Plasma Physics Performance

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Objectives

- Ensure adequate plasma power and H-mode operation with a reasonable confinement time.

- Maintain plasma stability against various magnetohydrodynamic (MHD) instabilities when reactor is operating.

- Determine plasma performance under steady-state operation of tokamak.

Assumed Parameters

- Taken from a variety of sources, including the textbook(s), academic reports and other DEMO studies.
    - I = 12 MA (ITER: 15 MA)
    - $q_0 = 1$
    - $T_i = T_e = 12.5$ keV (ITER: 10.5 keV)
    - $n_e = n_i = 1\text{x}10^{20}\ \text{m}^{-3}$ (ITER: $1.3\text{x}10^{20}\ \text{m}^{-3}$)

- Parameters chosen to optimize stability and power requirements within current physics capabilities.

Plasma Geometry

- Aspect Ratio:

$$A = \frac{R_0}{a}$$

- Elongation:

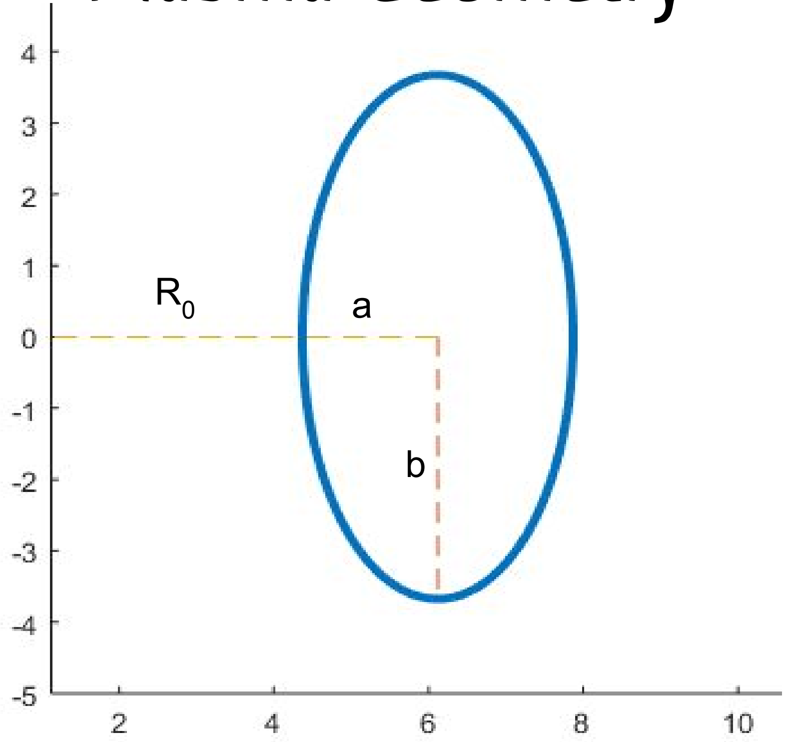
$$\kappa = \frac{b}{a}$$

- Plasma Volume:

