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Citation: Physics of Plasmas **23**, 062515 (2016); doi: 10.1063/1.4954379
View online: http://dx.doi.org/10.1063/1.4954379
View Table of Contents: http://scitation.aip.org/content/aip/journal/pop/23/6?ver=pdfcov
Published by the AIP Publishing

PHYSICS OF PLASMAS **23**, 062515 (2016)

A fluid model for the edge pressure pedestal height and width in tokamaks based on the transport constraint of particle, energy, and momentum balance

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(Received 20 May 2016; accepted 9 June 2016; published online 23 June 2016)

A fluid model for the tokamak edge pressure profile required by the conservation of particles, momentum and energy in the presence of specified heating and fueling sources and electromagnetic and geometric parameters has been developed. Kinetics effects of ion orbit loss are incorporated into the model. The use of this model as a "transport" constraint together with a "Peeling-Ballooning (P-B)" instability constraint to achieve a prediction of edge pressure pedestal heights and widths in future tokamaks is discussed. *Published by AIP Publishing*. [http://dx.doi.org/10.1063/1.4954379]

I. INTRODUCTION

The high pressure pedestal over roughly the outer 5% of the confined magnetic flux[1] that is characteristic of the High Confinement regime[2] (H-mode) is often subject to magnetohydrodynamic (MHD) instabilities[3,4] known as Edge Localized Modes (ELMs) that would deposit unacceptably large pulses of energy and particles on the divertor target plates of future tokamaks (e.g., ITER[5]). ELMs are generally understood[4] to be coupled MHD peeling-ballooning (P-B) modes, with the ballooning modes driven by the large edge pressure gradient, and the kink "peeling" modes driven by the large edge bootstrap current produced by this large pressure gradient. ELMs and their mitigation or pre-emption have been (e.g., Refs. 6–9) and remain a major area of fusion plasma physics research worldwide.

A model for the pressure profile in the edge pedestal based upon equating the "transport" constraints of particle, momentum, and energy balance (which determine the pressure profile) and a localized ballooning mode type of critical pressure gradient "P-B" constraint (which sets an instability limit on the pressure profile) was suggested a dozen years ago.[10] Subsequently, a global P-B constraint has been developed[11,12] as a limiting relation between the critical pressure at the top of the pedestal and the width of the edge pressure pedestal, and the transport physics of the edge pedestal has been extended to take into account electromagnetic forces,[13] the ion orbit loss of particles, energy, and momentum[14–16] and the calculation of the radial electric field.[17,18] The purpose of this paper is to present a new "transport" constraint for the edge pedestal pressure height and width that may be combined with the new "P-B" constraint[11,12] to constitute a "first-principles" model for the pressure profile in the tokamak edge pedestal.

II. TRANSPORT CONSTRAINT ON EDGE PEDESTAL HEIGHT AND WIDTH

The pressure profile in the edge plasma must satisfy the conservation of particles, energy, and momentum.

The radial particle continuity equation for the main ion species "j" (in the cylindrical approximation) is

$$\frac{1}{r}\frac{\partial\left(r\Gamma_{rj}(r)\right)}{\partial r} = N_{nbj}(r)(1-f_{nbj}^{iol}(r)) + n_e(r)\nu_{ionj}(r) - 2\frac{\partial F_j^{iol}(r)}{\partial r}\Gamma_{rj}(r), \tag{1}$$

where N_{nbj} and f_{nbj}^{iol} are the fast neutral beam source rate and the differential ion-orbit loss for fast beam ions of species "j," n_e is the electron density, ν_{ionj} is the ionization frequency of neutrals of species "j," $F_j^{iol}(r)$ is the cumulative ion-orbit loss of thermalized particles of species "j" over $0 < r' < r$ and Γ_{rj} is the outward radial particle flux of thermalized ions of species "j." The kinetic calculation of the ion-orbit loss parameters is based on conservation of canonical angular momentum, energy and magnetic moment and is described in Refs. 14–16. Equation (1) has the solution

$$r\Gamma_{rj}(r) = \int_0^r [N_{nbj}(1-f_{nbj}^{iol}) + n_e\nu_{ionj}]e^{-2[F_j^{iol}(r)-F_j^{iol}(r')]}r'dr'. \tag{2}$$

At steady-state, the fluid radial particle flux at any radius consists of all the thermalized source particles deposited out to that radius, but attenuated by the kinetic loss of outflowing particles that are able to access loss orbits that carry them out of the plasma.

