ECE 6453 HW #3 Solutions

1. a) $G_{L0} = 10^{13}$ EHP/cm^3 < N_D (10^{14} cm^{-3}) $\Rightarrow$ low-level injection

From minority continuity equation, we have

$$\frac{\partial \Delta n_p}{\partial t} = D_n \cancelto{0}{\frac{\partial^2 \Delta n_p}{\partial x^2}} + \cancel{\mu_n \mathcal{E}} \cancelto{0}{\frac{\partial \Delta n_p}{\partial x}} - \frac{\Delta n_p}{\tau_n} + G_0$$

(uniform distribution)

<i> @ steady state: $\frac{\partial \Delta n_p}{\partial t} = 0$,

$$\Rightarrow \Delta n_p = \underline{G_0 \cdot \tau_n} = G_{L0} = 10^{13}\ \text{cm}^{-3}$$

<ii> $t \geq 0$, light $\rightarrow$ G_{L0} reduced to half

$$\frac{\partial \Delta n_p}{\partial t} = -\frac{\Delta n_p}{\tau_n} + G_0/2$$

$$\Rightarrow \quad \Delta n_p(t) = A \exp(-t/\tau_n) + B$$

B.C. $t = 0$ $\Delta n_p = G_{L0}$

$t = \infty$ $\Delta n_p = G_{L0}/2$

$$\Rightarrow \boxed{\begin{aligned}\Delta n_p(t) &= \frac{G_{L0}}{2}\left[1 + \exp(-t/\tau_n)\right] \\ &= 5\times10^{12}\left[1 + \exp(-t\times10^{6})\right]\ (\text{cm}^{-3})\end{aligned}}$$

b) Minority carrier diffusion equation at work here.

under steady state:
$$0 = \frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + \underline{\underline{G_0 \exp(-\alpha x)}}$$

where α is the absorption coefficient of the semiconductor at the light wavelength.

Solve for this equation, we have

$$\Delta P_n(x) = \tau_p G_o \left[\frac{\alpha^2 L_p^2}{1-\alpha^2 L_p^2} \exp(-x/L_p) + \frac{1}{1-\alpha^2 L_p^2} \exp(-\alpha x) \right]$$

$$L_p = \sqrt{\tau_p D_p}.$$

At $t=0$, the light source turned off. The new equation to solve is:

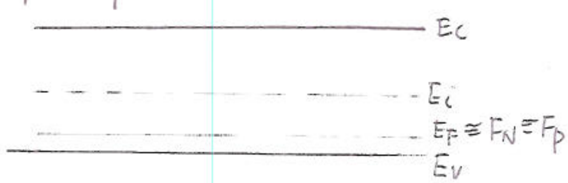
$$\frac{\partial \Delta P_n}{\partial t} = D_p \frac{\partial^2 \Delta P_n}{\partial x^2} - \frac{\Delta P_n}{\tau_p}$$

with B.C.'s

$$\begin{cases} \Delta P_n(x,0) = \tau_p G_o \left[\frac{\alpha^2 L_p^2}{1-\alpha^2 L_p^2} \exp(-x/L_p) + \frac{1}{1-\alpha^2 L_p^2} \exp(-\alpha x) \right] \\ \Delta P_n(x,\infty) = 0 \end{cases}$$

2. (a) Note that this is a majority carrier injection. Since it is low-level injection, $p \approx p_o$ and $n = n_o$ (no minority carrier injection).

Therefore $F_N = E_F \cong F_p$



(b) The majority continuity equation becomes:

$$\frac{\partial \Delta p}{\partial t} = D_p \frac{\partial^2 \Delta p}{\partial x^2} - \mu_p \mathcal{E} \frac{\partial \Delta p}{\partial x} - \underline{\mu_p p \frac{\partial \mathcal{E}}{\partial x}} - \frac{\Delta p}{\tau_p}$$

Look at each term on the right hand side.. we may first assume $\Delta p \propto e^{-x/L_p}$

〈i〉 $D_p \frac{\partial^2 \Delta p}{\partial x^2} \propto \frac{D_p}{L_p^2} \Delta p \approx 10^7 \Delta p$

〈ii〉 $-\mu_p \mathcal{E} \frac{\partial \Delta p}{\partial x} \propto \mu_p / L_p \, \mathcal{E} \Delta p \approx 4 \times 10^5 \mathcal{E} \Delta p$, $(\mathcal{E} \approx 0)$