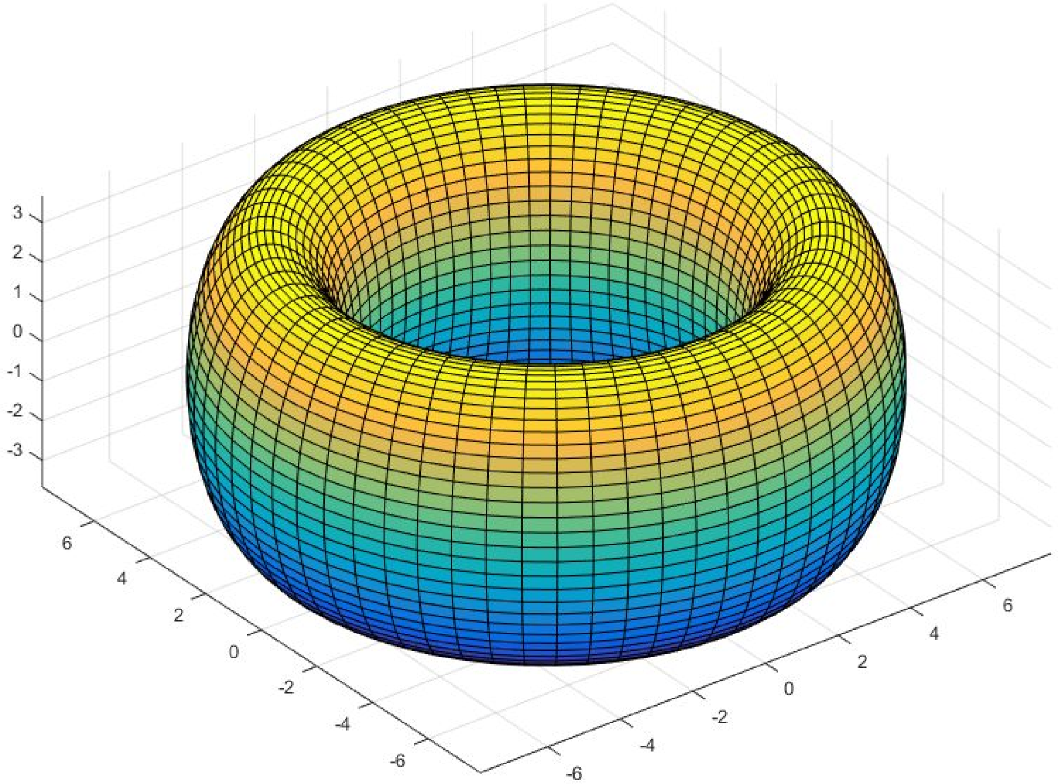
$$V_P = (\pi ab)(2\pi R_0)$$

GEOMETRY	**DEMO**	**ITER**
R_0 (*m*)	6.0	6.2
a (*m*)	1.75	2
K	2.1	1.7-1.85
delta	0	0.33-0.49
alpha	3.33	>3.1
A	3.43	3.1
b (*m*)	3.68	3.4-3.7
Plasma V (m^3)	761.7	840

Plasma Geometry



$$\kappa = \frac{b}{a} = 2.1$$



$$V_P = (\pi ab)(2\pi R_0) = 761.7 m^3$$

D-T Fusion Reactivity

$$\langle \sigma v \rangle = 10^{-6} \times C1 \times \theta \times e^{-3\xi} \times \left(\frac{\xi}{m_r c^2 T^3} \right)^{0.5} = 1.91 \cdot 10^{-22} \frac{m^3}{s}$$

- Reactivity is essential to fusion power.

$$\theta = \frac{T}{1.0 - \frac{T[C2 + T(C4 + T \times C6)]}{1.0 + T[C3 + T(C5 + T \times C7)]}}$$

$$\xi = \left(\frac{(B_G)^2}{4\theta} \right)^{1/3}$$

- Within 10% of graphical value.

Plasma Power

$$\mathrm{V_P} = (\pi \mathrm{ab})(2\pi \mathrm{R_0}) = 762\mathrm{m}^3$$

$$\mathrm{Rate} = \frac{1}{4}\mathrm{n}^2\langle\sigma\nu\rangle = 1.45 \cdot 10^{21}\mathrm{rxns/sec}$$

$$\mathrm{E_f} = 17.6\ \mathrm{MeV} = 17{,}600\ \mathrm{keV}$$

$$\mathrm{P_f} = \mathrm{Rate} \cdot \mathrm{V_P} \cdot \mathrm{E_f} \cdot \left(1.6 \cdot 10^{-16}\right) = 4100\ \mathrm{MW} > 1000\ \mathrm{MW}$$

Impurity Content

- ITER - 2% Beryllium
- Radiation cooling: predominant effect of impurity on plasma.
- Bremsstrahlung: power loss per unit volume from the electron acceleration.

$$P_{brems}(MW) = 4.8 \cdot 10^{-43} z^2 n_z(m^{-3}) n_e(m^{-3}) T_e^{\frac{1}{2}}(keV) = 4.1$$

$$P_{brems,ITER}(MW) = \ 4.1$$

- Concentrations up to ~10% of low-z impurities may be tolerable in the plasma.
- Magnetic divertors significantly reduce impurity content.

