GEORGIA INSTITUTE OF TECHNOLOGY

SCHOOL of ELECTRICAL & COMPUTER ENGINEERING

QUIZ #1

DATE: 16-Sep-11 COURSE: ECE-2025

NAME: _____ GT username: _____

LAST, FIRST (ex: gtbuzz7)

3 points | 3 points | 3 points

Recitation Section: Circle the date & time when your **Recitation Section** meets (not Lab):

	L05:Tues-9:30am (Richards)		L06:Thur-9:30am (Casinovi)
	L07:Tues-Noon (Richards)		L08:Thur-Noon (Casinovi)
	L09:Tues-1:30pm (Chang)		L10:Thur-1:30pm (Coyle)
L01:M-3pm (Barry)	L11:Tues-3pm (Chang)	L02:W-3pm (Clements)	L12:Thur-3pm (Baxley)
L03:M-4:30pm (Barry)		L04:W-4:30pm (Clements)	L14:Thur-4:30pm (Baxley)

- Write your name on the front page ONLY. **DO NOT** unstaple the test.
- Closed book, but a calculator is permitted.
- One page ($8\frac{1}{2}'' \times 11''$) of **HAND-WRITTEN** notes permitted. OK to write on both sides.
- JUSTIFY your reasoning CLEARLY to receive partial credit.
- You must write your answer in the space provided on the exam paper itself.
  Only these answers will be graded. Circle your answers, or write them in the boxes provided.
  If more space is needed for scratch work, use the backs of previous pages.

Problem	*Value*	*Score*
1	20	
2	20	
3	20	
4	20	
5	20	
No/Wrong Rec	-3	

PROBLEM Fa-11-Q.1.1:

(a) For each of the following complex numbers below, write the number in the standard polar form (i.e., $z = re^{j\theta}$ with $r \geq 0$ and $-\pi < \theta \leq \pi$). Note: z^* means the "conjugate" of z.

$$z_1 = -je^{j2\pi/3} \qquad z_2 = \left(\frac{6-2j}{7+4j}\right)^* \qquad z_3 = \frac{1}{60j}$$

$z_1 =$ _____

$z_2 =$ _____

$z_3 =$ _____

(b) Recall the special properties of the roots of unity. Specifically, note that we have used the following fact:

$$\sum_{k=0}^{N-1} e^{j2\pi k/N} = 0.$$

Determine the value of z when

$$z = -\sum_{k=0}^{N-2} e^{j2\pi k/N}$$

and $N = 92$. Write your answer in the standard polar form (i.e., $z = re^{j\theta}$ with $r \geq 0$ and $-\pi < \theta \leq \pi$).

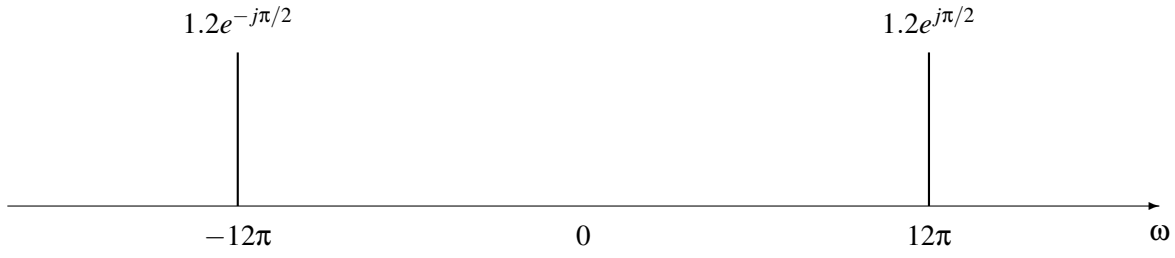
$z =$ _____

PROBLEM Fa-11-Q.1.2:

(a) Express the signal $x(t)$ as a sinusoid in standard form (i.e., $A\cos(\omega t+\varphi)$ with $A \geq 0$ and $-\pi < \phi \leq \pi$).

$$x(t) = 2.4\cos(400t + 0.8\pi) + \sqrt{5}\cos(400(t - 0.95\pi))$$

(b) The signal $y(t)$ has the spectrum shown below.



The signal $y(t)$ also has the following formula:

$$y(t) = 6.5\cos(12\pi t + \pi/3) + A\cos(\omega t + \phi).$$

Calculate the values of the unknown variables $A \geq 0, \omega \geq 0$ and $-\pi < \phi \leq \pi$ in the formula above.

