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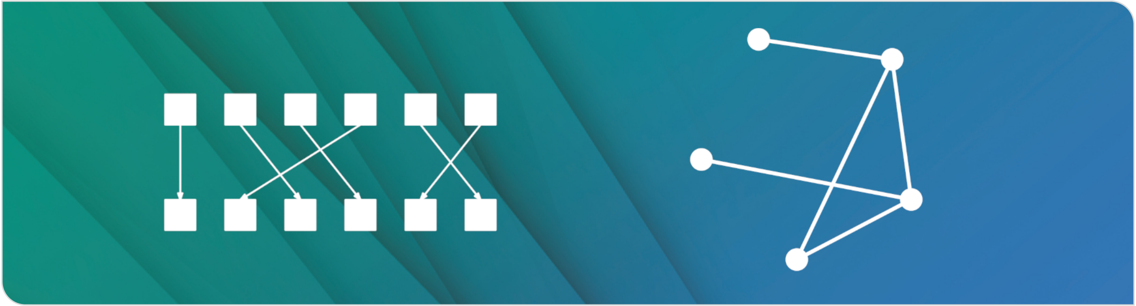
Mining Perfect Hash Functions

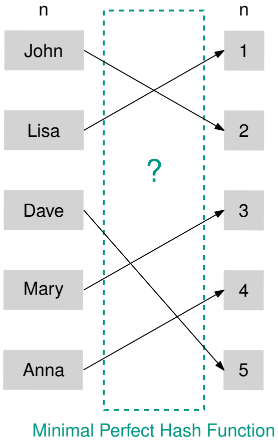
SPAA Workshop

Hans-Peter Lehmann, Stefan Hermann, **Peter Sanders**, Stefan Walzer | Jun 17, 2024

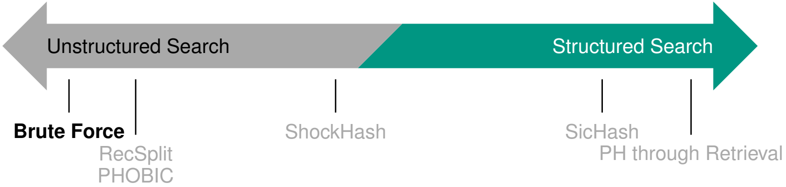


Minimal Perfect Hash Function (MPHF)

- Static set of n keys
- Data structure to injectively/bijectively map keys to the first n integers
- MPHF: Lower bound $1.44n$ bits $\ll$ space of input keys
- Goal: Near minimal space, constant time query, linear construction time
- Applications: databases, hash tables, AMQ, retrieval, replace pointers



Overview


Unstructured Search
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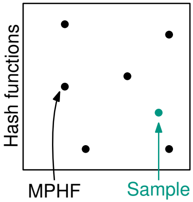
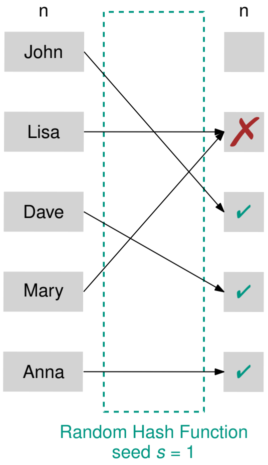
Brute-Force Construction

- Given a hash function
  try seeds 1, 2, 3, . . .
- Perfect hash function data structure:
  store successful seed s
- Expected tries: $n^n/n! \approx e^n$
  $\Rightarrow \approx n \log e \approx 1.44n$ bits (this is optimal)
- But exponential construction time, ≈linear query time





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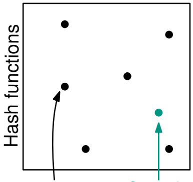

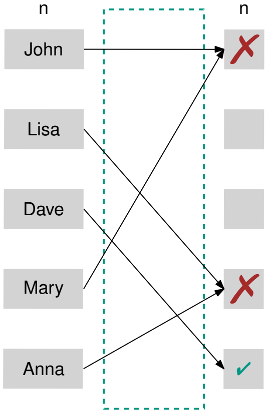

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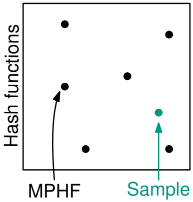

