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# Structure in the Edge Plasma Profiles in Tokamaks

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It is argued that the structure observed in radial profiles in the tokamak edge plasma is determined by the requirements of ion particle, momentum and energy conservation and the underlying transport mechanisms in the presence of sources and losses of particles, energy and momentum. The intent of this paper is to define a systematic formalism that can be employed for evaluating these transport coefficients from experimental inference and comparison with theory.

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## 1 Introduction

The radial profiles of density, temperature, rotation velocities, radial electric field, etc. in the edge of tokamak H-mode plasmas all exhibit an interesting structure [1-3]. The purpose of this paper is to demonstrate that these profiles are coupled by the particle, momentum and energy balance requirements and their structures are related to the sources and losses of ions, energy and momentum via diffusive and non-diffusive transport. Ion particle sources and losses determine the radial ion flux, which torques toroidal and poloidal rotation of the plasma, which in turn creates a  $\mathbf{U} \times \mathbf{B}$  electric field in the rotating plasma. The resulting electromagnetic ( $\mathbf{V} \times \mathbf{B}$  and  $E_{rad}$ ) forces produce an inward particle pinch that sets the requirement for ion radial pressure gradient needed in order to achieve the radial ion particle flux that is required by the continuity equation, as corrected to account for ion orbit loss. The ion energy balance requirement determines the total ion heat flux, and subtraction of the convective heat flux leaves the required ion conductive heat flux, which determines the required ion temperature gradient scale length, the inverse of which in turn can be subtracted from the inverse of the ion pressure gradient scale length to determine the ion density gradient scale length.

## 2 Radial Ion Flux

Integration of the steady-state particle continuity equation for the main ion species “j”

$$\nabla \cdot n_j \mathbf{V}_j \equiv \nabla \cdot \Gamma_j = S_{nbj} + n_{oj} n_e \langle \sigma_{ion} v \rangle_j \equiv S_{nbj} + n_e \nu_{ionj} \equiv S_j \quad (1)$$

determines the flux-surface averaged radial component of the ion particle flux,  $\Gamma_{rj}$ , in terms of the neutral beam and recycling neutral sources. However, not all of this particle flux flows in the plasma subject to plasma transport processes and momentum balance because some of the ions are either born on drift orbits that pass outward through the last closed flux surface (LCFS), or are carried radially outward in the flowing plasma until they access such orbits, at which point these ions are lost from the plasma across the LCFS. A cumulative (with radius) fraction  $F_{orbj}(r)$  of this total particle flux  $\Gamma_{rj}$  resulting from external sources is lost from the edge region across the separatrix by ion-orbit-loss of the thermalized plasma ions, thereby reducing the actual flux of particles being transported radially outward in the plasma from that calculated from Eq. (1) to  $\hat{\Gamma}_{rj} \equiv (1 - F_{orbj})\Gamma_{rj}$  [4,5]. In order to maintain charge neutrality, the loss of thermalized plasma ions by ion-orbit-loss must be compensated by an inward current of main plasma ions  $j_r^{iol}(r) = -e_j F_{orbj}(r) \Gamma_{rj}(r)$ , so that the net outward flux of main plasma ions is then  $\hat{\Gamma}_{rj} \simeq (1 - F_{orbj})\Gamma_{rj} - F_{orbj}\Gamma_{rj} = (1 - 2F_{orbj})\Gamma_{rj}$ .

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### 3 Ion Orbit Loss of Particles, Energy and Momentum

We are concerned with the calculation of the loss of thermalized plasma ions, their energy and their directed momentum by excursions from the flux surface on orbits that cross the last closed flux surface (LCFS) and do not return to the plasma. Such processes take place for the thermalized plasma ions primarily in the edge plasma. The tokamak plasma is a flowing plasma with a source in which the lost ions are constantly replenished by the outflow of plasma from inner flux surfaces [4,5]. The loss cone is cumulative with radius, and a source of particles into the incrementally increased loss cone at each radius is provided by the outward radial particle flux from regions with smaller loss cones, thus maintaining an ion orbit loss in equilibrium. This situation is different from the static plasma situation usually treated in the literature, in which scattering into the loss cone is required for an equilibrium with ion orbit loss.