Similarly, the radial energy balance equation for the main ion species is

$$\frac{1}{r}\frac{\partial\left(rQ_{rj}(r)\right)}{\partial r} = q_j^{nbi}(r)\left(1-\alpha e_{nbi}^{iol}(r)\right) - q_{je}(r) - n_j(r)n_{oj}(r)\langle\sigma\upsilon\rangle_{cx}\frac{3}{2}\left(T(r)_j - T_{oj}\right) - \frac{\partial E_j^{iol}(r)}{\partial r}Q_{rj}(r) \tag{3}$$

with solution

$$rQ_{rj}(r) = \int_0^r \left(q_j^{nbi}\left(1-\alpha e_{nbi}^{iol}\right) - q_{je} - n_j n_{oj}\langle\sigma\upsilon\rangle_{cx} \times\frac{3}{2}\left(T_j - T_{oj}\right)\right)e^{-\left[E_j^{iol}(r)-E_j^{iol}(r')\right]}r'dr', \tag{4}$$

[a)]Electronic mail: weston.stacey@nre.gatech.edu where Q_{rj} is the total radial energy flux of ions of species "j," q_j^{nbi} is the neutral beam (or other) heating rate of ion species "j," $\langle\sigma\upsilon\rangle_{cxj}$ is the charge exchange rate coefficient for species "j," $T_j = p_j/n_j$ (where p_j is the pressure) is the temperature, E_j^{iol} and e_j^{iol} are the ion orbit energy loss fractions for thermalized (cumulative) and fast beam (differential) ions of species "j," and the subscript oj refers to the neutral atoms of species "j." A calculation for MAST found $\alpha \simeq 0$ for co-current beam injection and $0.5 < \alpha < 1.0$ for counter-current beam injection.[19]

The pressure gradient for species "j," is determined from the radial momentum balance

$$\frac{1}{r}\frac{\partial(rp_j)}{\partial r} = n_j e_j E_r + n_j e_j (V_{\theta j}B_\varphi - V_{\varphi j}B_\theta) \tag{5}$$

and the toroidal momentum balance equations

$$n_j m_j(\nu_{jk} + \nu_{dj})V_{\varphi j} - n_j m_j \nu_{jk} V_{\varphi k} = n_j e_j E_\varphi^A + e_j B_\theta \Gamma_{rj} + M_{\varphi j} \tag{6}$$

and similar equations for the other impurity ion species "k." The ν_{jk} is an interspecies collision frequency, the ν_{dj} is a momentum loss frequency due to viscosity plus inertia plus charge-exchange, $M_{\varphi j}$ is the toroidal momentum source rate, m_j is the mass, and E, B, and V are the electric and magnetic fields and the fluid velocity, and the A superscript indicates the component of the field due to the time-dependent magnetic potential. Equations (5) and (6) and similar equations for the impurity species "k" may be solved for the ion pressure gradient "transport" constraints

$$\left(-\frac{1}{r}\frac{\partial(rp_j)}{\partial r}\right) = \frac{(e_j B_\theta)^2}{m_j(\nu_{jk} + \nu_{dj})}\left[\Gamma_{rj} - \Gamma_j^{pinch}\right] \tag{7}$$

where

$$\Gamma_j^{pinch} = -\frac{n_j E_\varphi^A}{B_\theta} - \frac{M_{\varphi j}}{e_j B_\theta} + \frac{n_j m_j(\nu_{jk} + \nu_{dj})}{e_j B_\theta} \times \left(\frac{E_r}{B_\theta} + \frac{B_\varphi}{B_\theta}V_{\theta j}\right) - n_j m_j \nu_{jk} V_{\varphi k} \tag{8}$$

is a radial "pinch" flux associated with electromagnetic, external, and friction forces.

Equation (7) can be integrated from some interior edge location "r" outward to the separatrix to obtain

$$rp_j(r) - r_{sep}p_j^{sep} = \int_r^{rsep} \frac{(e_j B_\theta)^2\left(\Gamma_{rj} - \Gamma_j^{pinch}\right)}{m_j(\nu_{jk} + \nu_{dj})} r' dr'. \tag{9}$$

A similar set of equations obtains for the impurity species "k" but with the "j" and "k" subscripts interchanged. The electron pressure can then be estimated from charge neutrality.

