GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL of ELECTRICAL and COMPUTER ENGINEERING

ECE 2025 Spring 2011
Problem Set #11

Assigned: 16-Apr-11
Due Date: 29-Apr-11

This homework is OPTIONAL. However, the topics will be covered on the final exam. If you turn it in, the grade will replace your lowest homework grade. In effect, you will get one more HW drop.

This Homework can be turned at the last lecture on ***Friday, 29-April before Noon,*** or earlier that week.

Final Exam will be given on Friday, 6-May at 8:00 AM.
One page ($8\frac{1}{2}'' \times 11''$) of **handwritten** notes allowed. Calculators OK.

Reading: In *SP First*, Chapter 11: *Continuous-Time Fourier Transform*
Chapter 12: *Filtering, Modulation and Sampling,* (applications of the Fourier Transform).

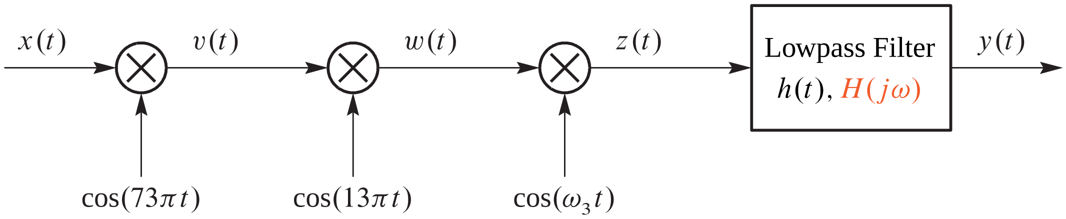
$\Longrightarrow$ **Please check `t-square` often. All official course announcements are posted there.**

Forgeries and plagiarism are a violation of the honor code and will be referred to the Dean of Students for disciplinary action. You are allowed to discuss HW exercises with other students, but you cannot give or receive any written material or electronic files. In addition, you are not allowed to copy material from old homeworks from previous semesters. Your submitted work must be your own original work.

ALL of the **STARRED** problems will have to be turned in for grading. A solution will be posted to the web. Some problems have solutions similar to those found on the CD-ROM.

Two-Parts in each HW Solution: Two distinct pieces of information are required for a complete solution:
Justification: Write a clear explanation of ***how*** you are solving the problem. This can be with or without mathematical formulas, but should convey your understanding of the solution.
Details: Carry out the solution of the particular problem. Details mean getting the algebra correct, making precise plots, and doing the numerical calculations.

PROBLEM 11.1*:
The system below involves the cascade of several modulators followed by a filter:



The signals are defined by

$$v(t) = x(t)\cos(73\pi t) \qquad w(t) = v(t)\cos(13\pi t) \qquad z(t) = w(t)\cos(\omega_3 t)$$

Suppose that the Fourier transform of $x(t)$ is

$$X(j\omega) = (b^2 - \omega^2)\{u(\omega + b) - u(\omega - b)\}$$

(a) Sketch a plot of the magnitude of the Fourier transform, $|X(j\omega)|$, for $b = 6\pi$ rad/s.

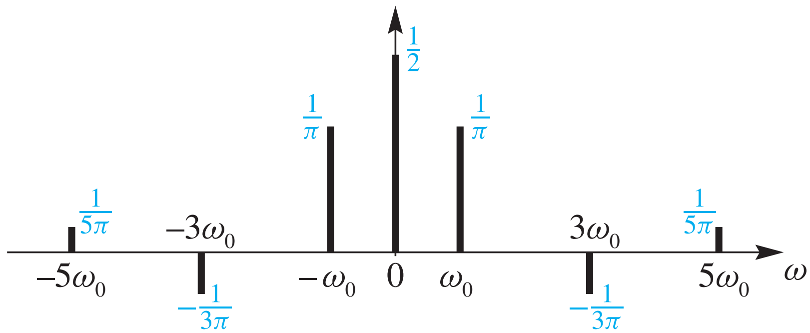
(b) Determine the Fourier transforms, $V(j\omega)$ and $W(j\omega)$. Give your answers as plots that sketch the magnitude of each one (still assuming $b = 6\pi$ rad/s).

(c) Determine the frequency ω_3 and define an ideal filter $H(j\omega)$ so that *exact recovery* of the output signal is accomplished, i.e., $y(t)$ is equal to the input $x(t)$. *Note:* this answer is not unique.

(d) Finally, determine the largest value of b for which exact recovery would be possible with an ideal filter for $H(j\omega)$. *Note:* when you change b, the input spectrum $X(j\omega)$ changes.

PROBLEM 11.2*:

Consider an LTI system whose frequency response $H(j\omega)$ is unknown. The system has a periodic input whose spectrum is shown below. *Note:* the Fourier transform of the input $X(j\omega)$ would be similar to the plot below, but would have impulses instead of lines and the area of the impulses would be 2π times the values below.



For each part of this problem, the output of the system is given and the frequency response must be determined by selecting from the list numbered 1–7 below. Choose the frequency response $H(j\omega)$ of the system that could have produced the specific output when the input is the signal with the spectrum above.