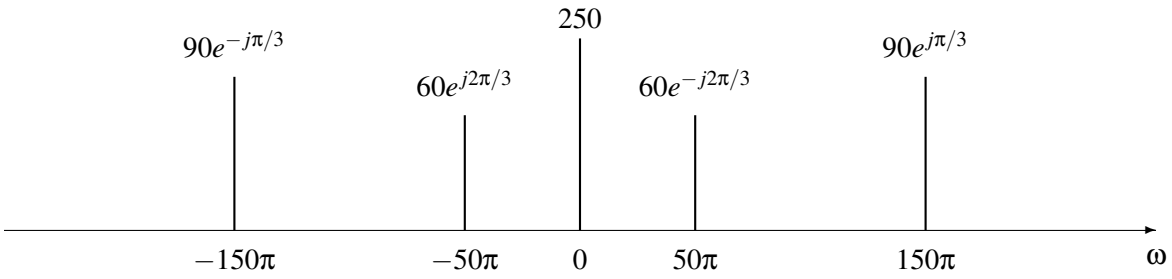
$A =$ _____

$\omega =$ _____

$\phi =$ _____

PROBLEM Fa-11-Q.1.3:

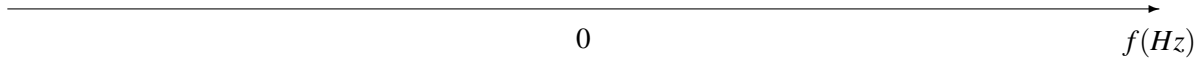
The signal $x(t)$ has the spectrum shown below.



(a) Write a formula for $x(t)$ in terms of a sum of sinusoids (i.e., $x(t) = A_0 + \sum_k A_k \cos(\omega_k t + \phi_k)$).

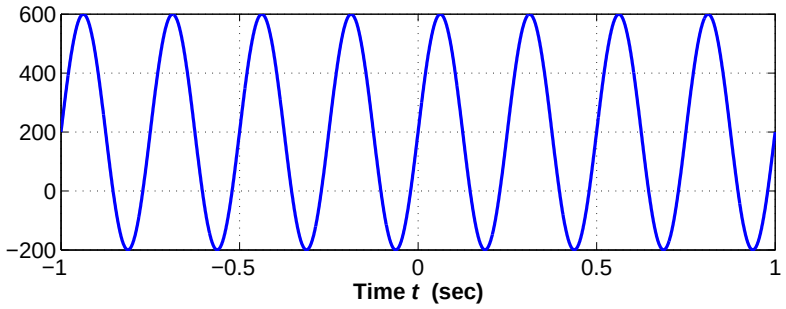
(b) Suppose that MATLAB is used to generate a signal $y(t)$ using the code below (where $y(t)$ is denoted `yy` in the code). Draw a sketch of the spectrum of the signal $y(t)$ on the axis below. **Be sure to label every aspect of your sketch carefully.**

```
dt = 1/1000;
tt = -1 :  dt :  1;
Fo = 0.5;
zz = 7 + sqrt(16)*exp(j*(2*pi*Fo*tt + 0.3*pi));
yy = real( zz );
```



PROBLEM Fa-11-Q.1.4:

A sinusoidal signal $x(t)$ is plotted below, which also has the formula $x(t) = A_0 + A_1 \cos(\omega t + \phi)$.



(a) Using the plot above, determine the values of the variables $A_0, A_1 \geq 0, \omega \geq 0$ and $-\pi < \phi \leq \pi$ in the formula above.

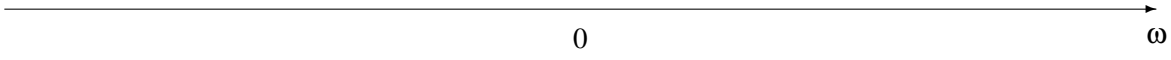
$A_0 =$ _____

$A_1 =$ _____

$\omega =$ _____

$\phi =$ _____

(b) In the space below, sketch the spectrum of the signal $x(t)$. **Be sure to label every aspect of your sketch carefully.**



PROBLEM Fa-11-Q.1.5:

Consider the following simultaneous sinusoidal equations which can be solved using phasor equations:

$$\begin{aligned} 84A_1\cos(\omega_0 t+\phi_1)+48A_2\cos(\omega_0 t+\phi_2-3) &= 0 \\ 44A_1\cos(\omega_0 t+\phi_1-1.5\pi)-62A_2\cos(\omega_0 t+\phi_2) &= \cos(\omega_0 t-5) \end{aligned}$$

(a) We will use the following MATLAB code to solve these simultaneous equations. Note that the variable assignments in the first three lines have been left for you to fill in below.

```
var1 = ??
var2 = ??
var3 = ??
YY = [84 48*exp(var1); 44*exp(-j*1.5*pi) var2];
xx = [0; exp(var3)];
zz = YY\xx                %solve xx=YY*zz for zz, equivalent to zz = inv(YY)*xx
```

Determine the values that should be in the three unassigned variables.

var1 = _____

var2 = _____

var3 = _____

(b) Assume that the correct variables were entered in part (a) and MATLAB returned the following result when the simultaneous equations were solved with the backslash operator (recall that MATLAB uses the notation `i` for the complex variable we denote *j*):

```
zz =
0.0018 - 0.0080i
0.0011 - 0.0142i
```

Calculate the values of the unknown variables $A_1 \geq 0, A_2 \geq 0, -\pi < \phi_1 \leq \pi$ and $-\pi < \phi_2 \leq \pi$.