〈iii〉 $\mu_p p \frac{\partial \mathcal{E}}{\partial x}$: $p = p_o + \delta p \approx p_o$, $\frac{\partial \mathcal{E}}{\partial x} = \frac{\rho}{\epsilon_s} \approx q \frac{\Delta p}{\epsilon_s}$ ← Poisson's equation

$$\Rightarrow \mu_p p \frac{\partial \mathcal{E}}{\partial x} = \frac{\sigma_p}{\epsilon_s} \Delta p \approx 5 \times 10^{10} \Delta p$$

From this estimation, one may see that the 3rd term dominate the overall transportation process for excess majority carriers. One can define a "relaxation time" as $(\sigma/\epsilon_s)^{-1} = \tau_{relax}$.

The equation can be simplified as

$$\frac{\partial \Delta P}{\partial t} \cong \frac{\delta_P}{\epsilon_s} \Delta P \approx -\frac{\Delta P}{\tau_{relax}}$$

$$\Rightarrow \Delta P_P(t) = \Delta P_{P_0} \exp(-t/\tau_{relax}).$$

This means that, for majority carrier injection, the excess carrier transport is due to a process called "relaxation" with a relaxation time constant of (ϵ_s/δ). The excess majority carrier will decay exponentially at very short period of time (in this case ~ 10ps) and appear at the end of the semiconductor bar. This is a process that is very different from the minority carrier injection.

(c) Sorry for the typo.. It should have been "excess electrons", not "excess hole injection". So... if $\Delta n(x,t) = 10^7 \delta(x)\delta(t)$...

$V = 20V$, $\mathcal{E} = V/l = 100\ V/cm$. From the velocity vs. electric field chart, we can assure that the low-field mobility approximation is valid, and the minority carrier continuity equation may still hold true. So, the equation used in Haynes-Shockley exp. can be applied.

$$\Rightarrow \Delta n_P(x,t) = \frac{N}{\sqrt{4\pi D_p t}} \exp\left(-\frac{(x-\mu_n \mathcal{E} t)^2}{4 D_n t} - t/\tau_n\right)$$

(d) $\Delta t = \Delta x / v_d = 1.059\ ns$

(e) As calculated in (d), the decrease of excess minority carrier through recombination is insignificant $e^{(\Delta t/\tau_n)} = e^{-1.059\times10^{-10}/10^{-7}} \approx 1$

The spreading of the pulse is mostly coming from diffusion ~~drift~~.

$FWHM = 2\sqrt{\ln 2}\sqrt{4D_n t_0}/v_d$, t_0 is the time interval between the excitation and the detection of peak amplitude at site x_i

$\therefore$ @ x_1, $FWHM = 0.63$ (ns)

@ x_2, $FWHM = 1.99$ (ns)

(e) $x_3 = 1mm$, $t_0 = 1.17 \times 10^{-7}$ (s) $\sim \tau_n$, one will expect the peak amplitude is $\propto \frac{1}{\sqrt{t}} exp(-t/\tau_n)$ of original value

(f) if $\mathcal{E} = V/l = \frac{2}{1 \times 10^{-4}} = 20 \, kV/cm$, The electron velocity will approach a saturation value. The drift current component is determined by $J_{drift} = q n v_{sat}$. So the continuity equation may be written as

$$\boxed{D_n \frac{\partial^2 \Delta n_p}{\partial x^2} + v_{sat} \frac{\partial \Delta n_p}{\partial x} + G - \frac{\Delta n_p}{\tau_n} = \frac{\partial \Delta n_p}{\partial t}}$$

Further simplification follows. Since $l = 1 \mu m \ll L_N \; (= \sqrt{\tau_n D_n})$ The metal contact at the end of the semiconductor bar will force an extraction of excess carriers. i.e. $\Delta n_p(x = 1 \mu m) = 0$. A linear approximation may be used to describe the excess carrier profile in this system. So.. $\frac{\partial^2}{\partial x} \Delta n_p \cong 0$

If there is no light or excess carrier excitation, $G = 0$.

The B.C. for solving this equation are:

$$\begin{cases} \Delta n_p(x,t)\big|_{x=0} = \Delta n_{p0} \\ \Delta n_p(x,t)\big|_{x=1\mu m} = 0 \end{cases}$$

Note: In real world, τ_n (minority carrier life time) $\sim 10^{-9}$ s range. So you are looking at a wider pulse and ~~slower~~ shorter diffusion length.