Fusion Power Balance

- Power balance: fusion-alpha power and external heating must be greater than radiation and transport power losses.

$$\frac{1}{4}\mathrm{n}^2\langle\sigma\nu\rangle_{\mathrm{f}}\mathrm{U}_\alpha + \mathrm{P}_\Omega \geq \mathrm{P}_{\mathrm{Rad}} + \mathrm{P}_{\mathrm{Trans}}$$

$$369\ \mathrm{MW} > 338\ \mathrm{MW}$$

Greenwald Density Limit

- Operational limit on central line-averaged density of plasma. Derived from plasma current and area.

$$\overline{n_G} = \overline{n_{e20}} = \frac{I_P}{\pi a^2} = 1.25 \cdot 10^{20} m^{-3}$$

- High current density is important for high plasma density; if plasma density (n_e) exceeds Greenwald (n_G), the current can quench disruptively.

$$\overline{n_e} = 1.0 \cdot 10^{20} m^{-3}$$

- Plasma density goal: 80-85% of the Greenwald Limit.

ITER Greenwald Limit

- ITER Greenwald density:

$$\overline{n_G} = 0.85 \cdot 10^{20} m^{-3}$$

- ITER nominal value:

$$\overline{n_e} = 1.3 \cdot 10^{20} m^{-3}$$

- Nominal plasma density within 75% of the nominal ITER value and under the Greenwald Limit.
- ITER nominal within factor of 2 of Greenwald.

Plasma Stability

- Tokamak plasmas subjected to several modes of instability.
- Mainly considered MHD instabilities
  - Ballooning
  - Kink
  - Flute
- Heavily related to beta: ratio of magnetic to kinetic pressure
- Safety Factor q_{95}: ratio of toroidal to poloidal fields

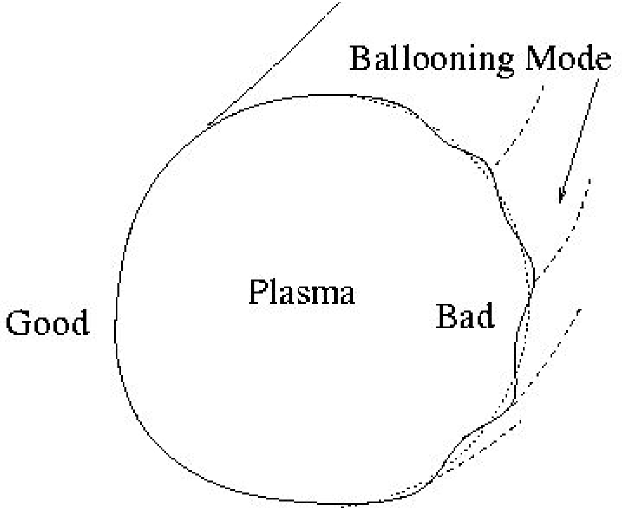
Kink Instabilities

- Occurs because the plasma is toroidally shaped, so poloidal field is stronger on the inside of the bend than on the outside. Suppressed by sufficiently strong toroidal field.
- Internal kink modes stabilized with q(o) - must be at least 1.
- External/surface kink modes stabilized by ensuring $q_{95} > 3q_0$:

$$q_{95} = \frac{5a^2B}{R_0I_P}\left(\frac{1+\ \kappa^2}{2}\right)\frac{\left(1.17-\ \frac{0.65}{A}\right)}{\left(1-\ \frac{1}{A^2}\right)^2} = 3.70 > 3.0$$