$A_1 =$ _____

$A_2 =$ _____

$\phi_1 =$ _____

$\phi_2 =$ _____

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NAME: **ANSWER KEY** GT username: **VERSION #2**

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PROBLEM Fa-11-Q.1.1:

(a) For each of the following complex numbers below, write the number in the standard polar form (i.e., $z = re^{j\theta}$ with $r \geq 0$ and $-\pi < \theta \leq \pi$). Note: z^* means the "conjugate" of z.

$$z_1 = -je^{j2\pi/3} \qquad z_2 = \left(\frac{6-2j}{7+4j}\right)^* \qquad z_3 = \frac{1}{60j}$$

$z_1 = e^{j\pi/6} = e^{j0.524}$

$z_2 = 0.785e^{j0.841}$

$z_3 = \frac{1}{60}e^{-j\pi/2}$

(b) Recall the special properties of the roots of unity. Specifically, note that we have used the following fact:

$$\sum_{k=0}^{N-1} e^{j2\pi k/N} = 0.$$

Determine the value of z when

$$z = -\sum_{k=0}^{N-2} e^{j2\pi k/N}$$

and $N = 92$. Write your answer in the standard polar form (i.e., $z = re^{j\theta}$ with $r \geq 0$ and $-\pi < \theta \leq \pi$).

$z = e^{-j\pi/46} = e^{-j0.0683}$

PROBLEM Fa-11-Q.1.2:

(a) Express the signal $x(t)$ as a sinusoid in standard form (i.e., $A\cos(\omega t+\varphi)$ with $A \geq 0$ and $-\pi < \phi \leq \pi$).

$$x(t) = 2.4\cos(400t + 0.8\pi) + \sqrt{5}\cos(400(t - 0.95\pi))$$

$$x(t) = 1.441\cos(400t + 1.365)$$

(b) The signal $y(t)$ has the spectrum shown below.

The signal $y(t)$ also has the following formula:

$$y(t) = 6.5\cos(12\pi t + \pi/3) + A\cos(\omega t + \phi).$$

Calculate the values of the unknown variables $A \geq 0, \omega \geq 0$ and $-\pi < \phi \leq \pi$ in the formula above.

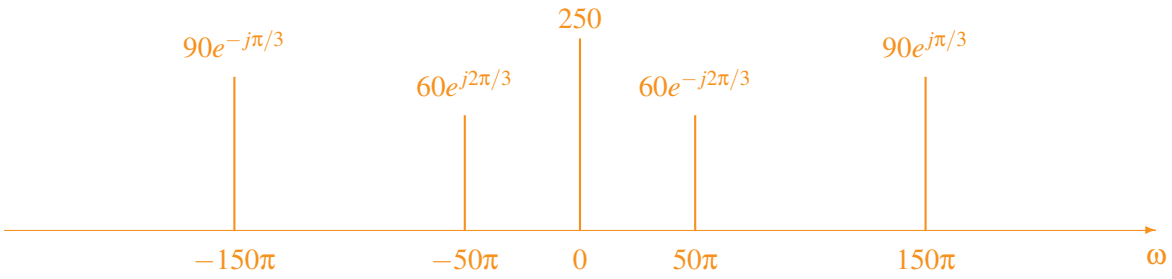
$A = 5.494$

$\omega = 12\pi$

$\phi = -2.204$

PROBLEM Fa-11-Q.1.3:

The signal $x(t)$ has the spectrum shown below.



(a) Write a formula for $x(t)$ in terms of a sum of sinusoids (i.e., $x(t) = A_0 + \sum_k A_k \cos(\omega_k t + \phi_k)$).

$$x(t) = 250 + 120\cos(50\pi t - 2\pi/3) + 180\cos(150\pi t + \pi/3)$$

(b) Suppose that MATLAB is used to generate a signal $y(t)$ using the code below (where $y(t)$ is denoted `yy` in the code). Draw a sketch of the spectrum of the signal $y(t)$ on the axis below. **Be sure to label every aspect of your sketch carefully.**

