Group 1:
Plasma

John Norris

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Overview

$$(\boldsymbol{fusion} + \boldsymbol{external})\boldsymbol{heating} \geq (\boldsymbol{radiation} + \boldsymbol{transport}\ )\boldsymbol{loss}$$

$$\frac{1}{5}P_{fus}\left(1+\frac{5}{Q_p}\right) \geq [(\rho_{Brems} + \rho_L + \rho_{Rec}) + \rho_{Tran}] \cdot V_{TP}$$

- Total Plasma Volume: $V_{TP} = 2\pi R\pi a^2\kappa = 451.192\ m^3$
  - where $R = 6.045\ m \quad \& \quad a = 1.375\ m \quad \& \quad \kappa = \frac{b}{a} = 2$
- Electricity Production Goal: $P_{Elec} = 1000\ MW$ (assumed)
- Plasma Amp.: $Q_p = 35$ (assumed)
- Thermal Eff.: $\eta_e = 35\% - 40\%$

$$\therefore\ P_{fus} = \frac{P_{Elec}}{\eta_e} = 2857.14\ - 2500\ \ MW$$

John Norris

Overview

- Plasma Density $(10^{20} m^{-3})$
  - $n_{GW} = \frac{I}{\pi a^2} \quad \Rightarrow \quad n_{e20} = k_{ITER} n_{GW} = 2.083 \ - 1.948$
    - $k_{ITER} = 85\%$
- Confinement Time (s)
  - $\tau_{IPB98} = (0.0562) I^{0.93} B_{\varphi}^{0.15} n_{e20}^{0.41} M^{0.19} R^{1.97} \kappa^{0.78} \left(\frac{a}{R}\right)^{-0.58} P_{heat}^{-0.69}$
    - where $B_{\varphi} = 11.249\ T$ & $\mathrm{M} = 2.014$ & 3.016 amu (for Deuterium & Tritium)
  - $\tau_e = H_{98} \tau_{IPB98} = 1.251 - 1.25$
    - $where\ \ H_{98} = 1.2$ (assumed)
- Current (MA)
  - $I = \sqrt{\frac{2a^2 \mathrm{P}fus}{\langle\sigma v\rangle\, U_{fus}\, R\, \kappa \left(10^{20} k_{ITER}\right)^2}} = 14.555 \ - 13.615$
    - $\langle\sigma v\rangle = 1.728 \cdot 10^{-22}\ m^3 s^{-1}$ (assumed maximal) @ $T_i = \frac{8.9}{8.1} T_e = 12\ keV$

      & $U_{fus} = 17.6\ MeV$

John Norris

Overview

- Impurities
  - *SS316 ($z \simeq 26$):* $\frac{n_{26}}{n_{e20}} = 0.05\%$ *&* *Be ($z = 4$):* $\frac{n_4}{n_{e20}} = 1.00\%$
- Transport Loss Density $(MW \cdot m^{-3})$
  - $\rho_{Tran} = \frac{3 n_{e20} T_i}{\tau_e} = 1.055 - 0.984$
- Radiation Loss Density $(MW \cdot m^{-3})$
  - $\rho_{Brems} \simeq 4.8 \cdot 10^{-43} \cdot \left(n_{e20}^2 \; \sum_i \frac{n_{zi}}{n_{e20}} \; z_i^2 \; T_e^{\frac{1}{2}} \right) = 0.038 - 0.033$
  - $\rho_L \simeq 1.8 \cdot 10^{-44} \cdot \left(n_{e20}^2 \sum_i \frac{n_{zi}}{n_{e20}} \; z_i^4 \; T_e^{\frac{-1}{2}} \right) = 0.050 - 0.043$
  - $\rho_{Rec} \simeq 4.1 \cdot 10^{-46} \cdot \left(n_{e20}^2 \sum_i \frac{n_{zi}}{n_{e20}} \; z_i^6 \; T_e^{\frac{-3}{2}} \right) = 0.057 - 0.050$

John Norris

Overview

$$(\boldsymbol{fusion} + \boldsymbol{external})\boldsymbol{heating} \geq (\boldsymbol{radiation} + \boldsymbol{transport}\ )\boldsymbol{loss}$$

$$\frac{1}{5}P_{fus}\left(1 + \frac{5}{Q_p}\right) \geq [(\rho_{Brems} + \rho_L + \rho_{Rec}) + \rho_{Tran}] \cdot V_{TP}$$

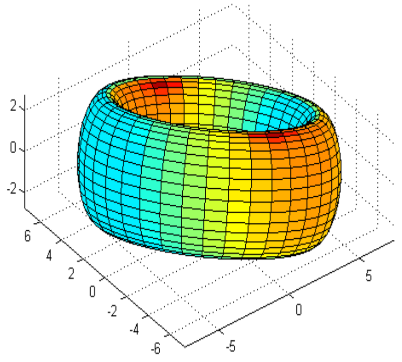
η_e	**Heating**	$\geq$	**Loss**
35%	653.061 MW	$\geq$	541.331 MW
40%	571.429 MW	$\geq$	501.236 MW

