GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2011
Problem Set #7

Assigned: 4-Mar-11
Due Date: Week of 14-Mar-11

Quiz #3 will be held in lecture on Friday 18-Mar-2011. It will cover material from Chapters 5 and 6, as represented in Problem Sets #6, and #7, as well as Lab #7.
Closed book, calculators permitted, and one hand-written formula sheet ($8\frac{1}{2}'' \times 11''$, both sides)

Reading: In *SP First*, Chapter 6: *Frequency Response of FIR Filters,*.

The *SP First* Toolbox for MATLAB has been posted on **`t-square`** under the "Lab Assignments" link. You can install it to get some useful functions and GUIs for manipulating complex numbers. The direct link to the toolbox is: `http://users.ece.gatech.edu/mcclella/SPFirst/Updates/SPFirstMATLAB.html`

The web site for the course uses **`t-square`**: `https://t-square.gatech.edu`
$\Longrightarrow$ Please check **`t-square`** daily. All official course announcements will be posted there.

ONLY the **STARRED** problems should be turned in for grading; a random subset of these will be graded.

Some of the problems have solutions that are similar to those found on the SP-First CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

Two-Parts in each HW Solution: Two distinct pieces of information are required for a complete solution:
Justification: Write a clear explanation of ***how*** you are solving the problem. This can be with or without mathematical formulas, but should convey your understanding of the solution.
Details: Carry out the solution of the particular problem. Details mean getting the algebra correct, making precise plots, and doing the numerical calculations.

PROBLEM 7.1:

When the frequency response is given, it is possible to determine the impulse response $h[n]$. Likewise, when the impulse response is given, it is possible to determine the Frequency response $H(e^{j\hat{\omega}})$. In the problems below, determine $h[n]$ or $H(e^{j\hat{\omega}})$. Express your answers for $h[n]$ as a sum of weighted and shifted impulses; for $H(e^{j\hat{\omega}})$ a sum of complex exponentials is sufficient.

(a) $h[n] = \pi\delta[n-7]$

(b) $h[n] = \sum_{k=0}^{3} (k - \delta[k-2])\delta[n-k]$

(c) $H(e^{j\hat{\omega}}) = 8\pi$

(d) $H(e^{j\hat{\omega}}) = 5e^{-j8\hat{\omega}}$

(e) $H(e^{j\hat{\omega}}) = 3je^{-j8\hat{\omega}}\sin(3\hat{\omega})$

(f) $H(e^{j\hat{\omega}}) = 7e^{-j8\hat{\omega}}\dfrac{\sin(2.5\hat{\omega})}{\sin(0.5\hat{\omega})}$

PROBLEM 7.2*:

The diagram in Fig. 1 depicts a *cascade connection* of two linear time-invariant systems; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

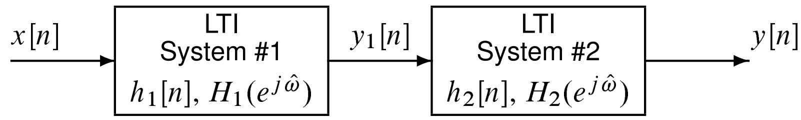


Figure 1: Cascade connection of two LTI systems.

Suppose that System #1 is a filter described by the impulse response

$$h_1[n] = \delta[n-3]$$

and System #2 is described by the difference equation

$$y_2[n] = y_1[n] + y_1[n-2],$$

(a) Determine the frequency response, $H_2(e^{j\hat{\omega}})$, of the second system.

(b) Determine the frequency response, $H(e^{j\hat{\omega}})$, of the overall cascade system.

(c) From the frequency response, $H(e^{j\hat{\omega}})$, determine the impulse response, $h[n]$, of the overall cascade system.

(d) When the input to this system is

$$x[n] = 29\cos(0.5\pi n + 0.25\pi) + 37\delta[n-2]$$

Use linearity, superposition, the impulse response, and the frequency response to determine $y[n]$.

