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W. M. Stacey

Fusion Research Center, Georgia Institute of Technology, Atlanta, GA, 30332, USA

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1 Introduction

We have previously derived [1,2] a generalized pinch-diffusion relation in the plasma edge region from momentum and particle balance. This pinch-diffusion relation was used to explain the steep pressure gradients in the edge of high confinement (H-mode) plasmas in terms of the requirements of momentum and particle conservation in the presence of recycling neutrals. The requirements of momentum and particle balance were manifest in the radial electric field and rotation velocity profiles acting through the pinch velocity term. While the implications of these previous results for particle transport in the plasma edge are implicit, they have not heretofore been explicitly set forth, which is thus the purpose of this paper.

2 Particle and Momentum Balance

The time-independent particle continuity equation for ion species 'j' is

$$\nabla \cdot \Gamma_j \equiv \nabla \cdot n_j \upsilon_j = S_j \tag{1}$$

where $S_j(r,\theta) = n_e(r,\theta) n_{j0}(r,\theta) < \sigma\upsilon >_{ion} \equiv n_e(r,\theta)\nu_{ion}(r,\theta)$ is the ionization source rate of ion species 'j' and n_{j0} is the local concentration of neutrals of species 'j'. The time-independent momentum balance equation for ion species "j" is

$$\nabla \cdot (n_j m_j \boldsymbol{\upsilon}_j \boldsymbol{\upsilon}_j) + \nabla p_j + \nabla \cdot \boldsymbol{\pi}_j = n_j e_j (\boldsymbol{\upsilon}_j \times \boldsymbol{B}) + n_j e_j \boldsymbol{E} + \boldsymbol{F}_j + \boldsymbol{M}_j - n_j m_j \nu^j_{elcxj} \boldsymbol{\upsilon}_j \tag{2}$$

where **E** represents the electric field, $\mathbf{F}_j$ represents the interspecies collisional friction, $\mathbf{M}_j$ represents the external momentum input rate, and the last term represent the momentum loss rate due to elastic scattering and charge exchange with neutrals of all ion species 'k'[$\nu_{atj} = \Sigma_k n^c_{k0}(< \sigma\upsilon >_{el} + < \sigma\upsilon >_{cx})_{jk}$].

The FSA radial component of Eq. (2) may be written to leading order as

$$E^0_r = \frac{1}{n^0_j e_j}\frac{\partial p^0_j}{\partial r} + \upsilon^0_{\phi j} B^0_\theta - \upsilon^0_{\theta j} B^0_\phi \tag{3}$$

*Corresponding author: e-mail: weston.stacey@nre.gatech.edu

3 Torque Representations

In order to evaluate the FSA (flux surface average) toroidal component of Eq. (2) it is necessary to evaluate the FSA toroidal viscous torque and inertial terms in that equation. The neoclassical viscous torque can be written as the sum of "parallel", "gyroviscous", and "perpendicular" components [3,4]. Since the flux surface average of the "parallel" component of the toroidal viscous torque vanishes identically, the flux surface averaged toroidal viscous torque may be written as the sum of the "gyroviscous" and "perpendicular" components

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\mathbf{\Pi}\right\rangle = \left\langle R^2\nabla\phi\cdot\nabla\cdot\mathbf{\Pi}\right\rangle_{gv} + \left\langle R^2\nabla\phi\cdot\nabla\cdot\mathbf{\Pi}\right\rangle_{\perp} \tag{4}$$

where

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\mathbf{\Pi}\right\rangle_{gv} = -\left\langle \frac{1}{Rh_p}\frac{\partial}{\partial l_\psi}\left(R^3 h_p \eta_4 \frac{\partial}{\partial l_p}\left(\upsilon_\phi/R\right)\right)\right\rangle \tag{5}$$

and

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\mathbf{\Pi}\right\rangle_{\perp} = -\left\langle \frac{1}{Rh_p}\frac{\partial}{\partial l_\psi}\left(R^3 h_p \eta_2 \frac{\partial}{\partial l_\psi}\left(\upsilon_\phi/R\right)\right)\right\rangle \tag{6}$$