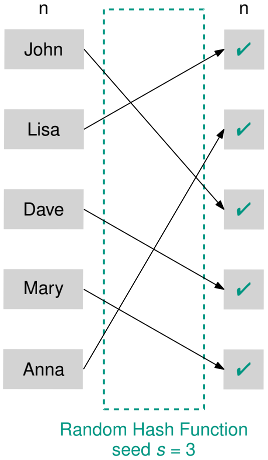

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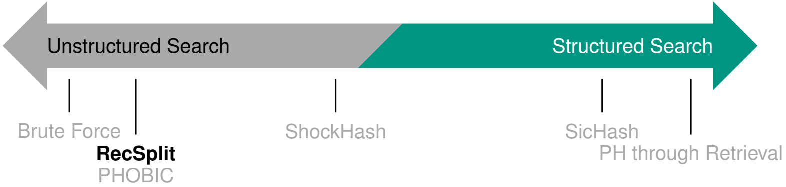
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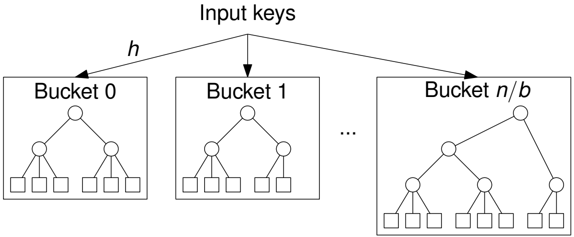
RecSplit [EGV20, BKLS23a]

- Randomly hash keys to buckets
  Store prefix sum of bucket sizes
  using Elias-Fano coding
- Tree structure within buckets
  - Brute-force search for splitting hash function
  - Specific shape depending only on bucket size
- Small leaves
  - Brute-force search for bijection hash function
  - Practicable for $\ell \leq 16$



Store seeds for leaves and inner nodes (variable-bitlength, geometrically distributed).
Overall: optimal $+\mathrm{O}(1)$ bits per node.

Bijections: Rotation Fitting

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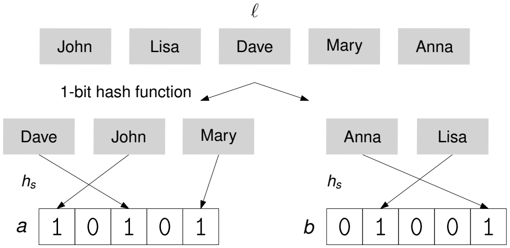
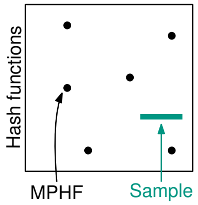


- Split keys into two subsets
- Determine function values independently
- Cyclically “rotate” word b
- Store seed and rotation $s \cdot \ell + r$
- Test $\approx \ell$ times fewer seeds
- Can use lookup tables



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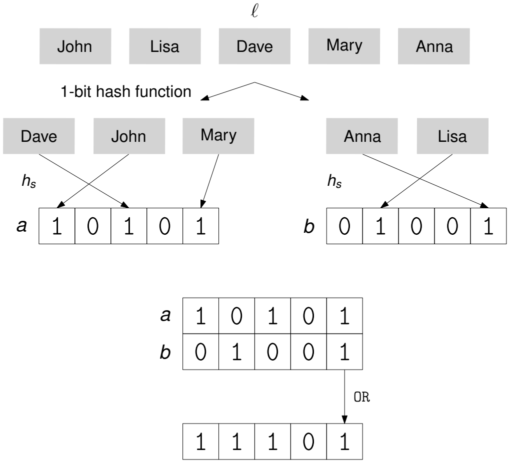

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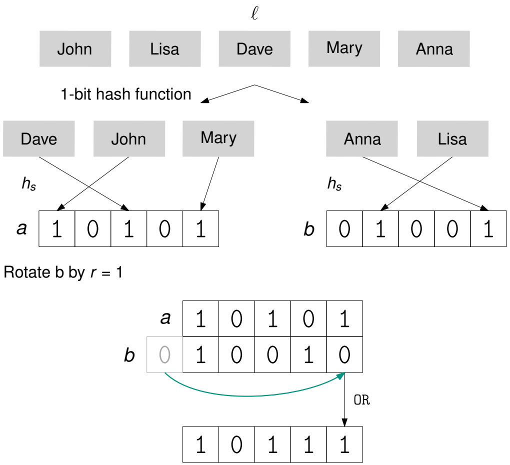

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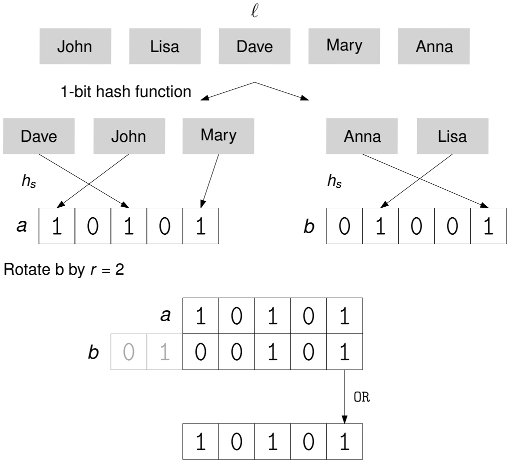