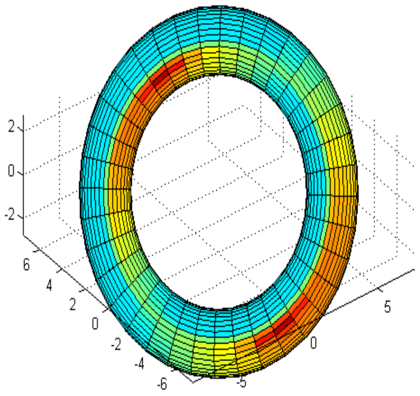
John Norris

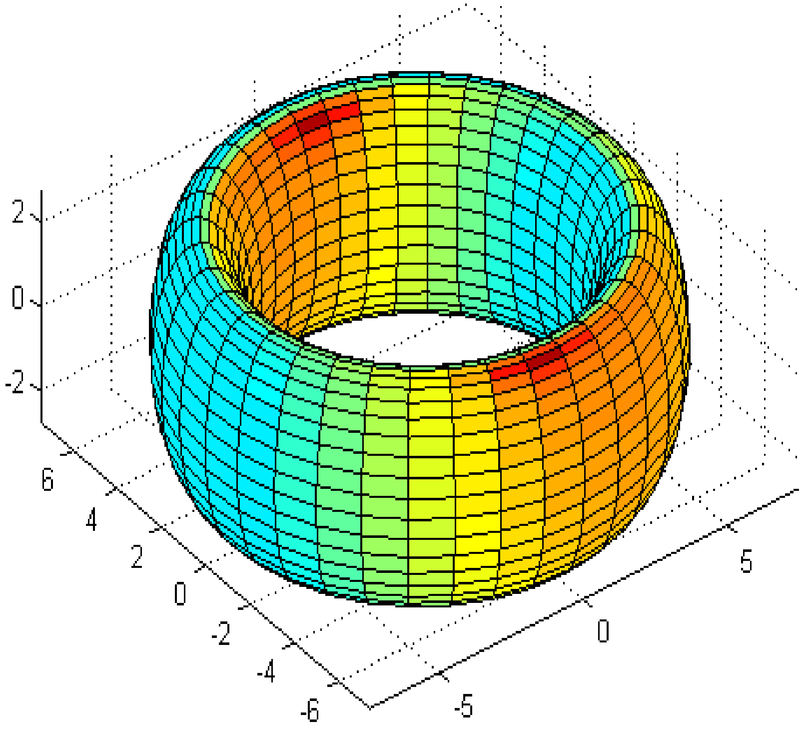
Additional Bounds

- Kink Stability Limit
  - $q_{95} = \frac{5a^2 B_\varphi}{2RI}\left(1+\kappa^2\left(1+2\delta^2-1.2\delta^3\right)\right)\left(1.17-\frac{0.65a}{R}\right)\left(1-\left(\frac{a}{R}\right)^2\right)^{-2} \geq 3$
    - where the plasma triangularity: $\delta = 0$ (assumed)
  - $q_{95} = 3.464 - 3.703$
- Troyon β limit
  - $\beta_C \leq \beta_N \frac{I}{ab} = \beta_N \frac{I}{a^2\kappa}$ & $\beta_N \leq 4.5$ $\therefore$ $\beta_C = 4.235 - 3.961$

