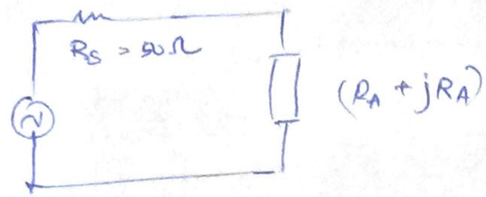
#1.



Out of 50 mW, 25 mW enters $\Rightarrow \frac{P_T}{P_s} = \frac{1}{2}$

$$\frac{P_T}{P_s} = \frac{4R_S R_A}{|\tilde{Z}_S + \tilde{Z}_A|^2} \Rightarrow \frac{1}{2} = \frac{4 \times 50 \times R_A}{|50 + R_A + jR_A|^2}$$

$$\Rightarrow \quad 400 R_A = (50 + R_A)^2 + R_A^2$$

(OR) $\quad 2R_A^2 + 100 R_A + 2500 = 400 R_A$

$$\Rightarrow \quad R_A^2 - 150 R_A + 1250 = 0$$

$R_A = 75 + 25\sqrt{7} = 141.44$

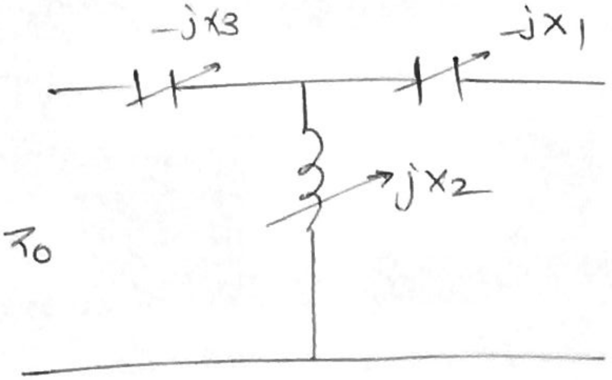
or

$75 - 25\sqrt{7} = 8.856$

$\therefore$ Antenna load impedance is $8.856 \pm j\,8.856\ \Omega$

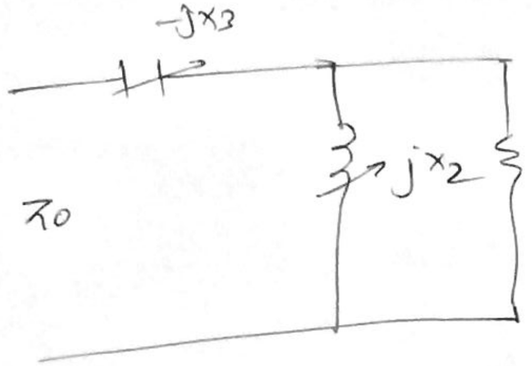
(OR)

$141.44 \pm j\,141.44\ \Omega$



$\tilde{Z}_A = R_A + jX_A$

chosing $X_1 = X_A$, the circuit simplifies as



$Z_{in} = Z_0$

$Z_L = -jX_3 + (jX_2 \parallel R_A)$

$Z_L = Z_0^*$ since Z_0 is real

imaginary part of $Z_L = 0$

$$Z_L = -jX_3 + \frac{R_A\, jX_2}{R_A + jX_2} = -jX_3 + \frac{R_A X_2\, j(R_A - jX_2)}{R_A^2 + X_2^2}$$

$$= \frac{R_A X_2^2}{R_A^2 + X_2^2} + j\left(\frac{R_A^2 X_2}{R_A^2 + X_2^2} - X_3\right)$$

$$X_3 = \frac{R_A^2 X_2}{R_A^2 + X_2^2} \quad \text{and} \quad \frac{R_A X_2^2}{R_A^2 + X_2^2} = Z_0$$

$$X_2 = \sqrt{\frac{Z_0 R_A^2}{R_A - Z_0}}$$

$$X_3 = \sqrt{Z_0(R_A - Z_0)}$$