(a) $y(t) = \frac{1}{2}$

(b) $y(t) = \frac{1}{2} + \frac{2}{\pi}\cos[\omega_0(t - \frac{1}{2})]$

(c) $y(t) = \frac{2}{\pi}\cos(\omega_0 t)$

(d) $y(t) = x(t) - \frac{1}{2}$

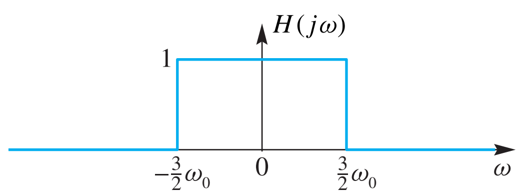
(e) $y(t) = x(t - \frac{1}{2})$

The possible filters are described by the following equations and graphs (some of these may not be used):

1. $H(j\omega) = \begin{cases} 0 & |\omega| < \frac{1}{2}\omega_0 \\ 1 & |\omega| \geq \frac{1}{2}\omega_0 \end{cases}$

2. $H(j\omega) = \begin{cases} 1 & |\omega| \leq \frac{1}{2}\omega_0 \\ 0 & |\omega| > \frac{1}{2}\omega_0 \end{cases}$

3.

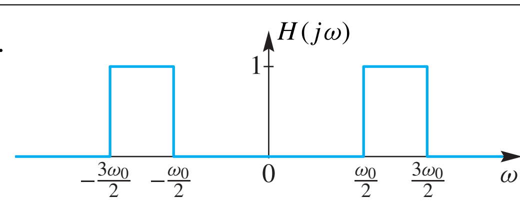


4. $H(j\omega) = \begin{cases} e^{-j\omega/2} & |\omega| \leq \frac{3}{2}\omega_0 \\ 0 & |\omega| > \frac{3}{2}\omega_0 \end{cases}$

5. $H(j\omega) = e^{-j\omega/2}$

6. $H(j\omega) = \frac{1}{2}[1 + \cos(\omega T_0)]$

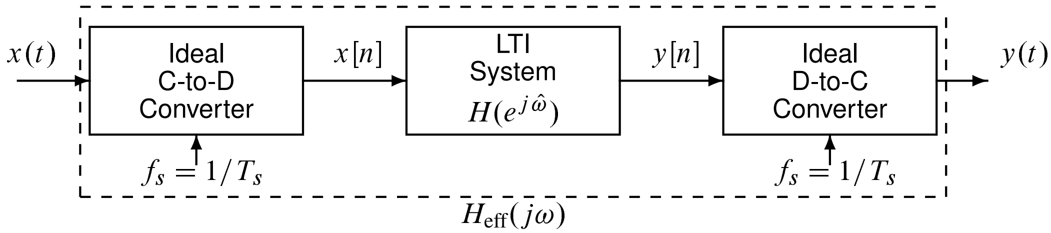
7.



PROBLEM 11.3*:

This type of problem has often appeared on the Final Exam.

Consider the following system for discrete-time filtering of a continuous-time signal:



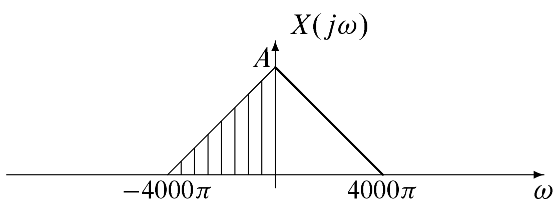
(a) Suppose that the discrete-time system is defined by the difference equation

$$y[n] = -0.9y[n-2] + x[n] + x[n-2] \qquad \Longleftrightarrow \qquad H(z) = \frac{1+z^{-2}}{1+0.9z^{-2}}$$

and the sampling rate of the two converters is $f_s = 8000$ samples/second. Assume that the input signal is bandlimited to less than 4000 Hz. Determine an expression for $H_{\text{eff}}(j\omega)$, the overall effective frequency response of the above system.

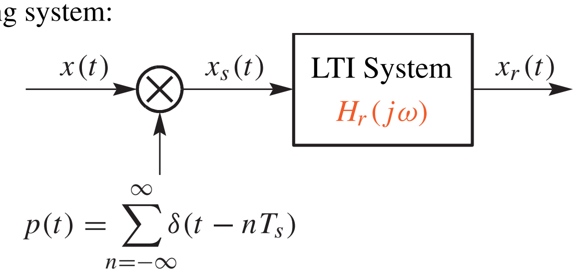
(b) The IIR filter define in part (a) is a notch filter. Determine the analog frequency (in hertz) that is removed by the notch in the frequency response.

(c) In this part, the *sampling frequency is a variable to be minimized.* Assume that the input signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$ as depicted below. For this input signal, what is the *smallest* value of the sampling frequency f_s such that the continuous-time Fourier transforms of the input and output satisfy the relation $Y(j\omega) = H_{\text{eff}}(j\omega)X(j\omega)$?



PROBLEM 11.4*:

The derivation of the Sampling Theorem involves the operations of impulse train sampling and reconstruction as shown in the following system:



Suppose that the Fourier transform of $x(t)$ is

$$X(j\omega) = 100|\omega|\{u(\omega+880\pi) - u(\omega-880\pi)\}$$

(a) Sketch a plot of the magnitude of the Fourier transform, $|X(j\omega)|$.

(b) For the input $X(j\omega)$, determine T_s for the smallest sampling rate $\omega_s = 2\pi/T_s$ so that the signal can be recovered from its sampled version $x_s(t)$. Recovery means that it is possible to define an ideal LPF $H_r(j\omega)$ so that $x_r(t) = x(t)$.

(c) If $X(j\omega)$ is the same input and $T_s = 1/660$ s, plot the Fourier transform $X_s(j\omega)$ and show where aliasing does and does not occur. There will be an infinite number of shifted copies of $X(j\omega)$, so draw a few to indicate the general pattern versus ω.