At a given location on each flux surface in the plasma edge there is a minimum ion speed  $V_{\min(\zeta_0)}$  (and energy) for which an ion with a given directional cosine  $\zeta_0$  with respect to the magnetic field can be lost, and all ions with that  $\zeta_0$  and speeds above this minimum will be lost. The minimum speed for loss  $V_{\min(\zeta_0)}$  decreases with increasing plasma radius (decreasing distance to the last closed flux surface). As a given volume of plasma flows outward across the plasma edge it first loses the highest energy ions and then loses successively lower energy ions as it flows across successively outward flux surfaces with successively lower ion temperatures. This loss is different for the different ion directions  $\zeta_0$ . So, the ‘hole’ in the plasma velocity distribution extends progressively down to lower  $V_{\min(\zeta_0)}$  with increasing radius, and the depth of the ‘hole’ is different for different  $\zeta_0$ ; i. e. the loss region is cumulative with increasing radius and directionally dependent.

We use the conservation of canonical toroidal angular momentum, of energy and of magnetic moment to write the orbit constraint for an ion introduced at a location “0” on flux surface  $\psi_0$  with parallel velocity  $V_{\parallel 0}$ ,

$$V_0^2 \left[ \left( \left| \frac{B}{B_0} \right| \frac{f_{\varphi 0}}{f_{\varphi}} \zeta_0 \right)^2 - 1 + (1 - \zeta_0^2) \left| \frac{B}{B_0} \right| \right] + V_0 \left[ \frac{2e(\psi_0 - \psi)}{Rm f_{\varphi}} \left( \left| \frac{B}{B_0} \right| \frac{f_{\varphi 0}}{f_{\varphi}} \zeta_0 \right) \right] + \left[ \left( \frac{e(\psi_0 - \psi)}{Rm f_{\varphi}} \right)^2 - \frac{2e(\phi_0 - \phi)}{m} \right] = 0 \quad (2)$$

where  $f_{\varphi} = |B_{\varphi}/B|$ ,  $R$  is the major radius and  $\psi$  is the flux surface value,  $\phi$  is the electrostatic potential,  $\zeta_0 = V_{\parallel 0}/V_0$  is the cosine of the initial guiding center velocity relative to the toroidal magnetic field direction, and  $V_0 = \sqrt{V_{\parallel 0}^2 + V_{\perp 0}^2}$ . Equation (2) can be solved for the minimum ion speed (energy) necessary for an ion located on an internal flux surface to cross the last closed flux surface at a given location (or to strike the chamber wall at a given location, etc.). Equation (2) implicitly assumes the electrostatic potential is either unchanged over the time required for a particle to escape the plasma ( $10^{-6}$ - $10^{-5}$ s) or is an average value in the presence of high frequency turbulence.

Since  $V_{0 \min}(\zeta_0)$  decreases with radius, cumulative (with increasing radius) particle, momentum and energy loss fractions can be defined

$$F_{orb} \equiv \frac{N_{loss}}{N_{tot}} = \frac{R_{loss}^{iol} \int_{-1}^1 \left[ \int_{V_{0 \min}(\zeta_0)}^{\infty} V_0^2 f(V_0) dV_0 \right] d\zeta_0}{2 \int_{-1}^1 \int_{V_0}^{\infty} V_0^2 f(V_0) dV_0} = \frac{R_{loss}^{iol} \int_{-1}^1 \Gamma\left(\frac{3}{2}, \varepsilon_{\min(\zeta_0)}\right) d\zeta_0}{2\Gamma\left(\frac{3}{2}\right)} \quad (3)$$

