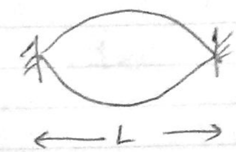
2\. C-String cello

Fundamental frequency = 65.4 Hz.

Since this is a standing wave

$$\frac{V}{2L} = f_0$$



$$V = 2Lf_0$$

Length: 4/4 cello; $L \approx 695\,mm = 0.695\,m$

$\therefore$ Propagation Velocity$(V) = 2Lf_0$

$$= 2 \times 0.695 \times 65.4 \quad m/s$$

$$= 90.906 \ m/s$$

3\. $$\vec{E}(\vec{r}) = (E_x\hat{x} + E_y\hat{y} + E_z\hat{z}) \exp(-j\frac{2\pi}{\lambda}\vec{k}\cdot\vec{r})$$

$$\Rightarrow \vec{E}_x(\vec{r}) = E_x \exp(-j\frac{2\pi}{\lambda}\vec{k}\cdot\vec{r})\,\hat{x}$$

$$\vec{k}\cdot\vec{r} = (\cos\phi_0 \sin\theta_0\,\hat{x} + \sin\phi_0 \sin\theta_0\,\hat{y} + \cos\theta_0\,\hat{z})(x\hat{x} + y\hat{y} + z\hat{z})$$

$$\vec{k}\cdot\vec{r} = x\cos\phi_0 \sin\theta_0 + y\sin\phi_0\sin\theta_0 + z\cos\theta_0$$

$$\therefore \vec{E}_x = \hat{x}\,E_x \exp(-j\frac{2\pi x}{\lambda}\cos\phi_0\sin\theta_0)\exp[(y\sin\phi_0\sin\theta_0 + z\cos\theta_0)(-\frac{2\pi}{\lambda}j)]$$

In other words,

$$\vec{E}_x = \hat{x}\,E_x \exp(-jkx\cos\phi_0\sin\theta_0)\exp[-jky\sin\phi_0\sin\theta_0 - jkz\cos\theta_0]$$

The scalar wave equation is

$$(\nabla^2 + k^2)(\vec{E}\cdot\vec{a}) = 0$$

or

(E_x) is

E_x is $|\vec{E}_x| = E_x \exp(-jkx\cos\phi_0\sin\theta_0)$ ~~const~~

$\exp(-jky\sin\phi_0\sin\theta_0)$ This ~~is independent of~~

$\exp(-jkz\cos\theta_0)$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\therefore \nabla^2 E_x = \Big[E_x(-jk)^2(\cos\phi_0\sin\theta_0)^2 + E_x(-jk)^2(\sin\phi_0\sin\theta_0)^2 + E_x(-jk)^2\cos^2\theta_0\Big] e^{-jkx\cos\phi_0\sin\theta_0 - jky\sin\phi_0\sin\theta_0 - jkz\cos\theta_0}$$

$$\nabla^2 E_x = -k^2 E_x[1]\, e^{-jk(x\cos\phi_0\sin\theta_0 + y\sin\phi_0\sin\theta_0 + z\cos\theta_0)}$$

$$k^2 E_x = k^2 E_x\, e^{-jk(x\cos\phi_0\sin\theta_0 + y\sin\phi_0\sin\theta_0 + z\cos\theta_0)}$$

$$\therefore \nabla^2 E_x + k^2 E_x = 0$$

$$\Rightarrow \boxed{(\nabla^2 + k^2)E_x = 0}$$