GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2011
Problem Set #9

Assigned: 1-Apr-11
Due Date: Week of 11-April-11

Quiz #3 will be held in lecture on Friday, 22-April.

Reading: In *SP First*, Ch. 9: *Continuous-Time Signals*.

The web site for the course uses **`t-square`**: `https://t-square.gatech.edu`
$\Longrightarrow$ Please check **`t-square`** daily. All official course announcements will be posted there.

ONLY the **STARRED** problems should be turned in for grading; a random subset of these will be graded.

Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss HW exercises with other students, but you cannot give or receive any written material or electronic files. In addition, you are not allowed to copy material from old homeworks from previous semesters. Your submitted work must be your own original work.

Some of the problems have solutions that are similar to those found on the SP-First CD-ROM.

Your homework is due in recitation at the beginning of class. After the beginning of your assigned recitation time, the homework is considered late and will be given a zero.

Two-Parts in each HW Solution: Two distinct pieces of information are required for a complete solution:
Justification: Write a clear explanation of ***how*** you are solving the problem. This can be with or without mathematical formulas, but should convey your understanding of the solution.
Details: Carry out the solution of the particular problem. Details mean getting the algebra correct, making precise plots, and doing the numerical calculations.

PROBLEM 9.1*:

In each of the following cases, use properties of the unit-impulse signal $\delta(t)$ and the unit-step signal $u(t)$ to simplify the expression ***as much as possible.*** Be careful to distinguish between multiplication and convolution. Convolution is denoted by a "star", as in $x(t) * \delta(t-a) = x(t-a)$ and multiplication is usually indicated by juxtaposition as in $x(t)\delta(t-a) = x(a)\delta(t-a)$. Provide some **explanation** or intermediate steps for each answer.

(a) Simplify $x(t) = \dfrac{\sin(13\pi(t-3.5))}{\sin(\pi(t-3.5))}\,\delta(t-3.5)$.

(b) Simplify $y(t) = \displaystyle\int_{-200}^{700} 3.3\,\delta(\lambda - t)\,d\lambda$. Give your answer as a plot vs. t.

(c) Simplify $z(t) = \left(\dfrac{d}{dt}\left\{0.7\cos(\tfrac{1}{2}\pi t)\,u(t-6)\right\}\right) * \delta(t-0.5)$.

(d) Simplify $s(t) = \displaystyle\int_{-\infty}^{\infty} 9e^{-0.4\tau^2}\,\delta(\tau - \tfrac{1}{2}t)\,d\tau$

PROBLEM 9.2*:
Some questions about convolving rectangles and impulses (where star $*$ denotes the convolution operator).
Recall: Convolving unit-step signals gives a unit-ramp signal: $u(t) * u(t) = tu(t)$

(a) $x_1(t) = u(t-\frac{1}{2}) * [\delta(t-3) - \delta(t-1)]$. Give your answer as a formula *and a plot.*

(b) $x_2(t) = \delta(t-\frac{1}{2}) * [u(t-3) - u(t-1)]$. Give your answer as a formula *and a plot.*

(c) $x_3(t) = u(t-\frac{1}{2}) * [u(t-3) - u(t-1)]$. Give your answer as a formula *and a plot.*

(d) $x_4(t) = [u(t-3) - u(t-1)] * [u(t-3) - u(t-1)]$. Give your answer as a formula *and a plot.*

PROBLEM 9.3*:
Use linearity and time-invariance to convolve the following two signals:

$$x(t) = \tfrac{1}{3} e^{-3(t-1)} [u(t) - u(t-\tfrac{1}{2})]$$
$$h(t) = \pi\,\delta(t-2.5) - \tfrac{1}{2} u(t-3.5)$$

(a) Expand the convolution $y(t) = x(t) * h(t)$ into several terms. Use superposition, and also exploit the result for convolving exponentials.

(b) Determine $y(\infty) = \lim_{t\to\infty} y(t)$, i.e., the value of the signal $y(t)$ as $t \to \infty$.

PROBLEM 9.4*:
A continuous-time system is defined by the "convolution-like" input/output relation

$$y(t) = \int_2^4 e^{2\tau - 6}\, x(t-\tau)\, d\tau$$

(a) Determine the step response of this system, i.e., the output when the input is $u(t)$. It will be necessary to deal with three cases.

(b) Obtain the impulse response, and show that it is equal to the derivative of the step response.

(c) Is this a stable system? Explain with a proof (if true) or counter-example (if false).

(d) Is it a causal system? Explain with a proof (if true) or counter-example (if false).

PROBLEM 9.5:
Make plots of the following *continuous-time* signals:

(a) $x_1(t) = 88u(t-2) - 88u(t-5)$

(b) $x_2(t) = \pi\,\delta(t - 2\pi)$

(c) $x_3(t) = \displaystyle\int_0^t \pi\,\delta(\tau - 2\pi)\, d\tau$

(d) $x_4(t) = e^{-(t-4)} u(t-4)$