Problem 7.1:

A linear, time-invariant system with an impulse response, *h[n]*, has the following frequency response:
If $h[n] = \sum_{k=0}^{M} b_k \cdot \delta[n-k]$ then $H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k \cdot e^{-jk\hat{\omega}}$

(a) If $h[n] = \pi\delta[n-7]$ then $H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k \cdot e^{-jk\hat{\omega}} = \pi e^{-j7\hat{\omega}}$

(b) If

$h[n] = \sum_{k=0}^{3} (k - \delta[n-2])\delta[n-k] = \delta[n-1] + (2-1)\delta[n-2] + 3\delta[n-3]$ then

$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k \cdot e^{-jk\hat{\omega}} = e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + 3e^{-j3\hat{\omega}}$;

(c) If $H(e^{j\hat{\omega}}) = 8\pi$ then $h[n] = 8\pi\delta[n]$;

(d) If $H(e^{j\hat{\omega}}) = 5e^{-j8\hat{\omega}}$ then $h[n] = 5\delta[n-8]$;

(e) If $H(e^{j\hat{\omega}}) = 3je^{-j8\hat{\omega}}\sin(3\hat{\omega}) = 3je^{-j8\hat{\omega}} \bullet \frac{e^{j3\hat{\omega}} - e^{-j3\hat{\omega}}}{2j} = \frac{3}{2}(e^{-j5\hat{\omega}} - e^{-j11\hat{\omega}})$ then

$h[n] = \frac{3}{2}\delta[n-5] - \frac{3}{2}\delta[n-11]$;

(f) If $H(e^{j\hat{\omega}}) = 7e^{-j8\hat{\omega}} \bullet \frac{\sin(2.5\hat{\omega})}{\sin(0.5\hat{\omega})} = 7e^{-j8\hat{\omega}} \bullet \frac{e^{j2.5\hat{\omega}} - e^{-j2.5\hat{\omega}}}{e^{j0.5\hat{\omega}} - e^{-j0.5\hat{\omega}}} = 7e^{-j8\hat{\omega}} \bullet \frac{e^{j2.5\hat{\omega}}}{e^{j0.5\hat{\omega}}} \bullet \frac{1 - e^{-j5\hat{\omega}}}{1 - e^{-j\hat{\omega}}}$ then

$= 7e^{-j6\hat{\omega}}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}}) = 7(e^{-j6\hat{\omega}} + e^{-j7\hat{\omega}} + e^{-j8\hat{\omega}} + e^{-j9\hat{\omega}} + e^{-j10\hat{\omega}})$

$h[n] = 7\delta[n-6] + 7\delta[n-7] + 7\delta[n-8] + 7\delta[n-9] + 7\delta[n-10]$.

Problem 7.2*:

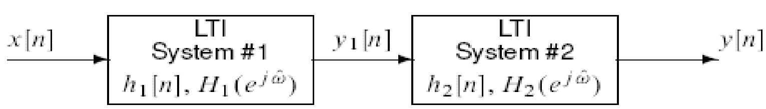


Figure 1: Cascade connection of two LTI systems.

System 1 impulse response: $h_1[n] = \delta[n-3]$, and System 2 difference equation:
$y_2[n] = y_1[n] + y_1[n-2]$

(a) Frequency response of System 2 is: $H_2(e^{j\hat{\omega}}) = 1 + e^{-j2\hat{\omega}}$ because $h_2[n] = \delta[n] + \delta[n-2]$;

(b) $H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}}) \bullet H_2(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}}(1 + e^{-j2\hat{\omega}}) = e^{-j3\hat{\omega}} + e^{-j5\hat{\omega}}$;

(c) $h[n] = \delta[n-3] + \delta[n-5] = h_1[n] * h_2[n]$;

(d) With input: $x[n] = 29\cos(0.5\pi n + 0.25\pi) + 37\delta[n-2] = x_1[n] + x_2[n]$,
then the output has two components. The second is a sum of two delayed version of $x_2[n]$ which is simply of the form: $37\delta[n-5] + 37\delta[n-7]$; and the first term is a sinusoidal signal with two frequency components at $\hat{\omega} = \pm 0.5\pi$, since $H(e^{j0.5\pi}) = e^{-j3 \bullet 0.5\pi}(1 + e^{-j2 \bullet 0.5\pi}) = 0$ and $H(e^{-j0.5\pi}) = 0$ as well, the first term produces an output of zero. Therefore the overall output is: $y[n] = 37\delta[n-5] + 37\delta[n-7]$.