Ballooning Instabilities

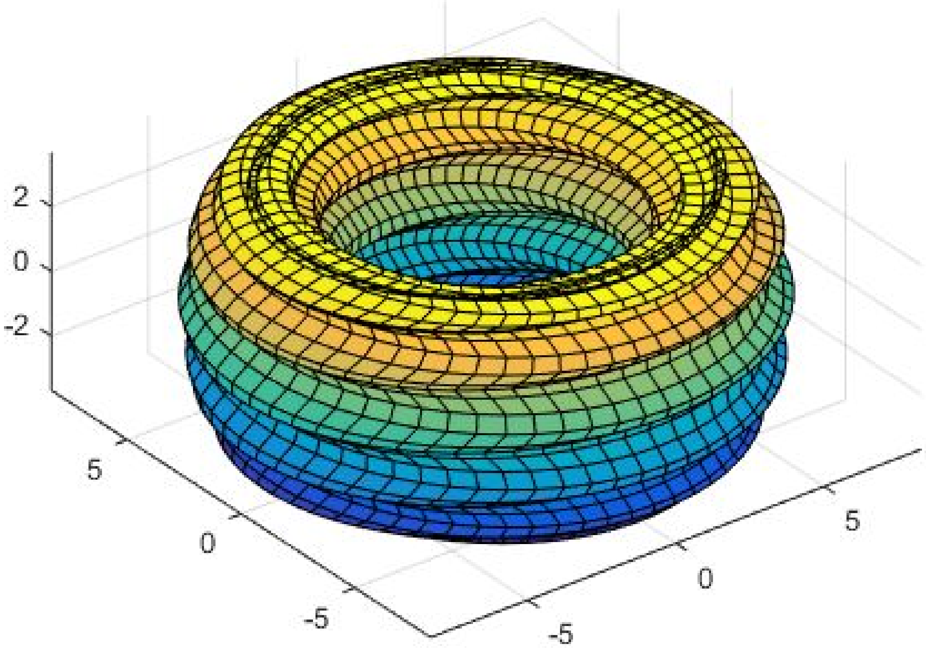
- Sufficiently large plasma pressure can induce critical pressure gradient and distort the plasma. (Kinetic pressure larger than poloidal magnetic pressure)
- Localized bulging in the plasma must be subdued



$$\beta = 1.6 < \beta_N \left(\frac{I_P}{aB}\right) = 6.9$$

Interchange/Flute Instabilities

- Common in tokamak plasmas when the magnetic field is concave towards the plasma (i.e. on the outboard side of tokamaks).
    - Due to pressure gradient - requires shear term to stabilize (Suydam Criterion)



$$\frac{dp}{dr} + \frac{rB_\Phi^2}{8\mu_0}\left(\frac{1}{q}\frac{dq}{dr}\right)^2 > 0$$

Confinement Time

- With sufficient power, edge transport is reduced, resulting in a doubling of confinement time. High confinement condition depends on amount of non-radiative power flowing across the last closed flux surface.

- Time for plasma particle to escape the last closed flux surface.

$$\tau_E^{IPB98} = 0.0562 I^{0.93} B^{0.15} P^{-0.69} M^{0.19} R^{1.97} A^{-0.58} \bar{n}_{e20}^{0.41} \kappa^{0.78} = 2.73s$$

$$\tau_{E,ITER}^{IPB98} = 6.2\ s$$

- H-mode confinement time is double that of L-mode.

H-Mode Threshold Power

- **P_{LH}**: threshold power required for H-Mode operation; if the power flux across the LCFS surpasses the threshold, the plasma is in H-mode operation. If the power flux fails to meet this threshold, it is in L-mode.

$$P_{LH}(MW) = \left(\frac{2.84}{M}\right) B^{0.82} \bar{n}_{e20}^{0.58} R a^{0.81} = 44.3$$

$$P_{LH,ITER}(MW) = 39.6$$

H-Mode Operation

- **P_{TR}**: power flux produced by the plasma that reaches the wall of the chamber across the last closed flux surface (LCFS).

$$P_{TR}\,(MW) = \left(\left[0.048\,\frac{T_e}{a^2}\right] + \left[\frac{3\bar{n}_e T_e (1.60218 \cdot 10^{-22})}{\tau_E}\right]\right) V_{plasma} = 321.4$$

$$P_{TR,ITER}\,(MW) = 61.7$$

Edge-Localized Modes

- ELM: intermittent, significant pulsed flow of particles across separatrix
  - May involve significant ejection of confined plasma energy - up to 10% of the plasma energy.

- Power across LCFS (P_{TR}) is above 120% of the L-H threshold power (P_{LH}): Type I ELMs
  - isolated, large amplitude: ejection of up to 10% of confined plasma energy
  - Up to 20% degradation in energy confinement time
  - Worst-case confinement time: $\tau_E' = 0.8 \cdot \mathrm{T}_E^{\mathrm{IPB98}} = 2.16\ s$
  - Produces a high heat flux on the divertor

Electron-Ion Collision Frequency

$$\Lambda = 12\pi\left[\frac{(\epsilon_0 kT)^3}{n_2 e_2^4 e_1^2}\right]^{1/2}$$

- Electrons transfer energy to ions at a specific frequency: number of electron-ion collisions per unit time.