Equation (9) identifies the variables upon which the pressure profile in the edge pedestal depends. The radial particle flux $\Gamma_{rj} > 0$ is given by Eq. (2), which indicates the intuitive result that increasing the neutral beam or gas fueling/recycling particle source will increase the pressure in the plasma edge. The second term, the particle pinch flux, can increase the edge pressure if inward, $\Gamma_j^{pinch} < 0$, or decrease the edge pressure if outward, $\Gamma_j^{pinch} > 0$. Examination of the pinch flux in DIII-D just before and after an L-H transition[20,21] reveals that Γ_j^{pinch} is weakly inward in the L-mode just prior to the transition but increases by an order of magnitude to become strongly inward in the H-mode, with this change being primarily due to the change in the E_r component from positive in the L-mode to strongly negative in the H-mode and the change in the $V_{\theta j}$ component from moderately inward in the L-mode to strongly inward in the H-mode.

Multiplying Eq. (5) by e/m and summing over main ions "j," impurity ions "k" and electrons leads to an Ohm's Law[17,18] for the determination of

$$E_r = -\eta j_r - \left\{\left[\frac{(V_{\theta j}B_\varphi - V_{\varphi j}B_\theta)}{(1 + n_k m_k/n_j m_j)}\right] - \left[\frac{(V_{\theta k}B_\varphi - V_{\varphi k}B_\theta)}{(1 + n_j m_j/n_k m_k)}\right]\right\} - \left\{\frac{\left(p_j L_{pj}^{-1} + p_k L_{pk}^{-1}\right)}{e(n_j + z_k n_k)}\right\}, \tag{10}$$

where η is the Spitzer resistivity and this first term is generally small compared to the other two terms. The second term is the motional electric field and is a dominant term in several DIII-D discharges that have been examined.[18] In the last term $L_{pj}^{-1} \equiv (-\partial p_j/\partial r)/p_j$. This expression, when evaluated with rotation velocities determined from experiment, agrees very well[18] with the conventional "experimental" electric field determined by using measured carbon density, temperature, and rotation velocity in the radial momentum balance of Eq. (5).

Thus, we conclude that the poloidal and toroidal rotation velocities are important in determining the pressure in the edge pedestal, contributing both directly and indirectly via their contribution to the radial electric field given by Eq. (10) to the pinch flux of Eq. (8). Unfortunately, present neoclassical models for the calculation of fluid rotation velocities are not in good agreement with measurements.[18] It is further noted that the ion orbit loss of momentum produces a preferential loss of counter-current directed particles, resulting in an intrinsic co-current rotation that must be taken into account in the calculation of fluid rotation velocities.[16]

III. EDGE MHD INSTABILITY CONSTRAINT

While Eq. (9) specifies the equilibrium edge pressure profile that would satisfy the particle, energy, and momentum balance constraints for a given set of tokamak parameters and heating and fueling rates, there are magnetohydrodynamic (MHD) and other instabilities that may prevent this equilibrium from being stable, e.g., the Peeling-Ballooning (P-B) modes that cause ELMs. Thus, the equations above constitute a necessary condition for the equilibrium pressure profile of Eq. (9) to exist for a given set of tokamak parameters and heating and fueling rates, but not a sufficient condition for it to be stable. A second constraint is needed in order to predict the maximum stable pedestal height and width. The numerical non-local MHD peeling-ballooning (P-B) stability calculations of Refs. 11 and 12 are clearly state-of-the-art and provide such a constraint on the maximum stable pedestal height as a function of the width of the pedestal, which is of the same form as the constraint of Eq. (9). Where the two constraints coincide should define the limiting pressure height and corresponding width for a particular plasma with given heating and fueling sources and magnetic and electric fields.

We note that the EPED model of Refs. 11 and 12, which has been successful in predicting the limiting pressure pedestal height and width for several present tokamaks, combines the P-B MHD constraint model together with a transport constraint based on the calculated onset of kinetic ballooning modes (KBM). Although the EPED model has successfully predicted pedestal height and width for several existing tokamaks, we suggest that the replacement of the KBM transport constraint with the more general transport constraint of this paper would put the EPED model on a firmer physics basis for the prediction of future tokamaks.

IV. SUMMARY

A fluid transport constraint for the edge pressure profile in tokamaks has been developed based on the conservation of particles, energy, and momentum. Electromagnetic forces and external momentum sources and the kinetic ion orbit loss of particles, energy, and momentum are taken into account. The constraint is formulated in terms of the pressure pedestal height as a function of the pedestal width in order to be compatible with the Peeling-Ballooning MHD stability constraint. The important role of plasma rotation in determining the edge pedestal pressure profile is discussed.

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