Plasma Geometry

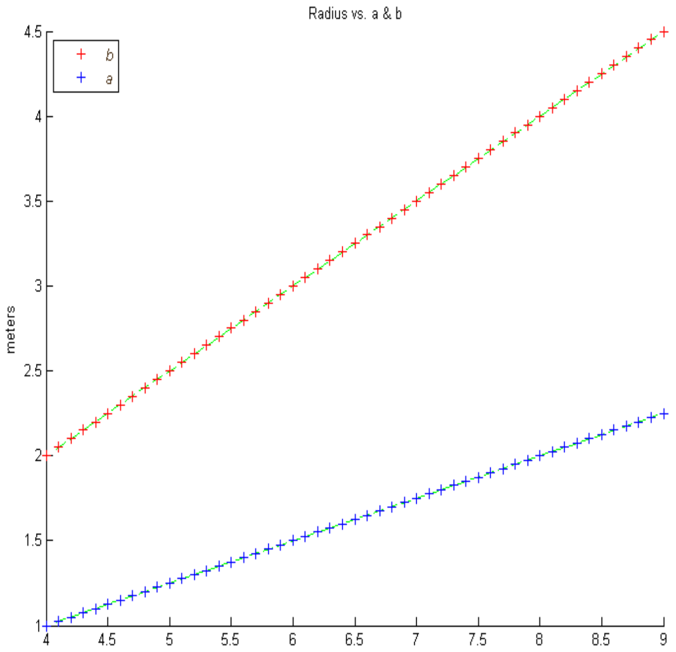


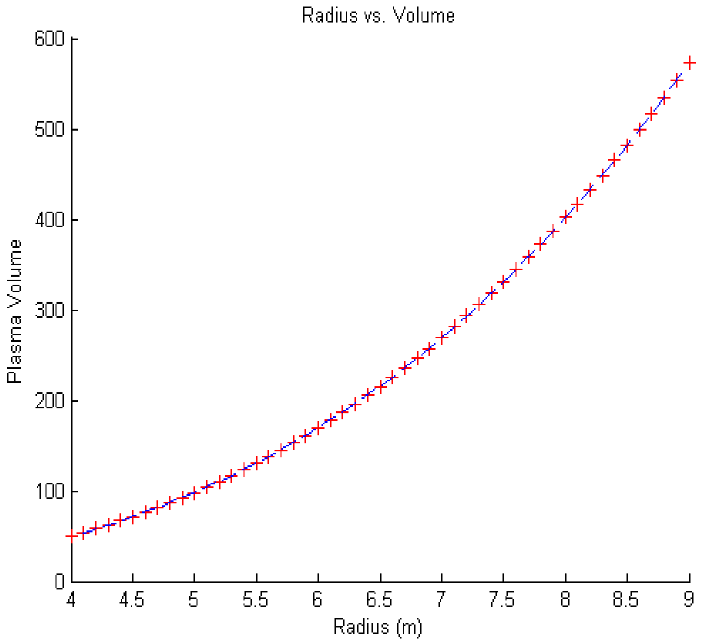




Plasma Geometry

- Depends on:
  - General Plasma Ratio
    - $\kappa,\ A$
  - Plasma Chamber
    - $a,\ \ b,\ r_w$
  - Coil Assemblage
    - $r_v,\ \Delta_m$
  - Shield & Blanket
    - $\Delta_B,\ \Delta_S$
- All of the parameters above were determined either iteratively or assumed





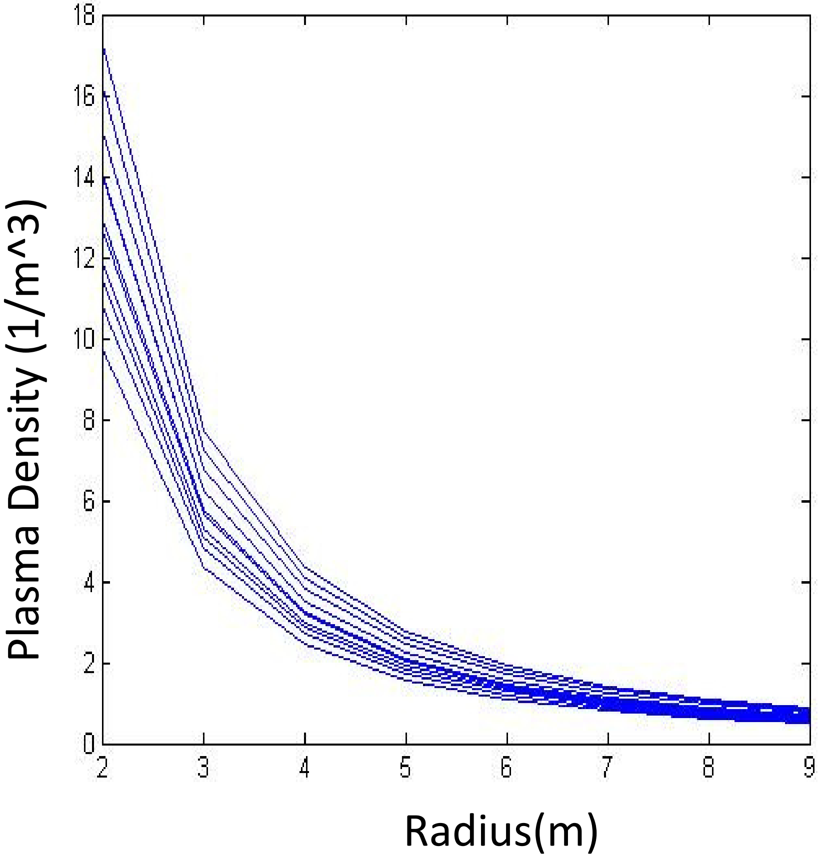
Christian Rey

Plasma Density

Correlations

- As Radius increases the Plasma density drops rather quickly
- Each line corresponds to a different current and as the current is increased so does the plasma density value
- In order to scale these values the plasma density value is divided by 10^20
- An increased plasma density greatly raises all calculated power values, and raises beta p and beta t values, the draw back being that the confinement times are greatly reduced