PROBLEM 7.3*:

Consider the linear time-invariant system given by the difference equation

$$y[n] = 1.25x[n-1] + 1.25x[n-2] + 1.25x[n-3] + 1.25x[n-4] = \sum_{k=1}^{4} \tfrac{5}{4} x[n-k]$$

(a) From the filter coefficients, write an expression for the frequency response $H(e^{j\hat{\omega}})$ using a sum of complex exponentials.

(b) Show that your answer in (a) can be expressed in a form that uses the *Dirichlet* formula:

$$H(e^{j\hat{\omega}}) = \alpha \frac{\sin(L\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j\beta\hat{\omega}}$$

and determine the value of the parameters L, α and β.

PROBLEM 7.4*:

Continue the previous question. Consider the linear time-invariant system given by the difference equation

$$y[n] = 1.25x[n-1] + 1.25x[n-2] + 1.25x[n-3] + 1.25x[n-4] = \sum_{k=1}^{4} \tfrac{5}{4}x[n-k]$$

(a) Using the frequency response formula for this system, determine the DC value of $H(e^{j\hat{\omega}})$.

(b) In addition, determine the frequencies where $H(e^{j\hat{\omega}})$ is zero, in the interval $-\pi \le \hat{\omega} \le \pi$.
Note: You can check your answer by doing a plot in MATLAB with `freekz( )` or `freqz( )`.

(c) Using the formula in the previous part, sketch the frequency response (magnitude only) as a function of frequency for $-\pi \le \hat{\omega} \le \pi$.
Note: You can check your plot by doing it in MATLAB with `freekz( )` or `freqz( )`.

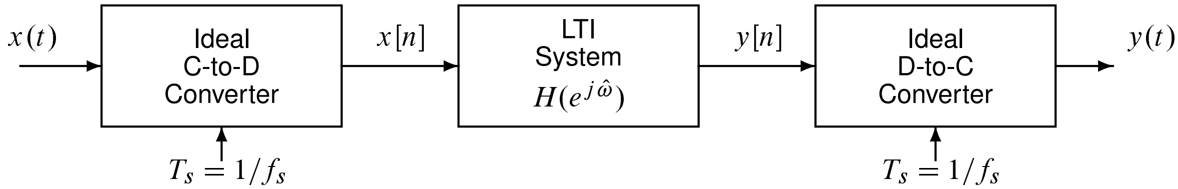
(d) Suppose that the input is $x[n] = 125 + 100\cos(\hat{\omega}_0 n + 0.25\pi)$, for $-\infty < n < \infty$. Find all possible nonzero frequencies $0 < \hat{\omega}_0 \le \pi$ for which the output $y[n]$ is a constant for all n, i.e.,

$$y[n] = c \qquad \text{for } -\infty < n < \infty$$

and find the value for c. (In other words, the cosine term in $x[n]$ is removed by the filter.)

PROBLEM 7.5*:

Consider the following system for discrete-time filtering of a continuous-time signal:



In this problem, assume that the frequency response of the discrete-time system is

$$H(e^{j\hat{\omega}}) = 7 - 7e^{-j5\hat{\omega}}$$

(a) Sketch the frequency response magnitude for $H(e^{j\hat{\omega}})$ over the frequency range $-\pi < \hat{\omega} \le \pi$.

(b) Assume that the input signal $x(t)$ is

$$x(t) = 37 + 29\cos(300\pi t - 0.2\pi) \quad \text{for } -\infty < t < \infty$$

For a sampling rate of $f_s = 200$ samples/sec, draw the spectrum of $x[n]$, the discrete-time signal after the C-to-D converter, which is also the input to the LTI system.

(c) For the same $x(t)$, and $x[n]$, as in the previous part, and the same sampling rate, determine a simple formula for the output $y(t)$ which is valid for $-\infty < t < \infty$.