1.

$$\tilde{A}_z(\vec{r}) = \frac{\mu}{4\pi r} e^{-jkr} \int_{-\frac{n\lambda}{4}}^{\frac{n\lambda}{4}} I_0 \exp(jkz\cos\theta)\, dz$$

$$= \frac{\mu}{4\pi r} e^{-jkr} I_0 \frac{2}{k\cos\theta} \sin\left(\frac{n\lambda}{4} k \cos\theta\right)$$ Using Integration by parts

$$= \frac{\mu I_0}{2\pi r k} \frac{e^{-jkr}}{\cos\theta} \sin\left(\frac{nk\lambda}{4}\cos\theta\right) = \frac{\mu I_0}{2\pi r k} \frac{e^{-jkr}}{\cos\theta} \sin\left(\frac{n\pi}{2}\cos\theta\right)$$

$$\vec{S} = \frac{k^2 \eta \sin^2\theta}{2\mu^2} |\tilde{A}_z(r)|^2 = \frac{I_0^2 \eta}{8\pi^2 r^2} \tan^2\theta \sin^2\left(\frac{n\pi}{2}\cos\theta\right)$$

$$P_T = \oint \vec{S}\cdot ds = \frac{I_0^2 \eta}{8\pi^2} \int_0^{2\pi}\int_0^{\pi} \tan^2\theta \sin\theta \sin^2\left(\frac{n\pi}{2}\cos\theta\right) d\theta\, d\phi$$

$$= \frac{I_0^2 \eta}{8\pi^2} 2\pi \int_0^{\pi} \sin\theta \tan^2\theta \cdot \sin^2\left(\frac{n\pi}{2}\cos\theta\right) d\theta$$

$$= \frac{\eta I_0^2}{4\pi} \int_0^{\pi} \sin\theta \tan^2\theta \sin^2\left(\frac{n\pi}{2}\cos\theta\right) d\theta$$

$$\text{Directivity } D(\theta,\phi) = \frac{|S|}{P_T} 4\pi r^2 = \frac{\frac{\eta I_0^2}{8\pi^2 r^2} 4\pi r^2 \tan^2\theta \sin^2\left(\frac{n\pi}{2}\cos\theta\right)}{P_T}$$

$$= \left[\frac{\frac{\eta I_0^2}{2\pi} \tan^2\theta \sin^2\left(\frac{n\pi}{2}\cos\theta\right)}{\frac{\eta I_0^2}{4\pi} \int_0^{\pi} \sin\theta \tan^2\theta \sin^2\left(\frac{n\pi}{2}\cos\theta\right) d\theta} \right]$$

$$= \left(\frac{2 \tan^2\theta \sin^2\left(\frac{n\pi}{2}\cos\theta\right)}{I(n)} \right)$$

To solve "I(n)" you can use MATLAB or some NUMERICAL INTEGRATION method.

I(n) varies with 'n'

$$D(dB) = 10 \log \left(\frac{2 \tan^2\theta \sin^2[\frac{n\pi}{2} \cos\theta]}{I(n)} \right)$$

for n = 1; $I(1) \approx 2.82$

n = 2; $I(2) \simeq 7.91$

n = 3; $I(3) \simeq 12.8$

n = 4; $I(4) \simeq 17.7$

n = 5; $I(5) \simeq 22.7$

	Radiation resistance (Ω)	Gain (dBi)	HPBW	Gain x HPBW
n = 1	169.3	2.4	70.8	127.4
n = 2	474.8	4.0	45.9	114.7
n = 3	768.4	5.4	32.4	113.4
n = 4	1062.6	6.5	24.8	109.12
n = 5	1362.72	7.4	20	108

Radiation resistance

$$\simeq \frac{2P_r}{I^2} \quad \text{and} \quad P_r(n=1) = I^2 \frac{\eta}{4\pi}(2.82) = I^2(86.64)$$

$\eta \simeq 377\Omega$

$$\therefore R_r(n=1) = 2 \times 86.64 = 169.3\,\Omega$$

Gain is obtained from MATLAB code

Gain = max(D)

HPBW is also obtained from MATLAB code. angular spread b/w the two points of D where gain drops by ½

GPeak = max(D)
HPBW = sum(D >= Gpeak/2) / M * 180

c) HWDP has radiation resistance of 73 Ω
peak gain of 2.2 dBi
HPBW ≃ 78°

(N=1) Radiation resistance ≃ 169.3 Ω
Peak gain = 2.4 dBi
HPBW ≈ 70.8°

this antenna has better peak gain performance.