ECE 2025 Spring 2011, Problem Set #6 Solutions

6.1* An LTI system is described by the difference equation:

$$y[n] = \frac{1}{2}x[n] - \pi x[n-1] + x[n-2] - 3x[n-4]$$

(a) To determine the impulse response of the system we imagine applying an input to the system that is the unit impulse $\delta[n]$. We can calculate the output as:

$$h[0] = \frac{1}{2}\delta[0] - \pi\delta[-1] + \delta[-2] - 3\delta[-4] = \frac{1}{2}$$

$$h[1] = \frac{1}{2}\delta[1] - \pi\delta[0] + \delta[-1] - 3\delta[-3] = -\pi$$

$$h[2] = \frac{1}{2}\delta[2] - \pi\delta[1] + \delta[0] - 3\delta[-2] = 1$$

$$h[3] = \frac{1}{2}\delta[3] - \pi\delta[2] + \delta[1] - 3\delta[-1] = 0$$

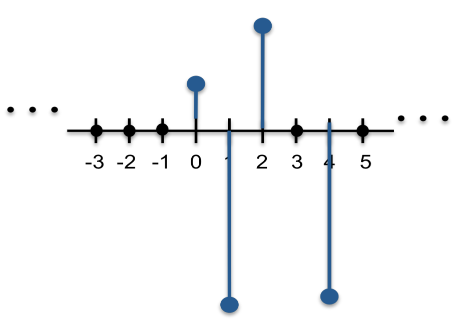
$$h[4] = \frac{1}{2}\delta[4] - \pi\delta[1] + \delta[2] - 3\delta[0] = -3$$

$$h[n] = 0,\ n \geq 5$$

Alternatively, we can see that the impulse response may be determined by inspection from $y[n]$ to yield:

$$h[n] = \frac{1}{2}\delta[n] - \pi\delta[n-1] + \delta[n-2] - 3\delta[n-4]$$

(b) Stem plot:



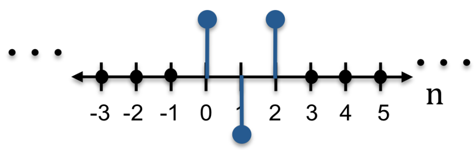
(c) The filter coefficients may be determined directly from the impulse response:

$b_k < 0$	b_0	b_1	b_2	b_3	b_4	$b_k > 4$
0	1/2	-π	1	0	-3	0

(d) The order of the filter (M) is the maximum delay in M samples used in creating each output signal. For the given signal, we need to include the n-4[th] sample. Therefore, $M = 4$.
The length of the filter (L) is the past L point used to computer the output. Thus, it takes the past 5 points to compute the output. Therefore, $L = 5$. Note that the intervening zero is counted.

6.2*

(a)



(b) We can re-write the input $x_1[n]$:

$$x_1[n] = \pi(\delta[n-1] + \delta[n-4])$$
$$x_1[n] = \pi(\delta[n-1] - \delta[n-2] + \delta[n-3] + \delta[n-2] - \delta[n-3] + \delta[n-4])$$

Using *invariance*, we know that we have the following input/output relationships:

$$x[n-1] \Rightarrow 7\delta[n-2]$$
$$x[n-2] \Rightarrow 7\delta[n-3]$$

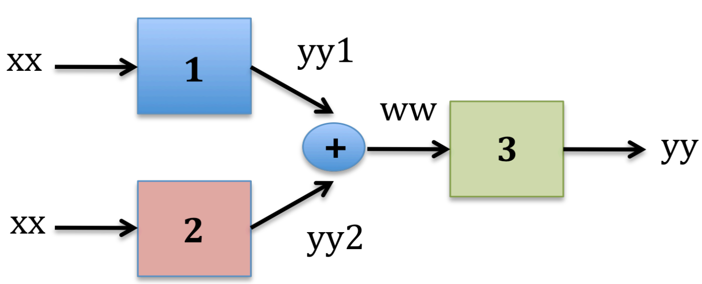
Using linearity, we can add each result and scale to obtain the total result:

$$y_1[n] = \pi(7\delta[n-2] + 7\delta[n-3]) = 7\pi\delta[n-2] + 7\pi\delta[n-3]$$

Note that the first input/output relationship only tells us that the cosine function gets multiplied by π at that particular frequency.