$$M_{orb} \equiv \frac{M_{loss}}{M_{tot}} = \frac{R_{loss}^{iol} \int_{-1}^1 \left[ \int_{V_{0 \min}(\zeta_0)}^{\infty} (mV_0 \zeta_0) V_0^2 f(V_0) dV_0 \right] d\zeta_0}{2 \int_{-1}^1 \int_{V_0}^{\infty} (mV_0) V_0^2 f(V_0) dV_0} = \frac{R_{loss}^{iol} \int_{-1}^1 \zeta_0 \Gamma\left(2, \varepsilon_{\min(\zeta_0)}\right) d\zeta_0}{2\Gamma(2)} \quad (4)$$

and

$$E_{orb} \equiv \frac{E_{loss}}{E_{total}} = \frac{R_{loss}^{iol} \int_{-1}^1 \left[ \int_{V_{0 \min}(\zeta_0)}^{\infty} \left(\frac{1}{2} m V_0^2\right) V_0^2 f(V_0) dV_0 \right] d\zeta_0}{\int_{-1}^1 \left[ \int_{V_0}^{\infty} \left(\frac{1}{2} m V_0^2\right) V_0^2 f(V_0) dV_0 \right] d\zeta_0} = \frac{R_{loss}^{iol} \int_{-1}^1 \Gamma\left(\frac{5}{2}, \varepsilon_{\min(\zeta_0)}\right) d\zeta_0}{2\Gamma\left(\frac{5}{2}\right)} \quad (5)$$

where  $\varepsilon_{\min}(\zeta_0) = mV_{0 \min}^2(\zeta_0)/2kT$  is the reduced energy corresponding to the minimum speed for which ion orbit loss is possible, and a Maxwellian ion distribution has been used for numerical evaluation. The quantities

$\Gamma(n)$  and  $\Gamma(n, x)$  in Eqs. (3-5) are the gamma function and incomplete gamma function. Most of the particles exiting the plasma across the separatrix will be lost from the plasma, but some will follow the orbit and return to the plasma without interacting with the wall or other particles in the SOL; the lost fraction that do not return to the plasma is designated  $R_{loss}^{iol}$ .

There is a preferential loss of particles with  $\zeta_0 > 0$ , which causes a residual  $\zeta_0 < 0$  intrinsic rotation in the edge plasma in the direction opposite to the toroidal magnetic field. Determining the minimum loss speed  $V_{\min(\zeta_0)}$  as described above leads to an expression for the equivalent net parallel co-field momentum loss rate (or counter-field momentum gain rate) due to ion orbit loss  $\Delta M_{\varphi j}^{iol} = |B_\varphi/B| nm\nu_{dj} \Delta V_{\parallel}$ , which can be used to compute the intrinsic rotation caused by ion orbit loss [6]

$$\begin{aligned} \Delta V_{\parallel}(\rho) &= R_{loss}^{iol} 2\pi \int_{-1}^1 d\zeta_0 \left[ \int_{V_{\min(\zeta_0)}}^{\infty} (V_0 \zeta_0) V_0^2 f(V_0) dV_0 \right]_{\rho} = 4\pi M_{orb}(\rho) \left[ \int_0^{\infty} (V_0) V_0^2 f(V_0) dV_0 \right]_{\rho} \\ &= 2 \frac{\Gamma(2)}{\pi^{\frac{1}{2}}} M_{orb}(\rho) V_{th}(\rho) = \frac{2}{\pi^{\frac{1}{2}}} M_{orb}(\rho) \sqrt{\frac{2kT_{ion}(\rho)}{m}} \end{aligned} \quad (6)$$

#### 4 Ion Rotation Velocities

The continuity Eq. (1) can be combined with the momentum balance equation to obtain a balance among the inertial, pressure, viscous, electric field,  $\mathbf{V} \times \mathbf{B}$ , collisional friction and external source terms

$$n_j m_j (\mathbf{V}_j \cdot \nabla) \mathbf{V}_j + \nabla p_j + \nabla \cdot \Pi_j = n_j e_j (\mathbf{E} + \mathbf{V}_j \times \mathbf{B}) + \mathbf{R}_j^1 + (\mathbf{S}_j^1 - m_j \mathbf{V}_j S_j) \quad (7)$$