$$\bar{v}_{ei} = \frac{2}{3(2\pi)^{1/2}} n_i \left(\frac{ze^2}{4\pi\epsilon_0}\right)^2 \frac{4\pi}{(m_e)^{1/2}(T_e)^{3/2}} ln(\Lambda) = 4.5 \times 10^3\ s^{-1}$$

Collisionality

- The collisionality expresses the electron-ion interaction in terms of their mean free path around the poloidal magnetic field

$$\nu^* = \bar{\nu}_{ei}\sqrt{\frac{m_i}{T_{ei}}}\epsilon^{-3/2}qR = 0.9$$

- ITER has $\nu^* < 0.2$, making the ELM a factor of two too large; $\nu^* = 0.9$ gives an approximately tolerable ELM size.

Electron/Ion Plasma Frequencies

- Frequencies at which electrons and ions oscillate within the plasma

$$\omega_{pi} = \left(\frac{4\pi n_i Z^2 e^2}{m_i}\right)^{1/2} = 104{,}000\,\frac{\mathrm{rad}}{s}\,(\mathrm{ITER}{:}\ 119{,}000\,\frac{\mathrm{rad}}{s})$$

$$\omega_{pe} = \left(\frac{4\pi n_e e^2}{m_e}\right)^{1/2} = 5.6 \cdot 10^{14}\,\frac{\mathrm{rad}}{s}\,(\mathrm{ITER}{:}\ 6.43 \cdot 10^{14}\,\frac{\mathrm{rad}}{s})$$

Electron/Ion Gyro-frequencies

- Angular frequency of particle motion - perpendicular to magnetic field

$$\Omega_\mathrm{e} = \frac{\mathrm{eB}}{\mathrm{m_e}} = 9.7 \cdot 10^{7}\, \frac{\mathrm{rad}}{s} \;\; (\mathrm{ITER{:}}\; 9.3 \cdot 10^{7}\, \frac{\mathrm{rad}}{s})$$

$$\Omega_\mathrm{i} = \frac{\mathrm{eB}}{\mathrm{m_i}} = 7.5 \cdot 10^{10}\, \frac{\mathrm{rad}}{s} \;\; (\mathrm{ITER{:}}\; 7.2 \cdot 10^{10}\, \frac{\mathrm{rad}}{s})$$

Debye Length

- Distance within which electrostatic forces/potentials are non-negligible.

$$\lambda_D = 743\sqrt{T/n} = 8.3 \cdot 10^{-6}\text{m (ITER: } 6.68 \cdot 10^{-6}\text{m)}$$

- Number of particles within a sphere of radius equal to Debye length:

$$\# = \frac{4}{3}\pi\lambda_D^3 n = 240{,}115 \text{ particles (ITER: } 162{,}131)$$

Conclusion

- All performance parameters comparable to those of ITER and other DEMO reactors.

- This design should be able to function as a steady-state reactor with regards to plasma performance.

Questions

Resources

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Resources

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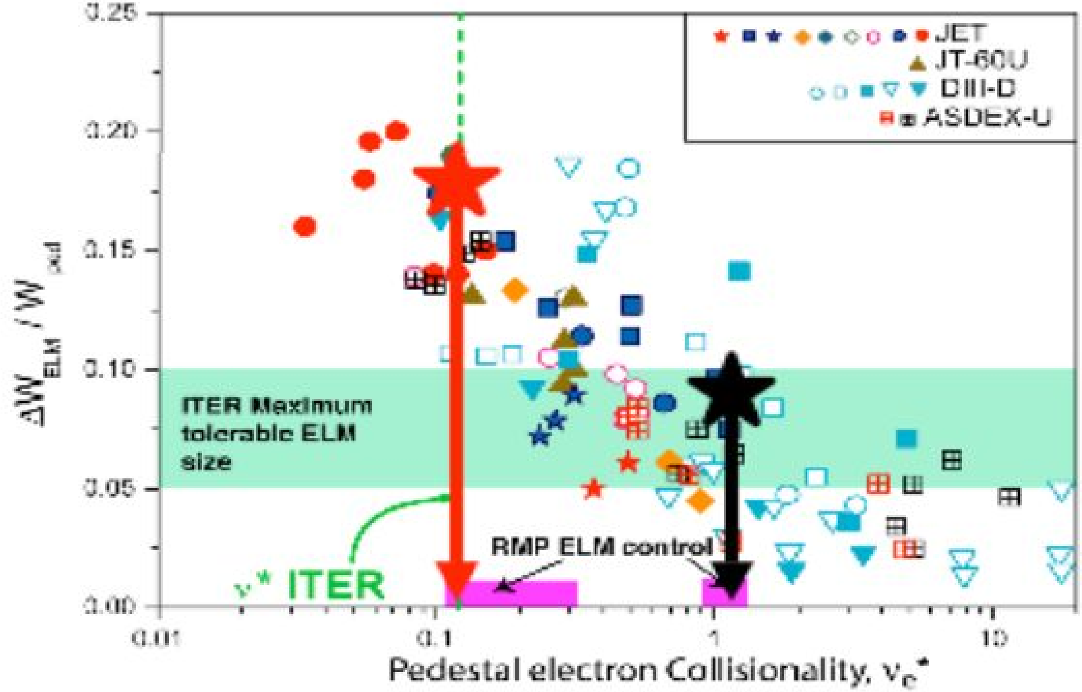
Collisionality



