter described by the difference equation

$$y_1[n] = \sum_{k=0}^{9} \alpha^k x[n-k],$$

and System #2 is described by the impulse response

$$h_2[n] = \delta[n] - \alpha\delta[n-1],$$

where α is a real number. Determine the impulse response sequence, $h_1[n]$, of the first system.

(b) Determine the impulse response sequence, $h[n] = h_1[n] * h_2[n]$, of the overall cascade system.

(c) Obtain a single difference equation that relates $y[n]$ to $x[n]$ in Fig. 1. Give numerical values of the filter coefficients for the specific case where $\alpha = 0.9$.

PROBLEM 6.5:

For each of the following systems, the signal $x[n]$ is the input and $y[n]$ is the output.

1. $y[n] = x[2n+1]$ (Time Expansion)
2. $y[n] = \left|\frac{1}{2}x[n-3]\right|$ (Magnitude)

(a) Find the impulse response for both systems. Give your answers as formulas.

(b) One of the systems is not linear; determine which one and justify your answer by giving a specific counterexample.

(c) One of the systems is not time-invariant; determine which one and justify your answer by giving a specific counterexample.

(d) One of the systems is not causal; determine which one and justify your answer by giving a specific counterexample.