6.3*

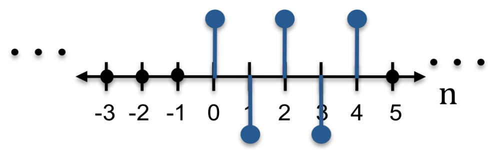
(a)



The code indicates the both *system 1* and *system 2* takes xx as an input and therefore they are in parallel. The code also illustrates that the output from each system is then summed ($ww = yy1 + yy2$) before being input into the last system, *system 3*, which produces the final output yy.

(b) We can readily plot the impulse response recognizing that the *conv* function is convolving the impulse response with the input. Thus:

$$h_1[n] = [1,-1,1,-1,1]$$



(c) We can look at the *conv* function to find impulse response. Thus the difference equation is:

$$yy_2[n] = 0x[n] + 0x[n-1] + x[n-2] = x[n-2]$$

(d) To find the overall impulse response:

$$h\ [n] = h_3[n] * \left(h_1[n] + h_2[n]\right)$$
$$h_3[n] = \delta[n] + \delta[n-1]$$

Since the convolution has the distributive property:

$$h\ [n] = h_3[n] * h_1[n] + h_3[n] * h_2[n]$$

Furthermore, (1) since the convolution has the associative property and (2) since convolution with a shifted impulse shifts the input by the same amount:

$$h\ [n] = h_1[n] * h_3[n] + h_2[n] * h_3[n]$$
$$h\ [n] = h_1[n] * (\delta[n] + \delta[n-1]) + h_2[n] * (\delta[n] + \delta[n-1])$$
$$h\ [n] = h_1[n] + h_1[n-1] + h_2[n] + h_2[n-1]$$
$$h\ [n] = \delta[n] + \delta[n-2] + \delta[n-3] + \delta[n-5]$$

(e) Using the overall impulse response above we can construct the following vector:

bb = [1 0 1 1 0 1]

6.4*

(a) The impulse response of the first system is:

$$h_1[n] = \sum_{k=0}^{9} \alpha^k \delta[n-k]$$

(b) To find the impulse response for the overall cascade system, and using the associative property of the convolution we know that:

$$h[n] = h_1[n] * h_2[n] = h_2[n] * h_1[n] = \left(\delta[n] - \alpha\delta[n-1]\right) * h_1[n]$$

Using linearity and the fact that convolution with an impulse results in a shifted input by the same amount that the impulse is shifted by.

$$h[n] = h_1[n] * (\delta[n] - \alpha\delta[n-1]) = h_1[n] - \alpha h_1[n-1]$$

$$h[n] = \sum_{k=0}^{9} \alpha^k \delta[n-k] - \sum_{k=0}^{9} \alpha^{k+1}\delta[n-k-1]$$

$$h[n] = \delta[n] + \sum_{k=1}^{9} \alpha^k \delta[n-k] - \sum_{k=0}^{8} \alpha^{k+1}\delta[n-k-1] - \alpha^{10}\delta[n-10]$$

$$h[n] = \delta[n] - \alpha^{10}\delta[n-10]$$

(c) We let $\alpha = 0.9$ and evaluate:

$$y[n] = 1.000x[n] - 0.349x[n-10]$$

6.5

(a) The impulse response for each system is:

$$h_1[n] = \delta[2n+1]$$

$$h_2[n] = \left|\frac{1}{2}x[n-3]\right|$$

(b) *System 2* is not linear. If we multiply the input by -1, we don't find –y[n].

$$-\left|\frac{1}{2}x[n-3]\right| \neq \left|-\frac{1}{2}x[n-3]\right|$$

(c) *System 1* is not time-invariant. If we shift the input by 1, we don't find a corresponding shift on the output.

$$x[2(n-1)+1] = x[2n-1] \neq x[2n+1-1]$$

(e) *System 1* is non-causal. If we are at sample n=0 for the input, to calculate the output we need an input value that is advanced in discrete time:

$$y[0] = x[1]$$