In a two-ion-species plasma (“j” and “k”), the toroidal component of the momentum balance equation for the main ion species “j” can be written

$$B_\theta e_j \widehat{\Gamma}_{rj} = n_j m_j (\nu_{dj} + \nu_{jk}) V_{\varphi j} - n_j m_j \nu_{jk} V_{\varphi k} - (M_{\varphi j} + n_j e_j E_\varphi^A) \quad (8)$$

and the equation for the impurity ion species “k” is obtained by interchanging the “j” and “k” subscripts. The quantity  $\nu_{dj} \equiv \nu_{viscj}^{\varphi} + \nu_{inertj}^{\varphi} + \nu_{ionj} + \nu_{elcxj} + \nu_{anomj}^{\varphi}$  is a composite toroidal momentum exchange frequency due to toroidal viscosity, toroidal inertia (Reynold’s stress), ionization, elastic scattering plus charge exchange and ‘anomalous’ viscosity (due to turbulence, non-axisymmetric toroidal magnetic field, etc.), respectively. This representation for the momentum exchange frequency is of course obvious for charge exchange and has been developed also for viscosity and the inertial term in Ref. 7. An expression for the momentum transport frequency associated with a given anomalous mechanism (e.g. ITG) would be calculated by dividing the anomalous expression for the radial transport flux of toroidal momentum by  $nmV_\varphi$ , which is a task that could be usefully undertaken by the transport community.

The quantity  $\nu_{dj} + \nu_{jk}$  represents the total momentum exchange frequency for ions of species “j”.  $M_{\varphi j} = M_{\varphi j}^{nbi} + M_{\varphi j}^{iol} + M_{\varphi j}^{anom}$  is the toroidal momentum input from neutral beams, from the directionally preferential ion-orbit-loss of ‘thermal’ plasma ion momentum [7], and from other sources.  $E_\varphi^A$  is the induced toroidal electric field, and the other quantities have their usual meaning.

Equation (8) and the corresponding equation for species “k” can be solved for

$$V_{\varphi j} = \frac{\left[ \frac{e_j B_\theta \widehat{\Gamma}_{rj} + M_{\varphi j} + n_j e_j E_\varphi^A}{n_j m_j (\nu_{jk} + \nu_{dj})} \right] + \frac{\nu_{jk}}{(\nu_{jk} + \nu_{dj})} \left[ \frac{e_k B_\theta \widehat{\Gamma}_{rk} + M_{\varphi k} + n_k e_k E_\varphi^A}{n_k m_k (\nu_{kj} + \nu_{dk})} \right]}{\left[ 1 - \frac{\nu_{jk} \nu_{kj}}{(\nu_{jk} + \nu_{dj})(\nu_{kj} + \nu_{dk})} \right]} \quad (9)$$

and a similar equation with the subscripts “j” and “k” interchanged for  $V_{\varphi k}$ .

The poloidal rotation velocities for the two ion species are determined by the poloidal component of the momentum balance equations for each species. Using the Shaing-Sigmar form of the parallel viscosity [8]  $\eta_{0j} = n_j m_j V_{thj} q R f_j (\nu_{jj}^*)$ ,  $f_j \equiv \varepsilon^{-3/2} \nu_{jj}^* / (1 + \varepsilon^{-3/2} \nu_{jj}^*) (1 + \nu_{jj}^*)$ ,  $\nu_{jj}^* \equiv \nu_{jj} q R / V_{thj}$  and neglecting

poloidal asymmetries over the flux surface in density and flow, the poloidal momentum balance equation for ion species “j” may be written [9]

$$(\nu_{viscj} + \nu_{jk} + \nu_{atomj}) V_{\theta j} - \nu_{jk} V_{\theta k} = -B_{\varphi} \left\{ \frac{e_j}{n_j m_j} \widehat{\Gamma}_{rj} + \nu_{viscj} \left( \frac{K^j T_j L_{Tj}^{-1}}{e_j B^2} \right) \right\} \quad (10)$$