Plasma Density vs Radius

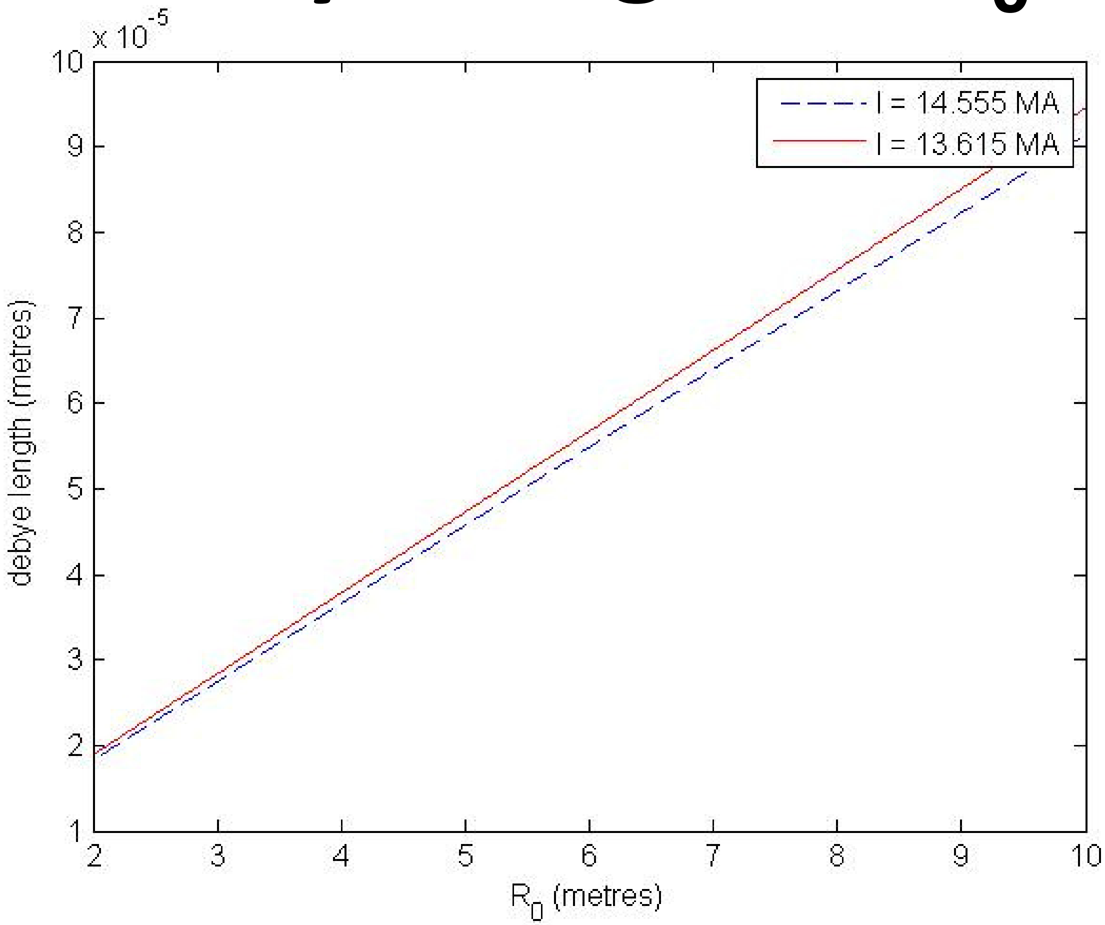


Timothy Seager

Debye Length

- $\lambda_e = \sqrt{\frac{\epsilon_0 k_B T_{e(K)}}{n_0 e^2}} = \sqrt{\frac{\epsilon_0 T_{e(eV)}}{n_0 e^2}} = 5.9 \cdot 10^{-5} - 6.1 \cdot 10^{-5}$ m
  - where k_B is Boltzmann constant
  - $n_0 = n_{e20} \cdot 10^{20}$ is the plasma density.
- This is also referred to as the *e*-folding distance of the shielded potential. This means that the electrostatic potential decreases *e* times at debye length.
- Most clothes have very small debye length for visible light and that’s why they are opaque. The same is not true for infrared however.

Debye Length vs R_0



Debye length vs. R_0 plotted for two bounds on the value of I.