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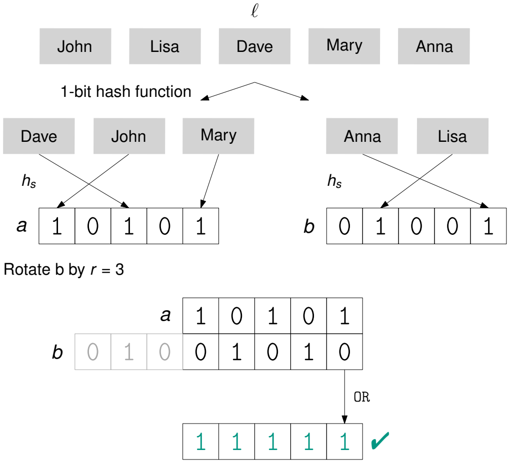
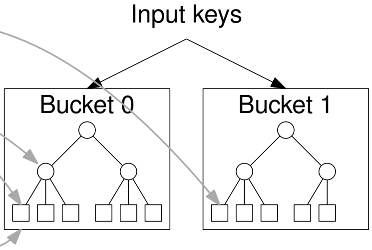


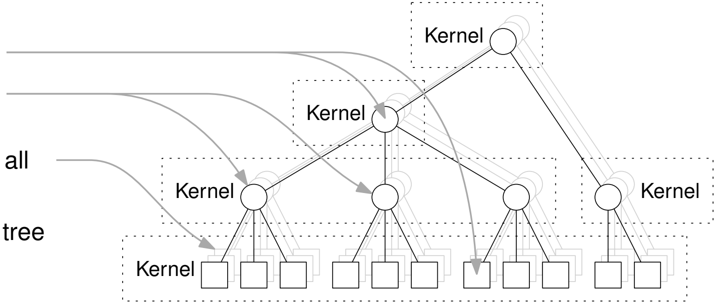
CPU Parallelization

- Bit parallelism
  - Bit operations rotate all keys of a leaf
- SIMD parallelism
  - Each lane tries a different hash function seed
- Multi-Threaded parallelism
  - Calculate different buckets in parallel



GPU Parallelization

- Threads try different seeds
- Groups of threads work on different tree nodes
- 2D grid of groups to calculate all trees with same shape
- Streams to calculate different tree shapes in parallel



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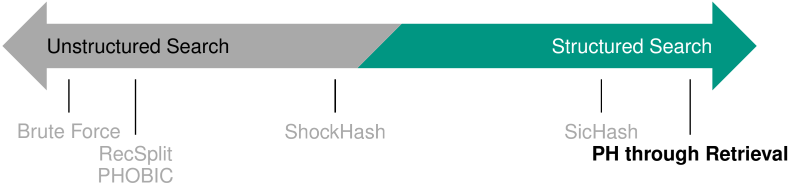
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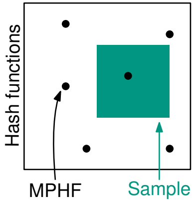
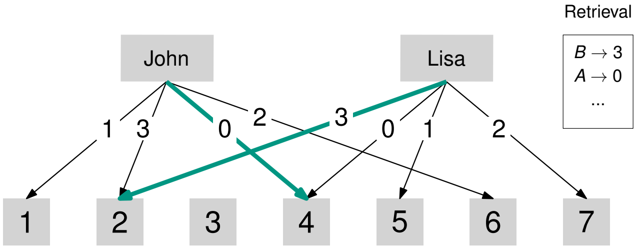
Perfect Hashing by Retrieval [DHSW22a]

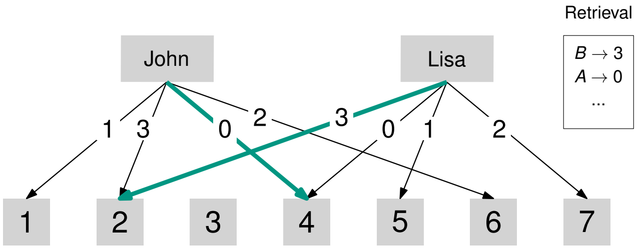
- Each object has 2^k choices
- Find collision-free mapping through perfect matching or cuckoo hashing
- Store static function $S \to \{0,1\}^k$ in retrieval data structure
- Space: $kn + o(n)$ bits





Perfect Hashing by Retrieval [DHSW22a]

- $k = 1$ bits: yields PHF with range $1..2n$
- $k = 2$ bits: yields PHF with range $1..1.024n$
  can be modified to an MPHF [PT21].
  Overall: $\approx$ 2.15 **bits per key**