in a right-hand (ψ, p, ϕ) toroidal flux surface coordinate system, where $\eta_2 = nT\tau/\left(\Omega\tau\right)^2$and $\eta_4 \approx (\Omega\tau)\eta_2 \approx$ *(10*^{3} $-$ 10^4*)*η_2, where $\Omega \equiv ZeB/m$ and τ is the collision time, so that the "gyroviscous" toroidal torque is generally a couple of orders of magnitude larger than the "perpendicular" toroidal viscous torque. Approximating the flux surface geometry by toroidal geometery and making a low order Fourier expansion $X\left(r,\theta\right) = X^0\left(r\right)\left[1 + X^c\cos\theta + X^s\sin\theta\right]$for the densities and rotation velocities allows Eqs. (3) and (4) to be written in a form exhibiting an explicit momentum transfer frequency

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\mathbf{\Pi}\right\rangle_{gvj} \approx \frac{1}{2}\eta_{4j}\frac{r}{R_0}\left(L_n^{-1} + L_T^{-1} + L_{\upsilon_\phi}^{-1}\right)\left[\left(4+\widetilde{n}_j^c\right)\widetilde{\upsilon}_{\phi j}^s + \widetilde{n}_j^s\left(1-\widetilde{\upsilon}_{\phi j}^c\right)\right]\upsilon_{\phi j} \equiv R_0 n_j^0 m_j \nu_{gvj}\upsilon_{\phi j} \tag{7}$$

and

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\mathbf{\Pi}\right\rangle_{\perp j} \approx R_0\eta_{2j}\left[L_{\upsilon_\phi}^{-1}\left(\frac{1}{r} - L_{\eta_2}^{-1}\right) - \frac{1}{\upsilon_{\phi j}}\frac{\partial^2\upsilon_{\phi j}}{\partial r^2}\right]\upsilon_{\phi j} \equiv R_0 n_j^0 m_j \nu_{\perp j}\upsilon_{\phi j} \tag{8}$$

where the poloidal asymmetry coefficients $\widetilde{n}_j^c \equiv n_j^c/\varepsilon$, etc. can be determined by solving the low order Fourier moments of the poloidal component of the momentum balance [4].

Turbulent, or "anomalous", toroidal viscous torque is usually assumed to be of the form of Eq. (6) with an enhanced viscosity coefficient η_{anom}, leading to

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\mathbf{\Pi}\right\rangle_{anomj} \approx R_0\eta_{anomj}\left[L_{\upsilon_\phi}^{-1}\left(\frac{1}{r} - L_{\eta_2}^{-1}\right) - \frac{1}{\upsilon_{\phi j}}\frac{\partial^2\upsilon_{\phi j}}{\partial r^2}\right]\upsilon_{\phi j} \equiv R_0 n_j^0 m_j \nu_{anomj}\upsilon_{\phi j} \tag{9}$$

Equation (1) can be used to write the inertial term in the FSA toroidal component of Eq. (2) as

$$\left\langle R^2\nabla\phi\cdot\nabla\cdot\left(n_j m_j \boldsymbol{\upsilon}_j\boldsymbol{\upsilon}_{\ j}\right)\right\rangle = \left\langle R^2\nabla\phi\cdot n_j m_j\left(\boldsymbol{\upsilon}_{\ j}\cdot\nabla\right)\boldsymbol{\upsilon}_j\right\rangle + R_0 n_j m_j \nu_{ionj}\upsilon_{\phi j} \tag{10}$$

and the same set of approximations can be used to write the first term on the right as

$$\left\langle R^2\nabla\phi\cdot n_j m_j\left(\boldsymbol{\upsilon}_j\cdot\nabla\right)\boldsymbol{\upsilon}_j\right\rangle \simeq \frac{1}{2}\left(\frac{\upsilon_{rj}}{R_o}\left\{\varepsilon\left(1+\widetilde{n}_j^c+\widetilde{\upsilon}_{\phi j}^c\right) - 2R_o L_{\upsilon_\phi j}^{-1}\right\} - \varepsilon\frac{\upsilon_{\theta j}^0}{R_o}\left\{\widetilde{\upsilon}_{\phi j}^s\left(1+\widetilde{n}_j^c+\widetilde{\upsilon}_{\theta j}^c\right) - \widetilde{\upsilon}_{\theta j}^s\left(1+\widetilde{\upsilon}_{\phi j}^c\right) - \widetilde{\upsilon}_{\phi j}^c\widetilde{n}_j^s\right\}\right) n_j m_j R_0\upsilon_{\phi j}^0 \equiv R_0 n_j m_j \nu_{nj}\upsilon_{\phi j}^0 \tag{11}$$