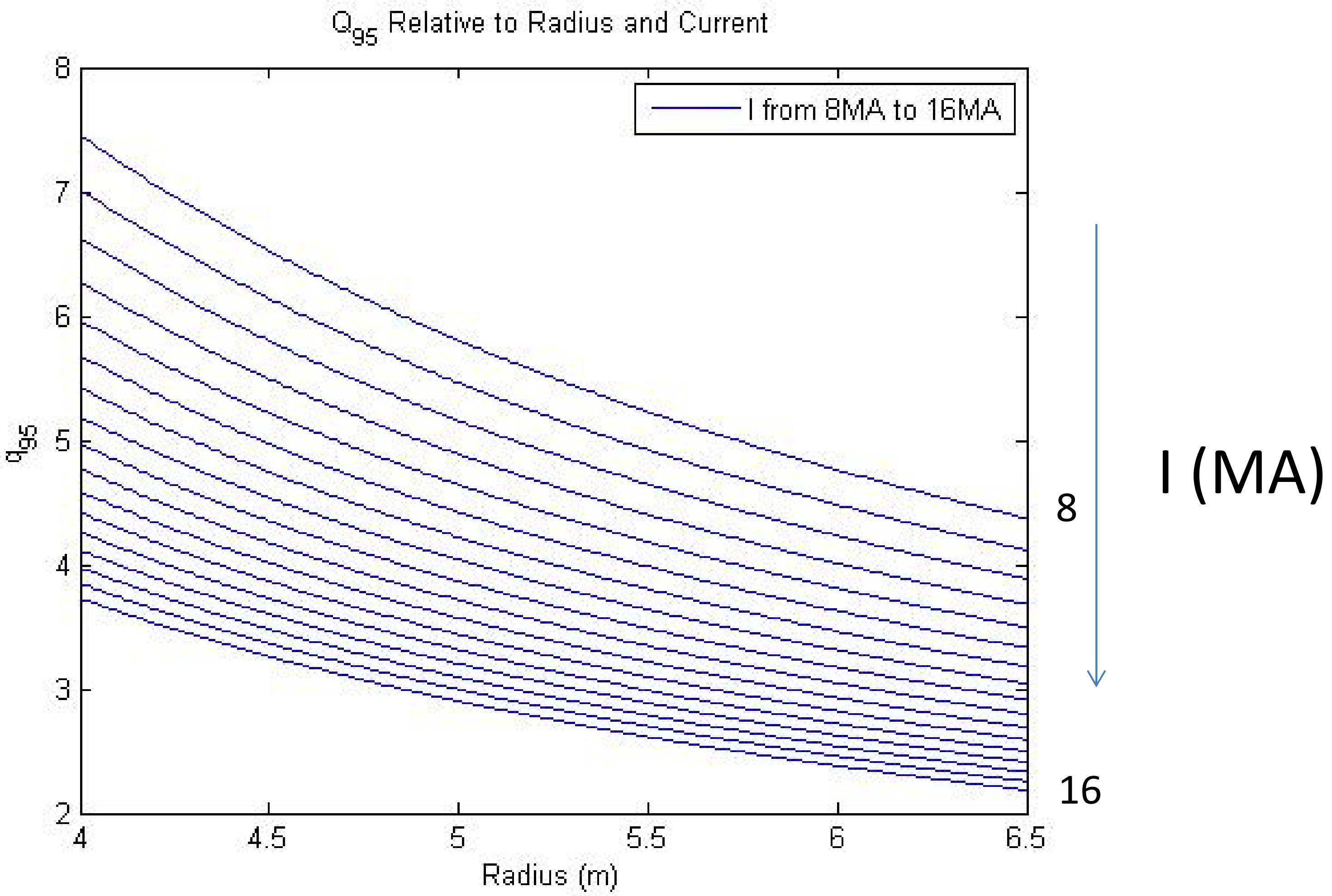
Fusion Plasma requires $\lambda_e \sim 10^{-4} - 10^{-5}$

Aishwarye Chauhan

Safety Factor

- Since the triangularity is assumed to be 0, the safety factor can be simplified:
    - $q_{95} = \frac{5a^2B}{2RI}(1+\kappa^2)\left(1+\frac{2}{3}\left(\frac{a}{R}\right)^2\right)$
- Combined limit of ballooning and kink modes
- Kink mode: distortion of the shape of the plasma, characterized by a sharp bend
- Ballooning mode: local bulges in the plasma due to imbalanced magnetic pressures