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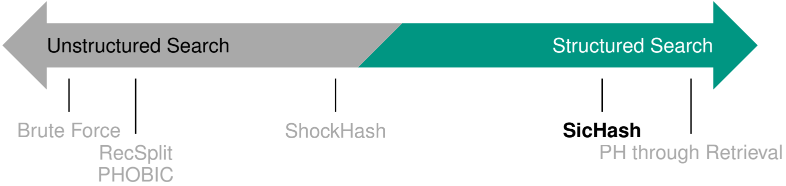
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SicHash

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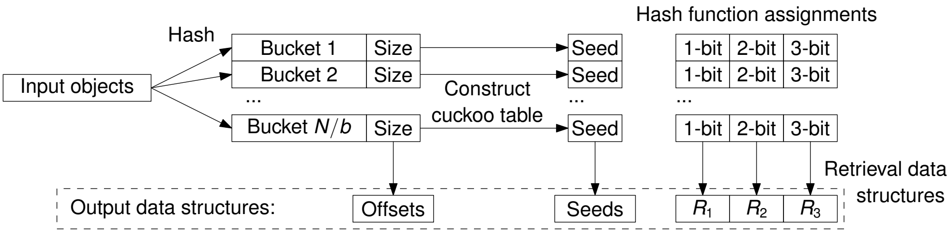
- Mix of 1/2/3-bit retrieval [DGM$^+$10] + Partitioning + Retries
- Around **2 bits per key** for MPHF



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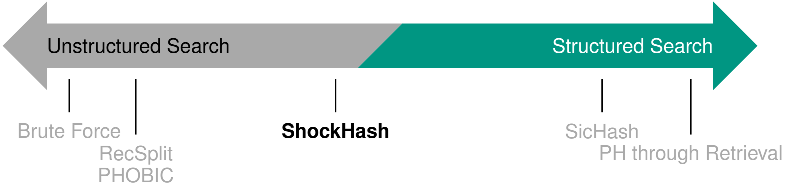

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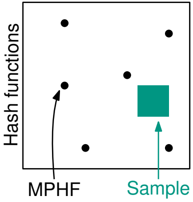
ShockHash

- Hybrid between RecSplit and 1-bit SicHash-MPHF
- Sample random graphs
- Store choice between two candidates [DHSW22a, LSW23c]

- Problem: Unlikely to work for $> n/2$ edges [PR04], here we use n
- **ShockHash**: Do it anyway, try **many** seeds

- Orientability check?
  Success probability?
  Space usage?



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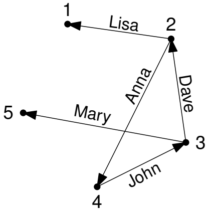
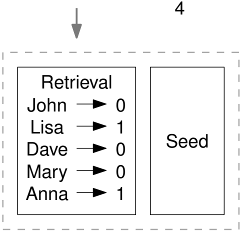
  Space usage?

x	$h_0(x)$	$h_1(x)$
John	③	4
Lisa	2	①
Dave	②	3
Mary	⑤	3
Anna	2	④

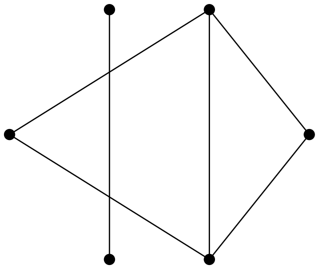




ShockHash data structure

Orientability Check

- 1-orientable if each component contains no more edges than nodes
  - Here: Tree with one additional edge
- Can be checked in linear time using connected components algorithms
- $\Rightarrow$ We check the 2^n different states of the retrieval data structure in linear time



Success Probability

$$\mathbb{P} \geq \frac{}{n^{2n}}$$

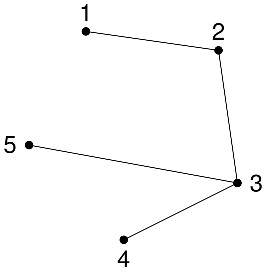
x	$h_0(x)$	$h_1(x)$
John	?	?
Lisa	?	?
Dave	?	?
Mary	?	?
Anna	?	?

Success Probability

- Labeled trees
  (Cayley's formula)

$$\mathbb{P} \geq \frac{n^{n-2}}{n^{2n}}$$

x	$h_0(x)$	$h_1(x)$
John	3	4
Lisa	2	1
Dave	2	3
Mary	5	3
Anna	?	?