Problem 7.3*:

(a)A system $y[n] = \sum_{k=1}^{4} \frac{5}{4} x[n-k]$ has an impulse response

$h[n] = 1.25\delta[n-1] + 1.25\delta[n-2] + 1.25\delta[n-3] + 1.25\delta[n-4]$

The frequency response: $H(e^{j\hat{\omega}}) = 1.25e^{-j\hat{\omega}} + 1.25e^{-j2\hat{\omega}} + 1.25e^{-j3\hat{\omega}} + 1.25e^{-j4\hat{\omega}}$.

(b)Now we can simplify it as follows:

$$H(e^{j\hat{\omega}}) = \frac{5}{4}e^{-j\hat{\omega}}(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}}) = \frac{5}{4}e^{-j\hat{\omega}} \bullet \frac{1 - e^{-j4\hat{\omega}}}{1 - e^{-j\hat{\omega}}}$$

$$= \frac{5}{4}e^{-j\hat{\omega}} \bullet \frac{e^{-j2\hat{\omega}}(e^{j2\hat{\omega}} - e^{-j2\hat{\omega}})}{e^{-j0.5\hat{\omega}}(e^{j0.5\hat{\omega}} - e^{-j0.5\hat{\omega}})} = \frac{5}{4}\frac{\sin(2\hat{\omega})}{\sin(\hat{\omega}/2)}e^{-j2.5\hat{\omega}}$$

Or it can be expressed with the Dirichlet formula: $H(e^{j\hat{\omega}}) = \alpha \frac{\sin(L\hat{\omega}/2)}{\sin(\hat{\omega}/2)} e^{-j\beta\hat{\omega}}$

with: $L = 4, \alpha = 1.25, \beta = 2.5$.

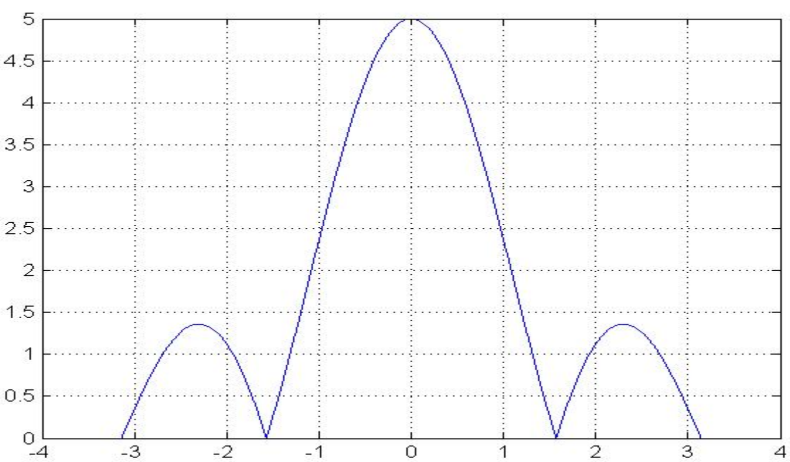
Problem 7.4*:

A system $y[n] = \sum_{k=1}^{4} \frac{5}{4} x[n-k]$ has frequency response: $H(e^{j\hat{\omega}}) = \frac{5}{4}\frac{\sin(2\hat{\omega})}{\sin(\hat{\omega}/2)}e^{-j2.5\hat{\omega}}$.

(a) The DC response is simply: $H(e^{j0}) = \frac{5}{4}\frac{\sin(2\hat{\omega})}{\sin(\hat{\omega}/2)}\Big|_{\hat{\omega}=0} = 5$ (by L'Hopital's Rule in calculus);

(b) Now the zero response happens at $\sin(2\hat{\omega}) = 0, \text{ and } \hat{\omega} \neq 0; \text{ or at } \hat{\omega} = \pm\pi/2, \text{ and } \hat{\omega} = \pm\pi$;

(c) The magnitude frequency response is displayed as follows:



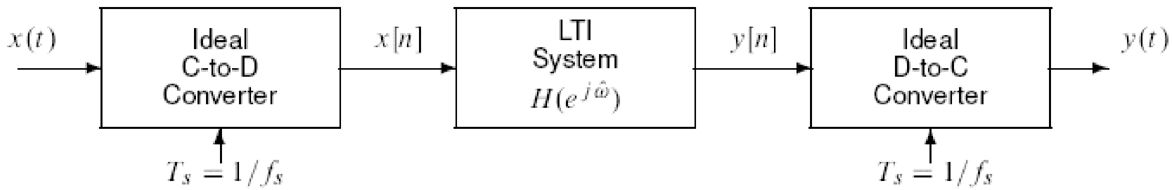
(d) If the input is: $x[n] = 125 + 100\cos(\omega_0 n + 0.25\pi)$ for $-\infty < n < \infty$ and $y[n] = c$, then all possible frequencies that satisfy the above input-output relation for a constant output (with the cosine component removed) is for $H(e^{j\omega_0}) = 0 \text{ and } \omega_0 \neq 0$, or equivalently,