4 Pinch-Diffusion Transport Relation

The above results may be used to write the FSA toroidal component of Eq. (2) as

$$n_j^0 m_j \nu_{jk}^0 \left((1+\beta_j)\, \upsilon_{\phi j}^0 - \upsilon_{\phi k}^0 \right) = n_j^0 e_j E_\phi^A + e_j B_\theta^0 \Gamma_{rj} + M_{\phi j}^0, \tag{12}$$

where

$$\beta_j \equiv \frac{\nu_{gvj}^0 + \nu_{\perp j}^0 + \nu_{anomj}^0 + \nu_{nj}^0 + \nu_{elcxj}^0 + \nu_{ionj}}{\nu_{jk}^0} \equiv \frac{\nu_{dj}^*}{\nu_{jk}^0} \tag{13}$$

Now, combining the radial and toroidal components of the FSA momentum balance equations—Eqs. (3) and (12)–yields a generalized pinch-diffusion relation [7] for the radial particle flux

$$\Gamma_{rj} \equiv \langle n_j \upsilon_{rj} \rangle = n_j D_{jj} \left(L_{nj}^{-1} + L_{Tj}^{-1} \right) - n_j D_{jk} \left(L_{nk}^{-1} + L_{Tk}^{-1} \right) + n_j \upsilon_{pj} \tag{14}$$

where the "diffusion coefficients" are given by

$$D_{jj} \equiv \frac{m_j T_j \left(\nu_{dj}^* + \nu_{jk} \right)}{\left(e_j B_\theta \right)^2}\ ,\ D_{jk} \equiv \frac{m_j T_k \nu_{jk}}{e_j e_k (B_\theta)^2} \tag{15}$$

and the pinch velocity is given by

$$n_j \upsilon_{pj} \equiv -\frac{M_{\phi j}}{e_j B_\theta} - \frac{n_j E_\phi^A}{B_\theta} + \frac{n_j m_j \nu_{dj}^*}{e_j B_\theta} \left(\frac{E_r}{B_\theta} \right) + \frac{n_j m_j f_p^{-1}}{e_j B_\theta} \left(\left(\nu_{jk} + \nu_{dj}^* \right) \upsilon_{\theta j} - \nu_{jk} \upsilon_{\theta k} \right) \tag{16}$$

A sum over the 'k' terms is understood when more than two ion species are present. The quantity $f_p^{-1} \equiv B_\phi / B_\theta$.

Subject to the assumption that there in a single impurity species (I) distributed with the same radial distribution and the same local temperature as the main ions (i), Eq. (14) can be written as a constraint on the main ion pressure gradient [1,2]

$$L_{pi}^{-1} \equiv -\frac{1}{p_i} \frac{dp_i}{dr} = \frac{\upsilon_{ri} - \upsilon_{pi}}{D_i} \tag{17}$$

and momentum balance can be used to reduce Eq. (16) to

$$\upsilon_{pi} = \frac{\left[-M_{\phi i} - n_i e_i E_\phi^A + n_i m_i \left(\nu_{iI} + \nu_{di}^* \right) \left(f_p^{-1} \upsilon_{\theta i} + {}^{E_r}\!/_{B_\theta} \right) - n_i m_i \nu_{iI} \upsilon_{\phi I} \right]}{n_i e_i B_\theta} \tag{18}$$

where the effective main ion diffusion coefficient in this approximation is

$$D_i = \frac{m_i T_i \nu_{iI}}{\left(e_i B_\theta \right)^2} \left[1 + \frac{\nu_{di}^*}{\nu_{iI}} - \frac{Z_i}{Z_I} \right] \tag{19}$$

We have previously found [1,2] that when the pinch velocity of Eq. (18) was evaluated from experiment, the radial particle flux was determined by solving the continuity Eq. (1) in the presence of recycling neutrals, and L_{Ti} was taken from experiment, that

$$\frac{-1}{n_i} \frac{\partial n_i}{\partial r} \equiv L_{ni}^{-1} = L_{pi}^{-1} - L_{Ti}^{-1} = \frac{\upsilon_{ri} - \upsilon_{pi}}{D_i} - L_{Ti}^{-1} \tag{20}$$

could be integrated inward from an experimental separatrix boundary condition to obtain a density profile with a pedestal structure that was in good agreement with the edge density profile obtained from Thomson scattering (when corrected for the presence of impurities). The pinch velocity term, determined primarily by the measured rotation velocity and radial electric field profiles, was found to be the dominant factor in determining the density profile.