D-T Reactivity Fitting Vectors

Fitting Vectors:	
C1	1.17E-09
C2	1.51E-02
C3	7.52E-02
C4	4.61E-03
C5	1.35E-02
C6	-1.07E-04
C7	1.37E-05
B_G (keV^0.5)	34.3827
rest mass (keV)	1124656

Other Plasma Parameters

- **Plasma Internal Inductance:** MHD stability poses Beta_N should be proportional to l_i.

$$l_i = \ln(1.65 + 0.89(q_{edge} - 1)) = 1.38189$$

- **Beta_N:** Chosen as 3. Values of Beta_N much lower than 3.5 will lend the plasma to large "Neoclassical Tearing Modes" (NTM) that must be stabilized.

(sig_nu) (m3/s)	1.72E-22
n_i,e (10^20 m^-3)	1
n_e20 (10^20 m^-3)	1.247255064
T_i (keV)	12.5
T_e (keV)	12.5
Z_eff	2.186
tau_E,IP,B98 (s)	2.669258936
l_i	1.381887504
T (K)	144982369

PLASMA	
I_p (MA)	12
f_BS (%)	80
q_95	3.62
q_0	1
B_phi_tfc (T)	12
B_phi_0 (T)	5.5
B_theta (T)	1.37

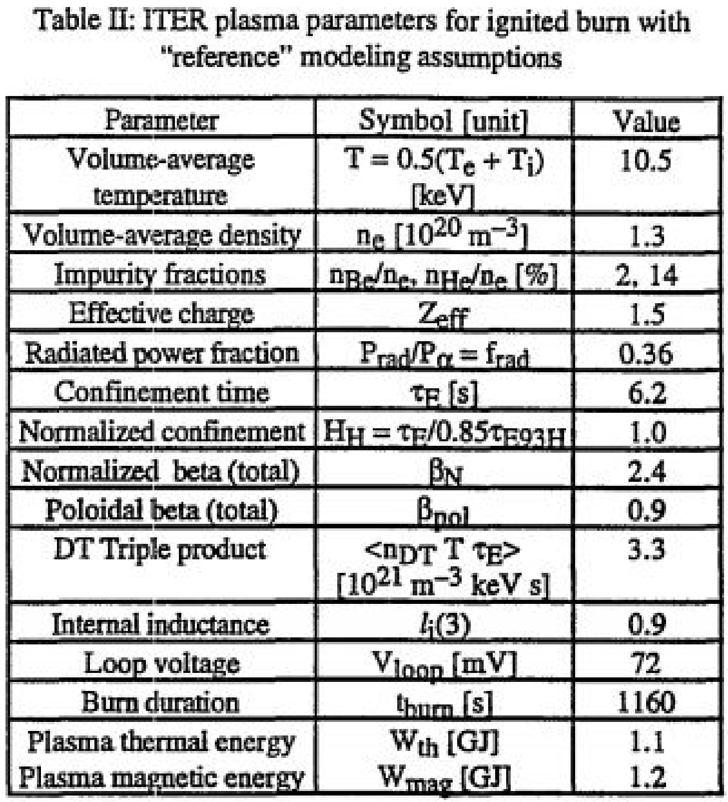
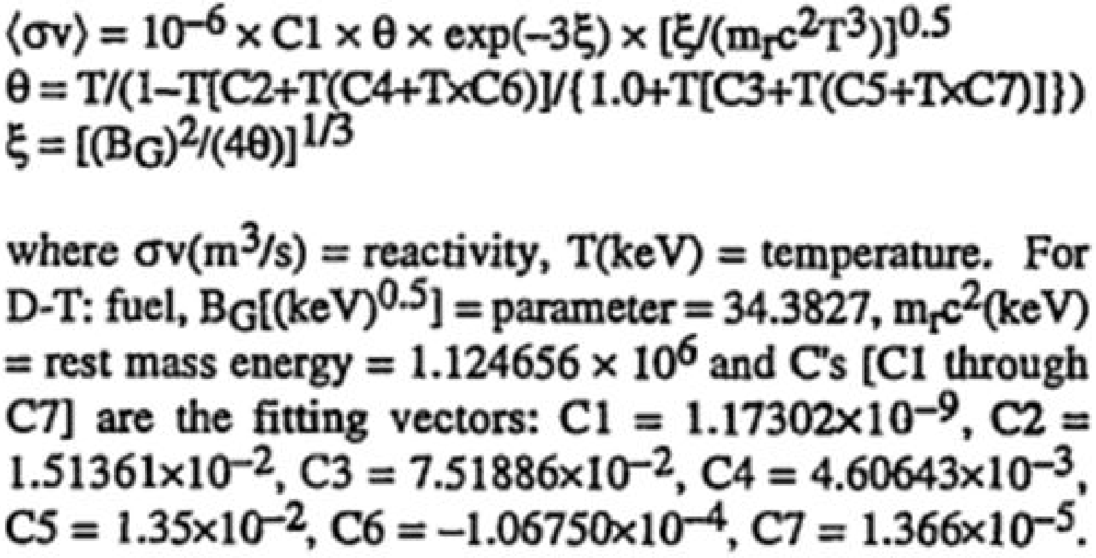


Table II: ITER plasma parameters for ignited burn with "reference" modeling assumptions

Parameter	Symbol [unit]	Value
Volume-average temperature	$T = 0.5(T_e + T_i)$ [keV]	10.5
Volume-average density	n_e [10^{20} m^{-3}]	1.3
Impurity fractions	n_{Be}/n_e, n_{He}/n_e [%]	2, 14