$\sin(2\omega_0) = 0, \text{ and } \omega_0 \neq 0; \text{ or at } \omega_0 = \pm k\pi/2, \text{ with } k \text{ a positive integer}$. Clearly the constant is: $c = 125 \bullet H(e^{j0}) = 125 \bullet 5 = 625$.

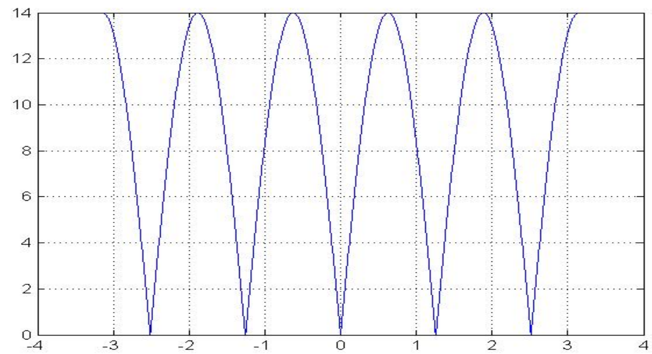
Problem 7.5*:

Consider the following

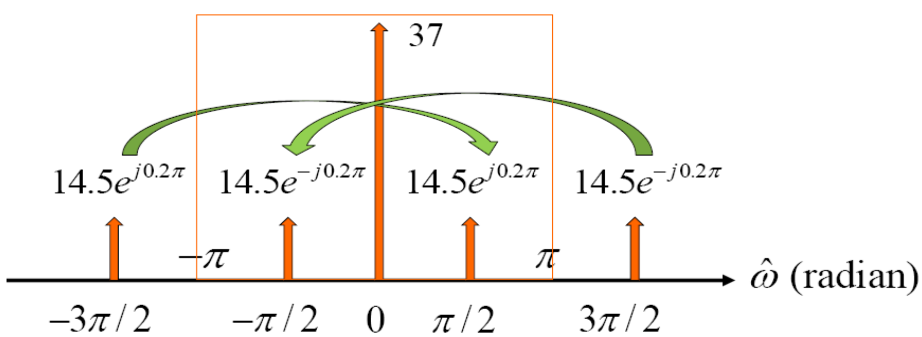
7system with the frequency response defined as: $H(e^{j\hat{\omega}}) = 7 - 7e^{-j5\hat{\omega}}$.



(a) The magnitude of the frequency response $|H(e^{j\hat{\omega}})|$ is displayed in the following:



(b) Now for the above system the input: $x(t) = 37 + 29\cos(300\pi t - 0.2\pi)$, and the sampling frequency: $f_s = 200Hz$. Since *x(t)* has a Nyquist rate of 300Hz because of the 150Hz sinusoidal component, folding occurs resulting in *x[n]* to have the following three spectrum components: (1) DC=37, (2) at $\hat{\omega} = 0.5\pi$ with a complex amplitude of $\frac{29}{2}e^{j0.2\pi}$, and (3) at $\hat{\omega} = -0.5\pi$ with a complex amplitude of $\frac{29}{2}e^{-j0.2\pi}$;



(c) Since the DC response is $H(e^{j0}) = 7 - 7e^{-j0} = 0$, and the frequency responses at $\hat{\omega} = \pm 0.5\pi$ are: $H(e^{\pm j0.5\pi}) = 7 - 7e^{\mp j2.5\pi} = 7 \pm 7j = 7\sqrt{2}e^{\pm j\pi/4}$, we therefore have the output signal spectrum with two components: (1) at $\hat{\omega} = 0.5\pi$ with a complex amplitude of $\frac{29}{2} \bullet 7\sqrt{2}e^{j(0.2+0.25)\pi}$, and (2) at $\hat{\omega} = -0.5\pi$ with a complex amplitude of $\frac{29}{2} \bullet 7\sqrt{2}e^{-j(0.2+0.25)\pi}$. Finally we have thus produced an amplified but aliased output signal: $y(t) = (29 \bullet 7\sqrt{2})\cos(100\pi t + 0.45\pi) = 203\sqrt{2}\cos(100\pi t + 0.45\pi), -\infty < t < \infty$.