where  $\nu_{viscj} \equiv q_j^f V_{thj} / R$ ,  $L_{Tj}^{-1} \equiv -T_j^{-1} \partial T_j / \partial r$  and  $K^j \equiv \mu_{01}^j / \mu_{00}^j$  (the  $\mu$ 's are the Hirshman-Sigmar coefficients [9-11]). A similar equation with the “j” and “k” sub/super-scripts interchanged obtains for the “k” ion species. The two Eqs. (10) can be solved for the poloidal rotation velocities

$$V_{\theta j} = \frac{\frac{-B_{\varphi}}{\nu_{\theta j}} \left\{ \left[ \nu_{viscj} \frac{K^j T_j}{e_j B^2} L_{Tj}^{-1} + \frac{e_j}{n_j m_j} \widehat{\Gamma}_{rj} \right] + \frac{\nu_{jk}}{\nu_{\theta k}} \left[ \nu_{visck} \frac{K^k T_k}{e_k B^2} L_{Tk}^{-1} + \frac{e_k}{n_k m_k} \widehat{\Gamma}_{rk} \right] \right\}}{\left[ 1 - \frac{\nu_{jk} \nu_{kj}}{\nu_{\theta j} \nu_{\theta k}} \right]} \quad (11)$$

where  $\nu_{\theta j} \equiv \nu_{viscj} + \nu_{jk} + \nu_{atomj} + \nu_{anom\theta j}$ . A similar equation with the “j” and “k” sub/super-scripts interchanged results for  $V_{\theta k}$ . A more accurate set of neoclassical poloidal rotation equations based on an elongated flux surface geometry and retaining poloidal asymmetries in density, velocity and electrostatic potential is given in Ref. 12.

## 5 Radial Electric Field

Multiplying the radial component of the momentum balance for each species (ions plus electrons) by  $e_{\sigma} / m_{\sigma}$  for the species and summing over species yields a generalized Ohm's law for the radial electric field

$$\begin{aligned} E_r &= \eta j_r - (\mathbf{u} \times \mathbf{B})_r + \frac{\nabla_r (p_j + p_k)}{e(n_j + z_k n_k)} \\ &= -\eta e_j \Gamma_{rj} F_{orbj} - \frac{(V_{j\theta} B_{\varphi} - V_{j\varphi} B_{\theta})}{(1 + n_k m_k / n_j m_j)} - \frac{(V_{k\theta} B_{\varphi} - V_{k\varphi} B_{\theta})}{(1 + n_j m_j / n_k m_k)} - \frac{(p_j L_{pj}^{-1} + p_k L_{pk}^{-1})}{e(n_j + z_k n_k)} \end{aligned} \quad (12)$$

where  $\mathbf{u}$  is the plasma mass velocity,  $\eta = 1.03 \times 10^{-4} Z_{eff} \ln \Lambda / T_e^{3/2}$  (eV)  $\Omega - m$  is the radial plasma resistivity which results from the derivation and  $j_r = -e_j \Gamma_{rj} F_{orbj}$  is the inward main ion current compensating the ion orbit loss. In several applications to tokamak edge plasmas, we have found that the first term is small compared to the other terms.

Effects such as turbulence and ion orbit loss enter this equation indirectly through their effect on the rotation velocities and pressure gradients. Equation (12) is the same as the sum of the “neoclassical” momentum balances over ions and electrons, so it contains the radial momentum balance for the main ion species, which is the equation which is commonly used for calculating the radial electric field.