Q_{95} Graph vs R_0



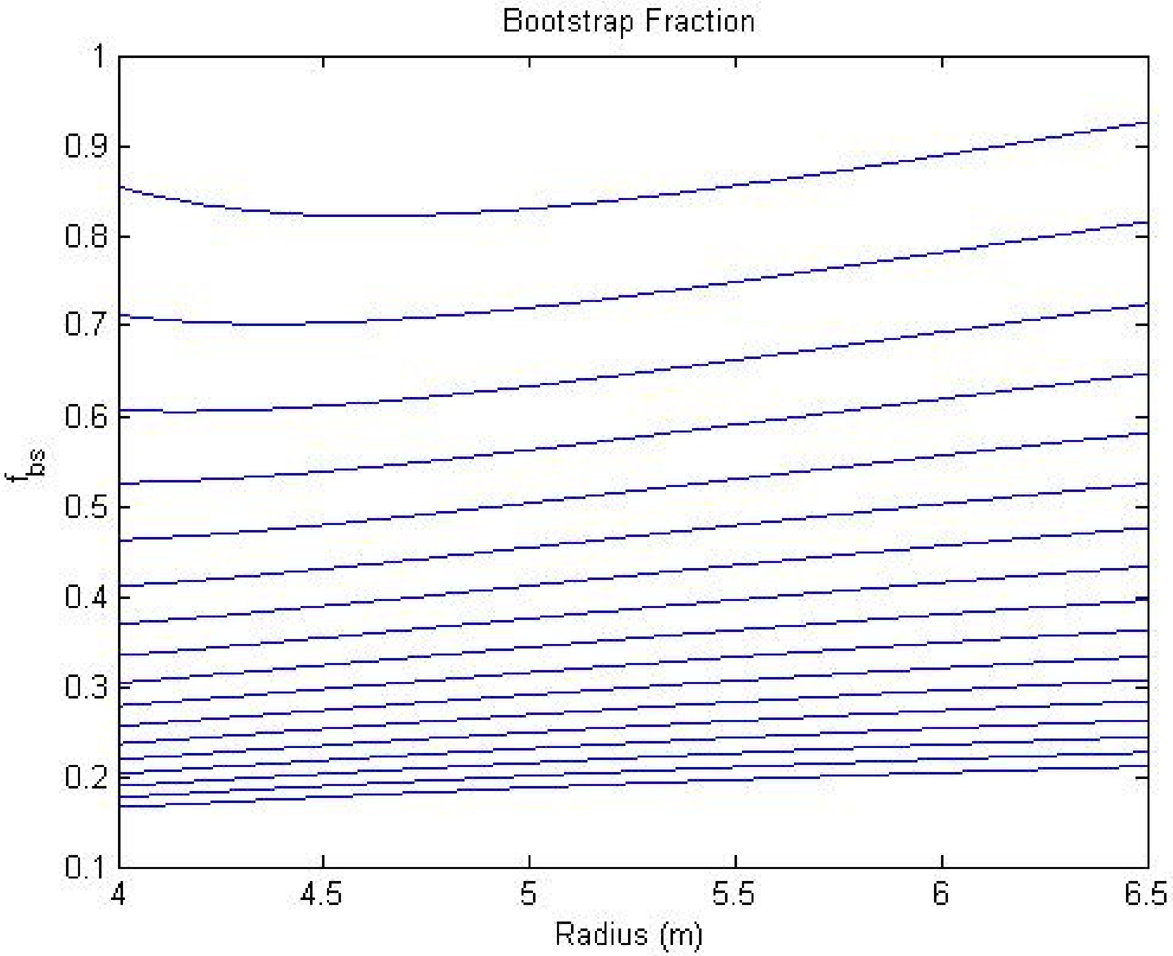
Jimmy McGowan

Bootstrap Current Fraction ($\boldsymbol{f_{bs}}$)

- Currents driven by plasma pressure gradients
- Provides a constant power output, rather than in pulses
- Current Fraction is the amount of current driving the plasma that is supplied by bootstrap effects
- Some reactors can reach around $f_{bs} = 0.9$
- Our reactor: $f_{bs} = 0.75$