Success Probability

- Labeled trees (Cayley's formula)
- Table rows can be in any order

$$\mathbb{P} \geq \frac{n^{n-2} \; (n-1)!}{n^{2n}}$$

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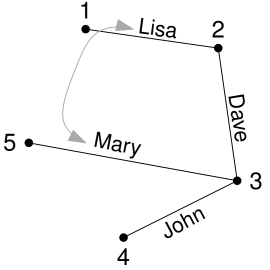

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$$\mathbb{P} \geq \frac{n^{n-2}\ (n-1)!\ 2^{n-1}}{n^{2n}}$$

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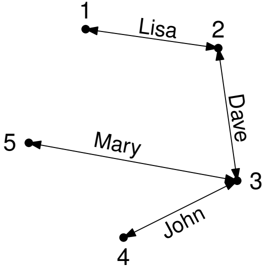

Success Probability

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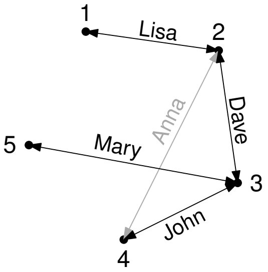

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$$\mathbb{P} \geq \frac{n^{n-2} \; (n-1)! \; 2^{n-1} \; n^2}{n^{2n}}$$

$$= \frac{n!}{n^n} \cdot \frac{2^{n-1}}{n}$$

Brute force

Almost 2^n times higher probability

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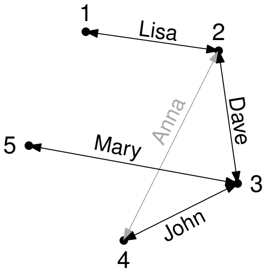
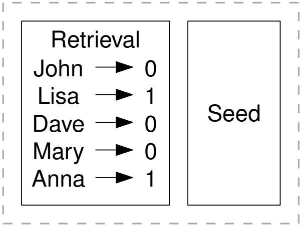


Space Usage

- $n + o(n)$ bits for 1-bit retrieval. Practice: $1.007n$ bits using BuRR [DHSW22b]
- Expected space for seed: $\approx \log\left(\frac{n}{2^{n-1}} \cdot \frac{n^n}{n!}\right) \approx n \log e - n$ bits
- Together: $\approx n \log e$ bits (optimal!)

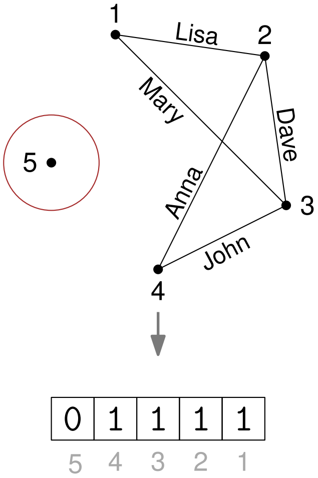


ShockHash data structure

Similar space as brute-force but nearly 2^n times faster!

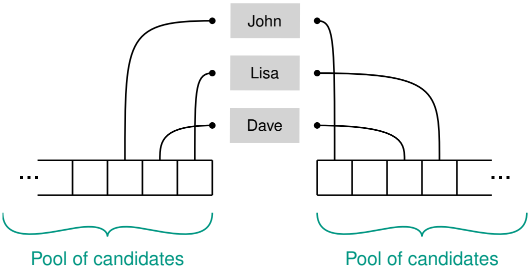
Filtering

- First implementation dominated by orientability check
  $\Rightarrow$ Filter seeds that can't work
- Efficiently in registers
- Filter passed with probability only 0.84^n
- Main ingredient for making ShockHash feasible in practice



Bipartite ShockHash

- Extension of ShockHash
- Test all combinations from a set of seed candidates
- Filter halves before combining them
- Brute-force: $e^n \approx 2.72^n$
  $\Rightarrow$ ShockHash: 1.36^n
  $\Rightarrow$ Bipartite ShockHash: 1.166^n



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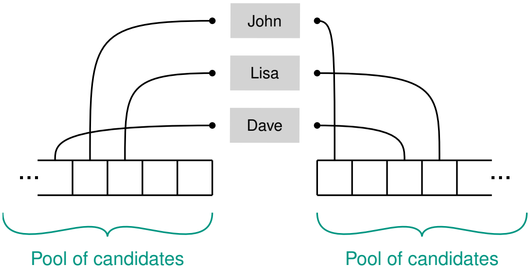

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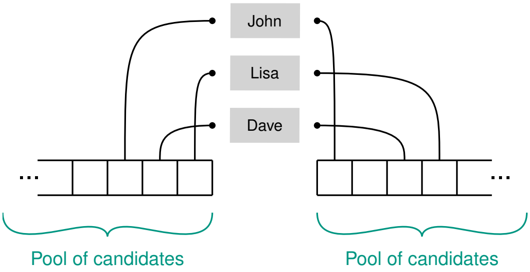

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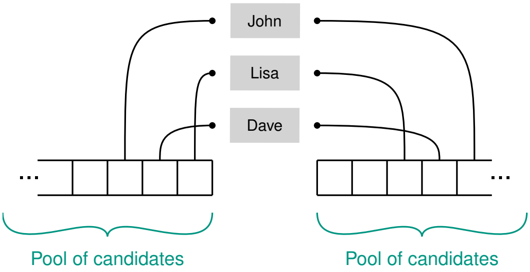