## 6 Ion Pressure, Temperature and Density Gradients

The toroidal and radial components of the momentum balance equations for the two ion species “j” and “k” can be solved to obtain a pinch-diffusion relation for the radial particle fluxes, the flux surface averaged component of which may be written [13]

$$\widehat{\Gamma}_{rj} = n_j D_{jj} (L_{nj}^{-1} + L_{Tj}^{-1}) - n_j D_{jk} (L_{nk}^{-1} + L_{Tk}^{-1}) + n_j V_{rj}^{pinch} \quad (13)$$

where the “self-diffusion” coefficient is  $D_{jj} \equiv (m_j T_j (\nu_{dj} + \nu_{jk})) / (e_j B_{\theta})^2$ , the “other-species-diffusion” coefficient is  $D_{jk} = (m_j T_k \nu_{jk}) / e_j e_k B_{\theta}^2$ , and the pinch velocity representing electromagnetic and other external forces is

$$n_j V_{rj}^{pinch} = -\frac{M_{\varphi j}}{e_j B_{\theta}} - \frac{n_j E_{\varphi}^A}{B_{\theta}} + \frac{n_j m_j}{e_j B_{\theta}} \left[ (\nu_{jk} + \nu_{dj}) \left( \frac{E_r}{B_{\theta}} + \frac{B_{\varphi}}{B_{\theta}} V_{\theta j} \right) - \nu_{jk} V_{\varphi k} \right] \quad (14)$$

Equation (13) and a similar equation with subscripts “j” and “k” interchanged for ion species “k” can be solved for the value of the pressure gradient scale length  $L_{pj}^{-1} \equiv -(\partial p_j / \partial r) / p_j$  that is required by momentum balance

$$L_{pj}^{-1} = \frac{(e_j B_\theta)^2 \left[ \left\{ \frac{\hat{\Gamma}_{rj}}{n_j} - V_{rj}^{pinch} \right\} + \frac{m_j/e_j}{m_k/e_k} \frac{\nu_{jk}}{(\nu_{dk} + \nu_{kj})} \left\{ \frac{\hat{\Gamma}_{rk}}{n_k} - V_{rk}^{pinch} \right\} \right]}{m_j T_j (\nu_{dj} + \nu_{jk}) [1 - (\nu_{jk} \nu_{kj} / (\nu_{dj} + \nu_{jk}) (\nu_{dk} + \nu_{kj}))]} \quad (15)$$

If we take advantage of the fact that usually  $L_{Tj}^{-1} \simeq L_{Tj}^{-1}$  for different ion species and further assume  $L_{nk}^{-1} \simeq L_{nj}^{-1}$ , we obtain a simplified expression for the main ion pressure gradient scale length that can be used to obtain a simple expression for the density gradient scale length

$$L_{nj}^{-1} \equiv L_{pj}^{-1} - L_{Tj}^{-1} = \frac{\hat{\Gamma}_{rj} - n_j V_{rj}^{pinch}}{n_j D_j} - L_{Tj}^{-1}, \text{ where } D_j \equiv \frac{m_j T_j \nu_{jk}}{(e_j B_\theta)^2} \left[ 1 + \frac{\nu_{dj}}{\nu_{jk}} - \frac{e_j}{e_k} \right] \quad (16)$$

The temperature gradient scale lengths for the ion species (and the electrons) are determined by the heat conduction relations [13]

$$L_{Tj}^{-1} = \frac{(Q_{rj} (1 - E_{orbj}) - 1.5 T_j \Gamma_{rj} (1 - 2 F_{orbj}))}{n_j T_j \chi_j} \quad (17)$$

where  $Q_{rj}$  is the total ion heat flux determined by integrating the ion energy balance equation.

## 7 Conclusion

One must be able to evaluate the transport coefficients ( $\nu_{dj} \equiv \nu_{viscj}^\varphi + \nu_{inertj}^\varphi + \nu_{ionj} + \nu_{elcxj} + \nu_{anomj}^\varphi$ ,  $\nu_{\theta j} \equiv \nu_{viscj} + \nu_{jk} + \nu_{atomj} + \nu_{anom\theta j}$ ,  $\chi_j$ ) in order to solve the above fluid equations for the various profiles in the edge plasma. It is possible to infer some of these from experiment for comparison with theory, and the purpose of this paper is to suggest a framework in which this can be done consistently.

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