1) Let A be a 4×4 matrix and $\tilde{A}$ be a matrix in echelon form obtained from A by row reduction. Show that $\tilde{A}$ is either an upper triangular matrix with nonzero diagonal entires, or the last row must be a row of zeros.
2) Find $\bar{y}_1,\ \bar{y}_2,\ \bar{y}_3$ form a basis for there span. Is $\bar{x}$ in the span$\{\bar{y}_1,\ \bar{y}_2,\ \bar{y}_3\}$ if so find its coordinate vector.

$$\bar{y}_1 = \begin{pmatrix}1\\2\\1\\1\end{pmatrix} \quad \bar{y}_2 = \begin{pmatrix}2\\0\\1\\1\end{pmatrix} \quad \bar{y}_3 = \begin{pmatrix}3\\2\\2\\0\end{pmatrix} \quad \bar{x} = \begin{pmatrix}1\\-2\\0\\-1\end{pmatrix}$$

3) Determine whether the following set of vectors is linearly independent or not, $\bar{v}_1 = \begin{pmatrix}1\\2\\1\\3\end{pmatrix}, \bar{v}_2 = \begin{pmatrix}3\\1\\1\\1\end{pmatrix}, \bar{v}_3 = \begin{pmatrix}1\\-3\\1\\-5\end{pmatrix}$

4) Consider the matrix

$$A = \begin{pmatrix}1 & 1 & -1 & 0 & -1\\ 1 & 0 & 1 & 1 & 0\\ 1 & 1 & -1 & 1 & -2\\ 1 & 0 & 1 & 1 & 0\end{pmatrix}$$

The Col(A) is a subspace of what vector space? Write down the definition of Col(A) Find a basis for Col(A). Write down the definition of the Nul(A). It is a subspace of what vector space? Find a basis for Nul(A). What is rank A? For an $m\times n$ matrix A why do the pivotal columns form a basis for Col(A)?

5) Find the inverse if it exists of the following matrices

$$A = \begin{pmatrix}4 & 2\\ 1 & 5\end{pmatrix} \quad B = \begin{pmatrix}1 & 2\\ 2 & 4\end{pmatrix}$$

$$C = \begin{pmatrix}1 & 1 & 0 & 1\\ 1 & 2 & 1 & 0\\ 1 & 0 & 1 & 1\\ 2 & 2 & 0 & 1\end{pmatrix}$$

6) Let A be a 4×3 matrix. Find the matrix representation of the elementary row operation that would take 3 times the 2nd row of A and subtract it from the 4th row of A