Effective charge	Z_{eff}	1.5
Radiated power fraction	$P_{rad}/P_\alpha = f_{rad}$	0.36
Confinement time	τ_E [s]	6.2
Normalized confinement	$H_H = \tau_E/0.85\tau_{E93H}$	1.0
Normalized beta (total)	β_N	2.4
Poloidal beta (total)	β_{pol}	0.9
DT Triple product	$<n_{DT}\ T\ \tau_E>$ [10^{21} m^{-3} keV s]	3.3
Internal inductance	$l_i(3)$	0.9
Loop voltage	V_{loop} [mV]	72
Burn duration	t_{burn} [s]	1160
Plasma thermal energy	W_{th} [GJ]	1.1
Plasma magnetic energy	W_{mag} [GJ]	1.2



$$\langle\sigma v\rangle = 10^{-6} \times C1 \times \theta \times \exp(-3\xi) \times [\xi/(m_r c^2 T^3)]^{0.5}$$
$$\theta = T/(1-T[C2+T(C4+T\times C6)]/\{1.0+T[C3+T(C5+T\times C7)]\})$$
$$\xi = [(B_G)^2/(4\theta)]^{1/3}$$

where $\sigma v(m^3/s)$ = reactivity, T(keV) = temperature. For D-T: fuel, $B_G[(keV)^{0.5}]$ = parameter = 34.3827, $m_r c^2$(keV) = rest mass energy = 1.124656×10^6 and C's [C1 through C7] are the fitting vectors: $C1 = 1.17302\times10^{-9}$, $C2 = 1.51361\times10^{-2}$, $C3 = 7.51886\times10^{-2}$, $C4 = 4.60643\times10^{-3}$, $C5 = 1.35\times10^{-2}$, $C6 = -1.06750\times10^{-4}$, $C7 = 1.366\times10^{-5}$.