Jimmy McGowan

Bootstrap Current



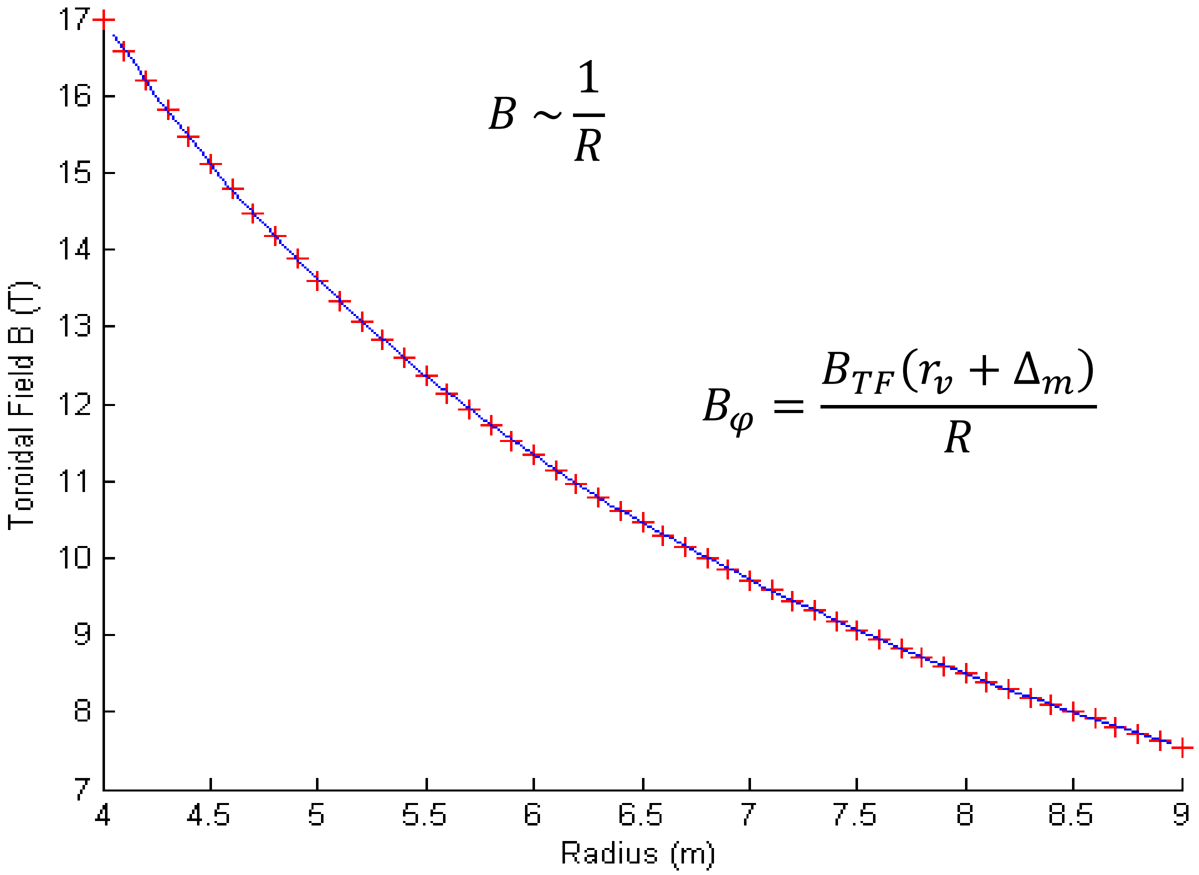
Jimmy McGowan

Toroidal Magnetic Field

- $B_\varphi = B_{TFC}\left(1 - \frac{r_w + \Delta_{BS}}{R_0}\right) = 11.249\ T$
  - $B_{TFC} = 20$ T represents the magnetic field in the toroidal F.C's
  - $r_w \approx a + 0.01 = 1.385\ m$ represents the radius of the plasma chamber from the center of the plasma to the first wall (on the plasma's minor axis)
  - $\Delta_{BS} = \Delta_B + \Delta_S = 0.67 + 059 = 1.26\ m$ represents the total widths of the shield and blanket (again on the plasma's minor axis)

B_φ decreases with increasing R_0.