5 Generalized Radial Diffusion Theory

Since diffusion theory is generally used to describe ion particle transport in plasma edge codes [5,6], it is of interest to compare the radial transport theory implied by the above relations with the form of diffusion theory commonly used in the plasma edge codes. Using the generalized pinch-diffusion relation of Eq. (17) in the continuity Eq. (1), which governs Γ_{rj}, yields the coupled set of generalized diffusion equations that determine the particle distribution in the edge plasma for ion species "j", $\nabla \cdot \mathbf{\Gamma}_j = S_j$, the radial component of which can be written for each species in the slab limit appropriate in the plasma edge

$$-\frac{\partial}{\partial r}\left(D_{jj}\frac{\partial n_j}{\partial r}\right)-\frac{\partial}{\partial r}\left(D_{jk}\frac{\partial n_k}{\partial r}\right)-\frac{\partial}{\partial r}\left(D_{jj}\frac{n_j}{T_j}\frac{\partial T_j}{\partial r}\right)-$$
$$\frac{\partial}{\partial r}\left(D_{jk}\frac{n_j}{T_k}\frac{\partial T_k}{\partial r}\right)+\frac{\partial\left(n_j \upsilon_{pj}\right)}{\partial r}=S_j \qquad (21)$$

Again, the "jk" subscript indicates a sum over "k". Note that the 'self-diffusion' coefficient D_{jj} involves all the momentum transport rates for species "j" (i.e. atomic physics, viscous, anomalous, etc. as well as the interspecies collisional momentum exchange frequency for species "j"). There is an Eq. (21) for each ion species in the plasma, and they are coupled.

The generalized diffusion theory of Eq. (21), which was rigorously derived from momentum balance and the continuity equation for each ion species in the plasma, is different in several respects from the usual ad hoc form of diffusion theory [Eq. (21) but retaining only the first term on the left side] that is commonly used to represent radial particle transport in plasma edge fluid codes. First, the diffusion equation for species "j" depends not only on the density gradient of species "j", but on the density gradients for all other ion species as well. Second, the diffusion equation for species "j" depends on the temperature gradients for all ion species. This implies that, when used in the predictive mode, the diffusion equations for all the ion densities and the heat balance equations for all the ion temperatures are coupled and must be solved simultaneously.

The second major difference is that there is a convection term with a pinch velocity [Eq.(16)] that depends on the poloidal rotation velocities for all the ion species and on the radial electric field, the induced toroidal electric field, and the neutral beam (or any other) external momentum input or torque. As discussed above, we have previously found [1,2] that the pinch velocity was the dominant term in the pinch-diffusion relation insofar as the determination of the edge density profile. Thus, we anticipate that the convective last term on the left in Eq. (21) will have a major effect on the calculation of the ion particle profile in the edge plasma. This implies that when Eq. (21) is used in the predictive mode, the rotation equations must also be solved simultaneously with the particle and heat diffusion equations. Solution of the rotation equations in the plasma edge has been discussed elsewhere [7], but remains to be carried out simultaneously with the particle and energy transport equations.

6 Diffusion Coefficients and Pinch Velocities

The profiles of ν_d^* (inferred from experiment [2]), ν_{iI} and $\nu_{Ii}(n_I m_I \nu_{Ii} = n_i m_i \nu_{iI}$ by momentum conservation) were used (together with the experimental temperature profile) to calculate the profiles of the diffusion coefficients defined by Eqs. (15) for a DIII-D H-mode shot, as shown in Fig. 1. The sharp increase in the experimentally inferred ν_d^* just inside the separatrix results in a sharp increase in the "self-diffusion" coefficients D$_{ii}$ and D$_{II}$ just inside the separatrix. Because the main ion self-diffusion coefficient D$_{ii}$ $>>$ D$_{iI}$, the first and third terms in Eq. (21), involving the main ion density and temperature gradients, are much more important than the second and fourth terms involving the impurity ion density and temperature gradients, in the main ion diffusion equation. On the other hand, since the impurity self-diffusion coefficient D$_{II}$ $<<$ D$_{Ii}$, the second and fourth terms involving the main ion density and temperature gradients are much more important in the impurity ion equation than are the terms involving the impurity ion density and temperature gradients.