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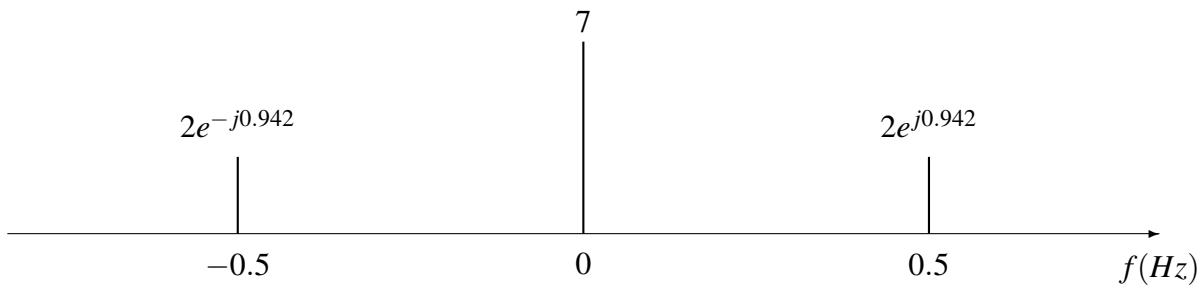
dt = 1/1000;
tt = -1 :  dt :  1;
Fo = 0.5;
zz = 7 + sqrt(16)*exp(j*(2*pi*Fo*tt + 0.3*pi));
yy = real( zz );
```



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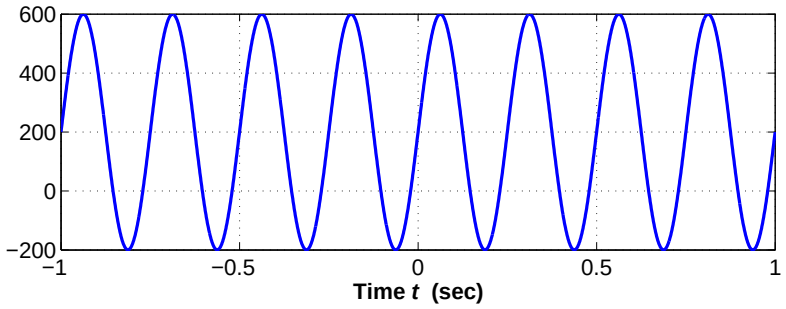


(a) Using the plot above, determine the values of the variables $A_0, A_1 \geq 0, \omega \geq 0$ and $-\pi < \phi \leq \pi$ in the formula above.

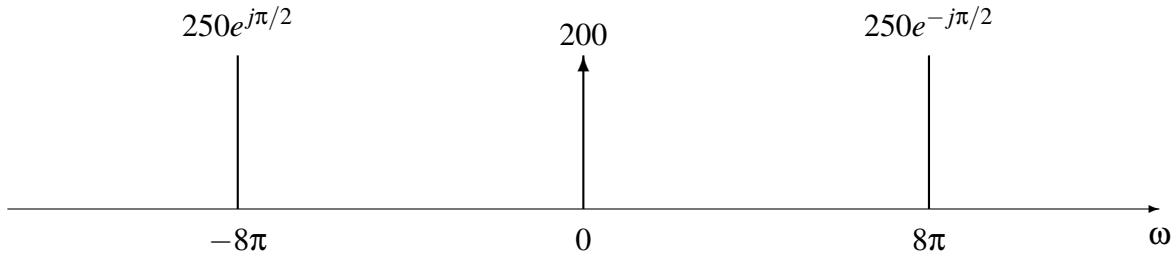
$A_0 = 200$

$A_1 = 400$

$\omega = 8\pi$

$\phi = -\pi/2$

(b) In the space below, sketch the spectrum of the signal $x(t)$. **Be sure to label every aspect of your sketch carefully.**



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YY = [84 48*exp(var1); 44*exp(-j*1.5*pi) var2];
xx = [0; exp(var3)];
zz = YY\xx                  %solve xx=YY*zz for zz, equivalent to zz = inv(YY)*xx
```

Determine the values that should be in the three unassigned variables.

var1 $= -j3$ (alternately in MATLAB code as $-\mathtt{j}*3$)

var2 $= -62$

var3 $= -j5$ (alternately in MATLAB code as $-\mathtt{j}*5$)

(b) Assume that the correct variables were entered in part (a) and MATLAB returned the following result when the simultaneous equations were solved with the backslash operator (recall that MATLAB uses the notation `i` for the complex variable we denote j):

```
zz =
0.0018 - 0.0080i
0.0011 - 0.0142i
```

Calculate the values of the unknown variables $A_1 \geq 0, A_2 \geq 0, -\pi < \phi_1 \leq \pi$ and $-\pi < \phi_2 \leq \pi$.

$A_1 = 0.0081$

$A_2 = 0.0143$

$\phi_1 = -1.3539$

$\phi_2 = -1.4955$