Toroidal Magnetic Field

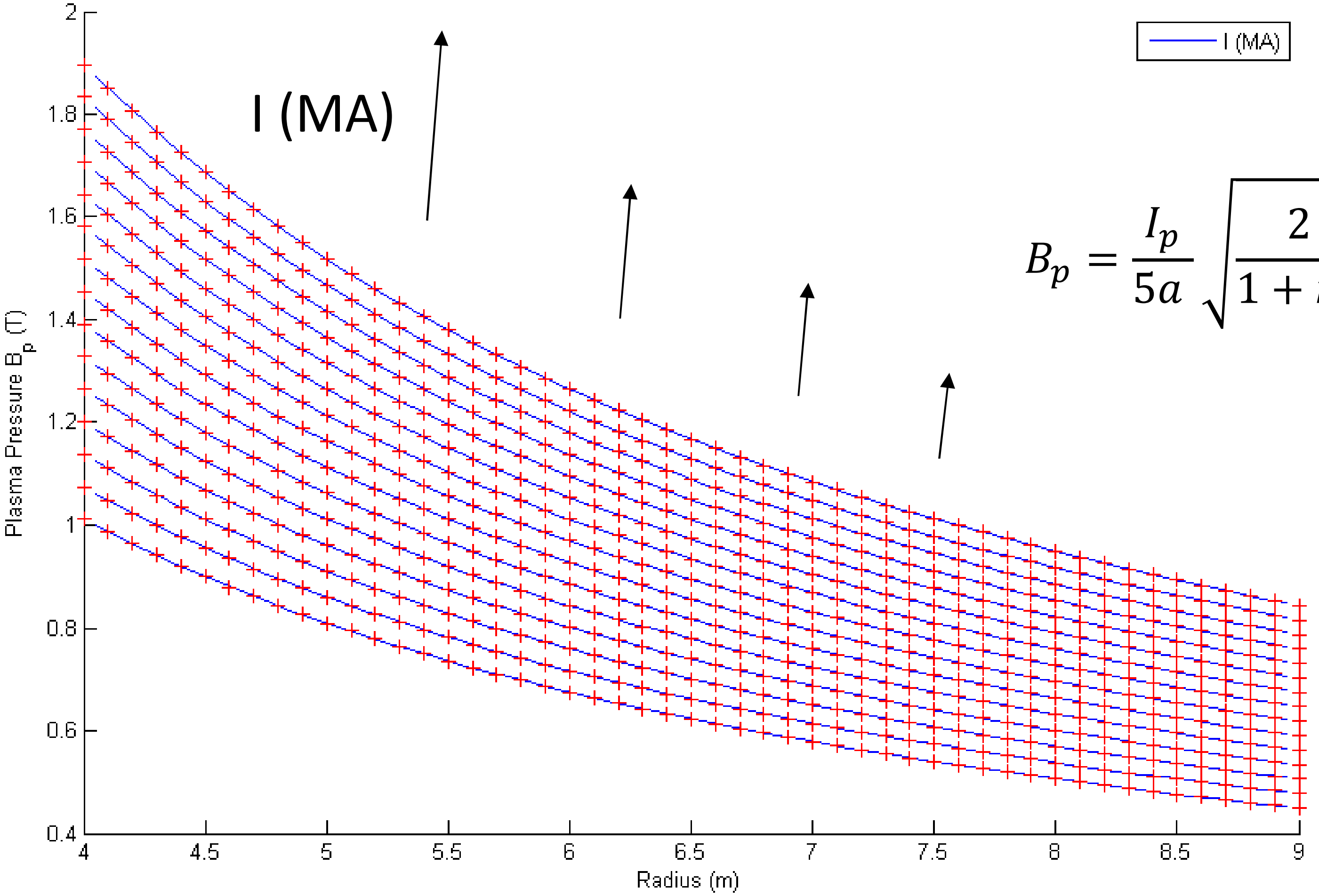


$$B \sim \frac{1}{R}$$

$$B_\varphi = \frac{B_{TF}(r_v + \Delta_m)}{R}$$

Christian Rey

Plasma Pressure

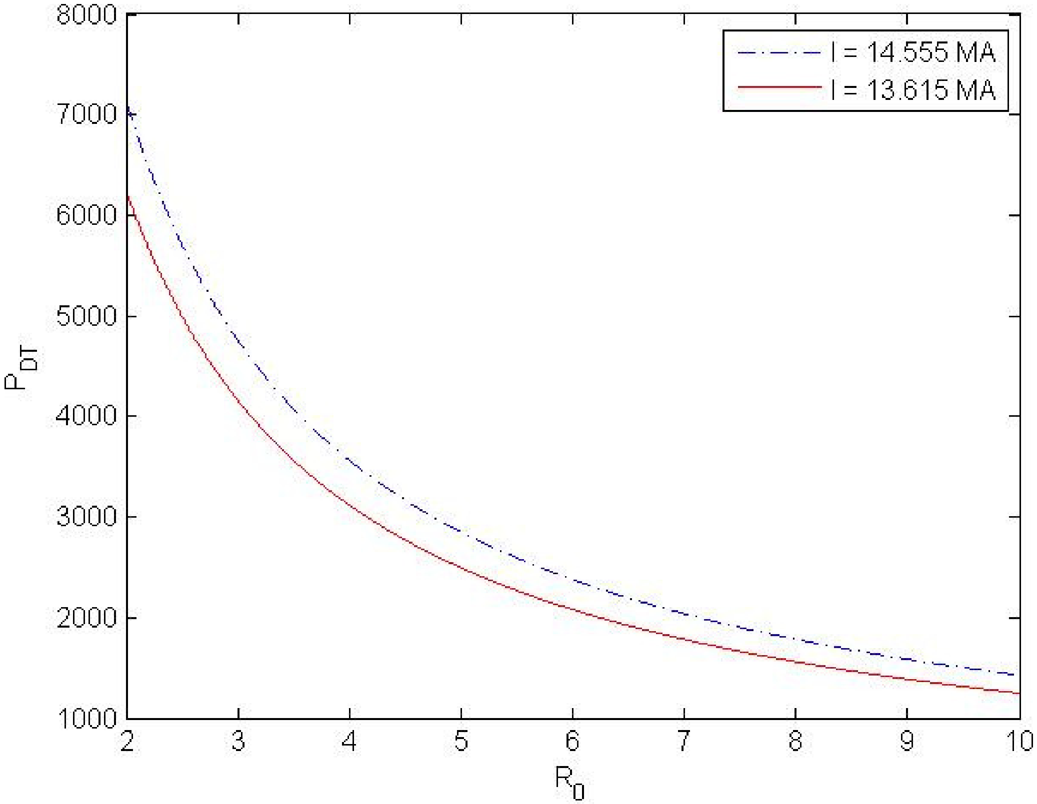


$$B_p = \frac{I_p}{5a}\sqrt{\frac{2}{1+\kappa^2}}$$

Christian Rey

Fusion Power (P_{DT})

- The fusion power for a 50-50 D- T plasma with elliptical cross section:
  - $P_{DT} = \frac{1}{4} n^2 \langle \sigma v \rangle U_{fus} V_{TP}$
    - where V_{TP} is the total plasma volume, and
    - U_{fus} is the total energy release by a fusion reaction
- For a constant value of I , P_{DT} is inversely proportional to R_0



Fusion Power (P_{DT}) vs. R_0

Increasing value of R_0 leads to very rapid decrease in the Fusion Power.

Aishwarye Chauhan

REFERENCES

[1] Stacey, Weston M. *Fusion: An Introduction to the Physics and Technology of Magnetic Confinement Fusion*. Weinheim: Wiley-VCH, 2010. Print.