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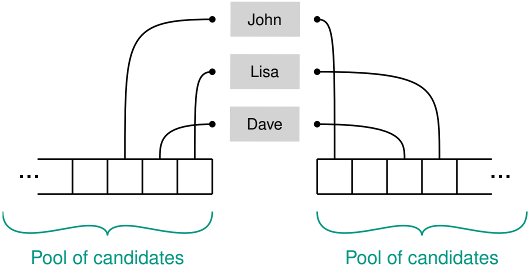

ShockHash on the GPU

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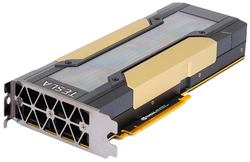
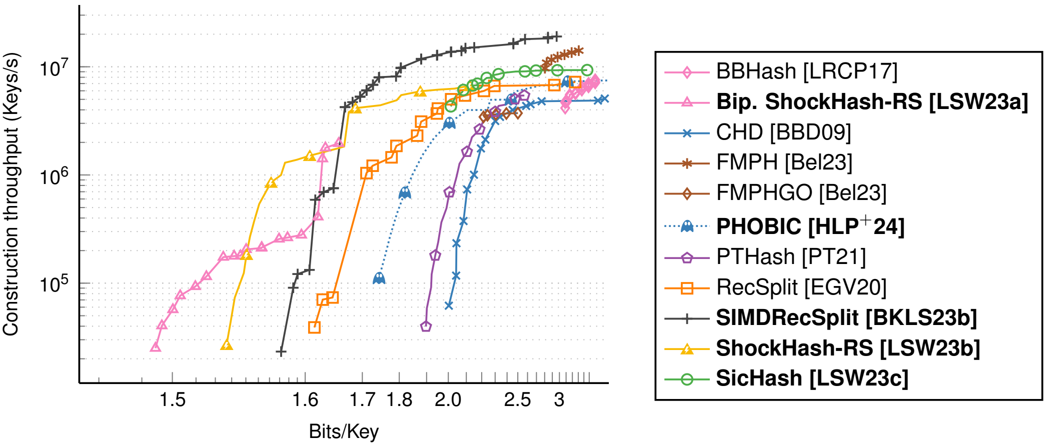
- Cuckoo hashing hard because of irregular memory access
- Filtering is easy and dominates asymptotically
- Hybrid implementation planned
  - Filtering and bit fiddling on the GPU
  - Cuckoo hashing on the CPU



Evaluation (100M keys, single threaded)



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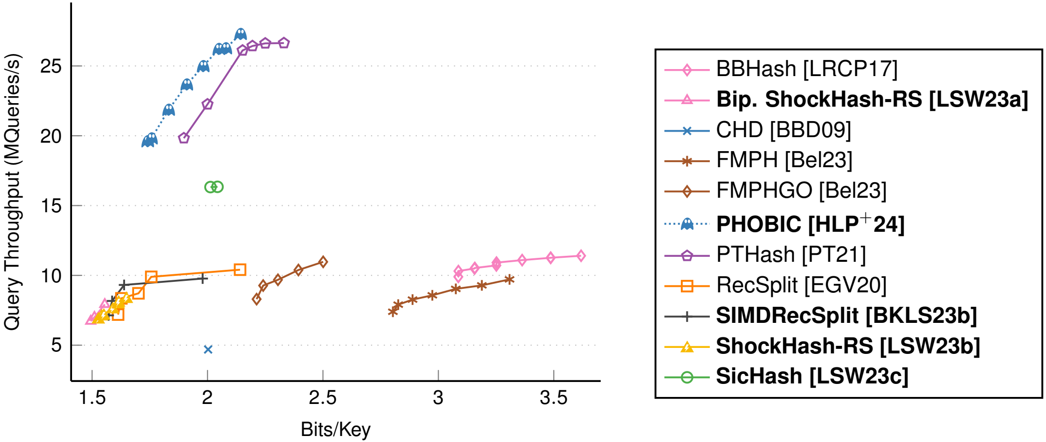

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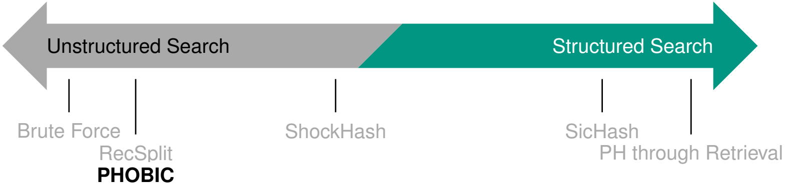
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PH through Retrieval