The contributions of the various components of the deuterium pinch velocity given by Eq. (18) are shown in Fig. 2 for a DIII-D H-mode shot. The normalized radius is in terms of poloidal flux. The inward pinch velocity is quite large in the edge.

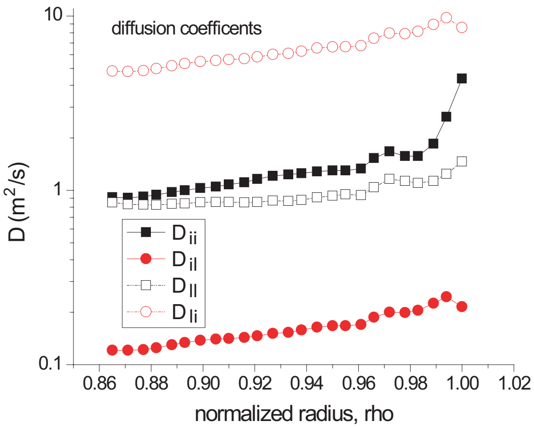

Fig. 1 Generalized diffusion coefficients in the edge of DIII-D H-mode shot 92976.

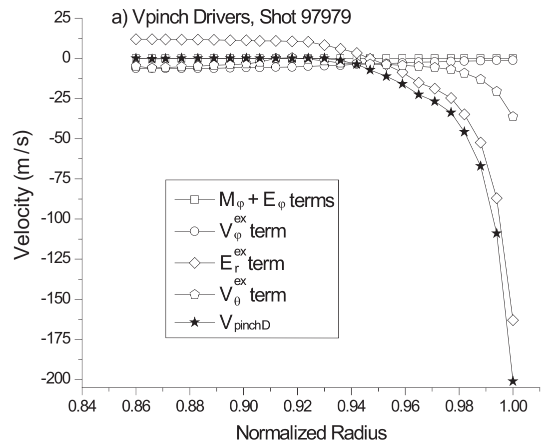

Fig. 2 Pinch velocity in the edge of DIII-D H-mode shot 97979.

7 Summary and Conclusions

The requirements of conservation of ion momentum and particle density lead directly to a generalized diffusion equation for each ion species, with diffusion-like terms involving the gradients of all ion densities and temperatures and a convective term involving a ”pinch velocity” consisting of rotation velocities, the radial electric field and other terms. The definitions of the pinch velocity and of the diffusion coefficients follow directly from the derivation from momentum balance.

These equations are quite different than the diffusion equations normally used to analyze the radial transport of particles in tokamak edge transport codes (e.g. [5] and [6]). For example, in these references the radial particle transport was modeled using only the first diffusion term on the left in Eq. (21) and neglecting the pinch term. The value of the ”self-diffusion” coefficient was inferred from experiment by adjusting it to force the calculation (with three of the diffusion terms and the pinch term of Eq. (21) set to zero) to ’match’ the experimental density profile. It is clear from Figs. 1 and 2 that this type of diffusion approximation and fitting procedure neglects a lot of physics.

References

[1] W. M. Stacey, Phys. Plasmas, 11, 4295 (2004).
[2] W. M. Stacey and R. J. Groebner, Phys. Plasmas, 13, 012513 (2006).
[3] W. M. Stacey and D. J. Sigmar, Phys. Fluids, 28, 2800 (1985).
[4] W. M. Stacey, R. W. Johnson and J. Mandrekas, Phys. Plasmas, 13, 062508 (2006).
[5] L. D. Horton, A. V. Chankin, Y. P. Chen, et al., Nucl. Fusion, 45, 856 (2005).
[6] G. D. Porter, R. Isler, J. Boedo and T. D. Rognlien, Phys. Plasmas, 7, 3663 (2000).
[7] W. M. Stacey, Contrib. Plasma Phys., 46, 597 (2006).

