

1. Let

$$A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{pmatrix} \quad \bar{b} = \begin{pmatrix} 1 \\ 7 \\ 1 \end{pmatrix}$$

- Row reduce A to row echolon form.
- Row reduce A to reduced echolon form.
- Find the pivotal columns of A , List the pivotal and free variables.
- Is the system consistent? If so find all the solutions.

2. Determine whether the vectors $\bar{v}_1 \in \text{span}[\bar{v}_2, v_3]$ where,

(a)

$$\bar{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \bar{v}_2 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \quad \bar{v}_3 = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$$

(b)

$$\bar{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \bar{v}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \bar{v}_3 = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

4 Let $A = [\bar{a}_1, \bar{a}_2, \bar{a}_3]$ with

$$\bar{a}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \bar{a}_2 = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \quad \bar{a}_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}.$$

Let $\bar{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

- What is $A\bar{x}$
- Find all \bar{x} so that $A\bar{x} = 0$. Can you describe this set geometrically?
- If they exist find all \bar{x} so that $A\bar{x} = \bar{b}$ where $\bar{b} = \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix}$. How is this set related to the set in (b)

5 If A is an $m \times n$ matrix whose columns span R^m . Explain why $m \geq n$. What can you say when $m = n$?