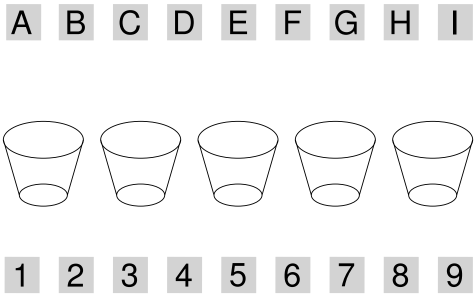
State-of-the-art [BBD09, FCH92, PT21]

A	B	C	D	E	F	G	H	I

1	2	3	4	5	6	7	8	9

State-of-the-art [BBD09, FCH92, PT21]

- Choose a number of buckets proportional to the number of keys



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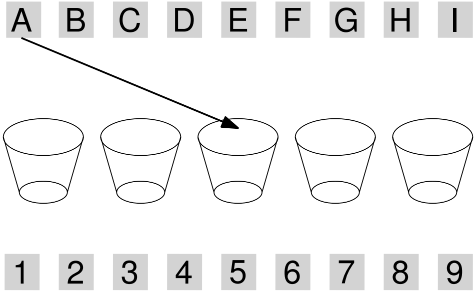

- Choose a number of buckets proportional to the number of keys
- Distribute keys to buckets using random hash function

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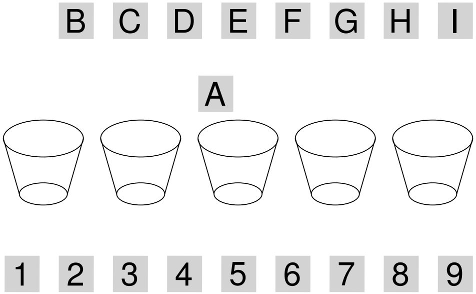

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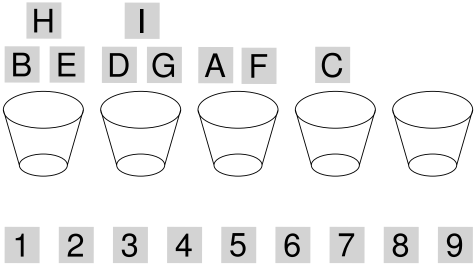
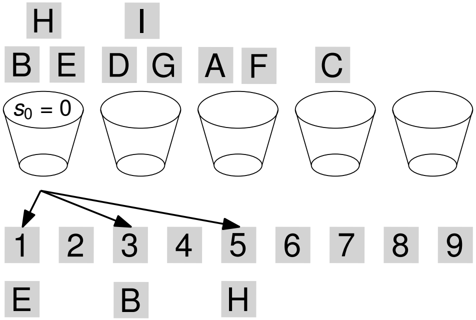


State-of-the-art [BBD09, FCH92, PT21]

- Choose a number of buckets proportional to the number of keys
- Distribute keys to buckets using random hash function
- For each bucket: Search for a seed such that there are no collisions



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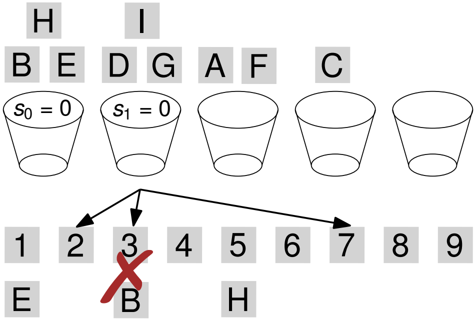

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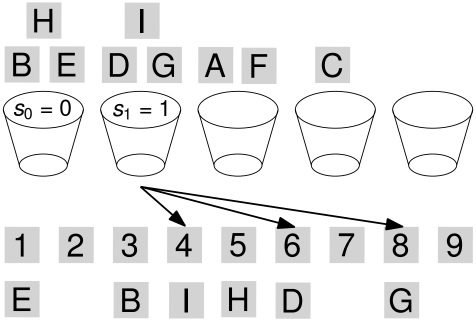

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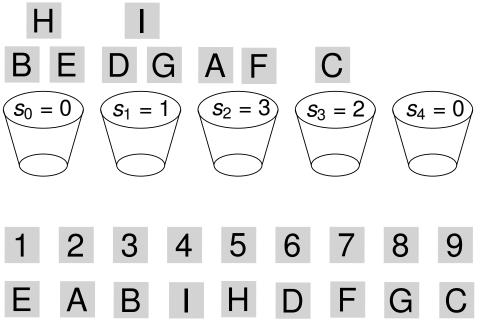

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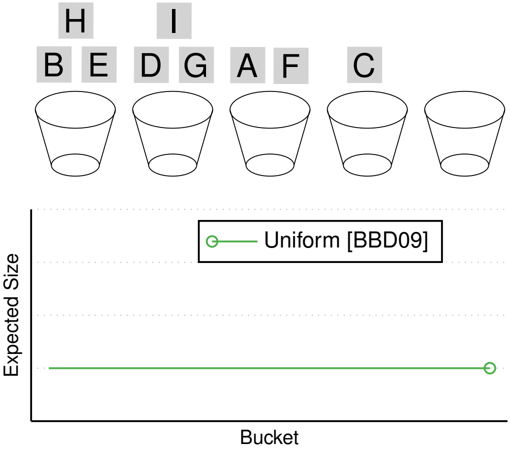
State-of-the-art [BBD09, FCH92, PT21]

- The first buckets are easier to insert
- Therefore insert in non-increasing size

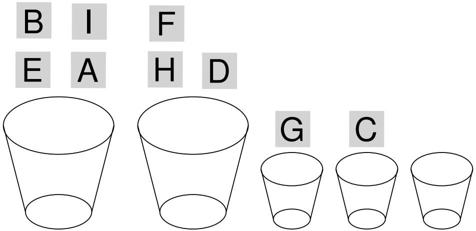
idea: buckets should have the same success probability in expectation

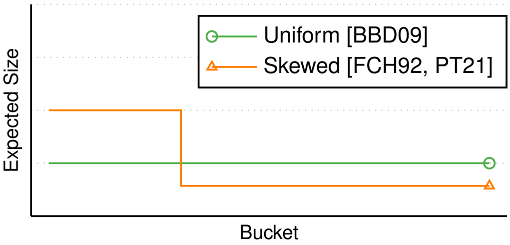


State-of-the-art [BBD09, FCH92, PT21]

- The first buckets are easier to insert
- Therefore insert in non-increasing size
- Exaggerating this effect by intentionally making some buckets even larger is helpful
- Previous state-of-the-art was to make 30% of the buckets larger

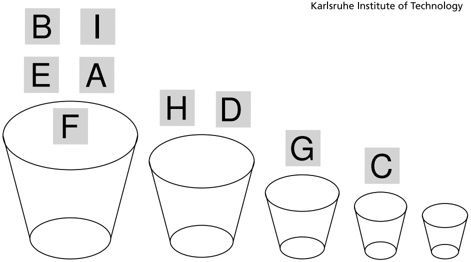
idea: buckets should have the same success probability in expectation

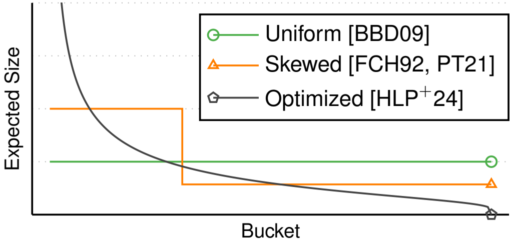




PHOBIC [HLP+24]

- The first buckets are easier to insert
- Therefore insert in non-increasing size
- Exaggerating this effect by intentionally making some buckets even larger is helpful
- Previous state-of-the-art was to make 30% of the buckets larger
- We determined optimal bucket sizes idea: buckets should have the same success probability in expectation

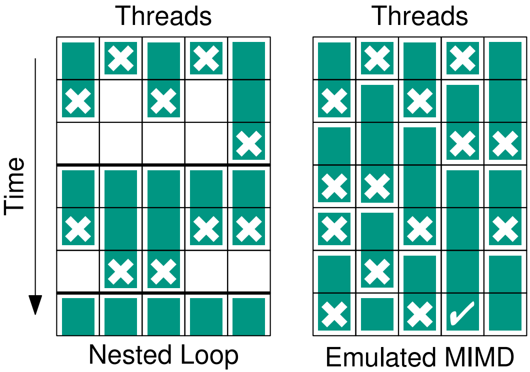




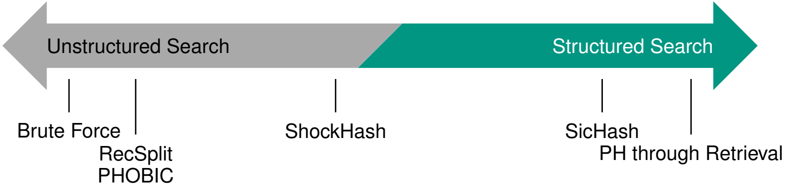
$$-\frac{1}{ln(1-x)}$$

PHOBIC on GPU

- Threads try different seeds
- Groups of threads work on different partitions
- MIMD on SIMD emulation [San94]
- 62x Speedup



Conclusion



- Tradeoff between space efficient brute-force and larger linear time algorithms
- Engineering wide range of tradeoffs
- Supported by GPUs and parallelization
- Future work
  - Combine ShockHash and PHOBIC
  - Hybrid ShockHash GPU implementation
  - How close can we get to the optimal space without construction time deteriorating?
  - More use of data structures to accelerate search
  - Hardness proofs for achieving lower bound $(+O(1))$?
  - Better performance (construction and query) when more bits are allowed (3–8)

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