ECE 3075A
Random Signals

Lecture 1

Introduction to Probability Theory & Its Applications in Electrical & Computer Engineering

School of Electrical and Computer Engineering
Georgia Institute of Technology
Summer, 2003

Syllabus

- Lecturer: Professor B.H. Juang
- GTA: TBA
- Lecture time: MWF 1600-1710 @ Van Leer C240
- Office hours: MF 1500-1555 @ Bunger Henry 310
- Text: Cooper & McGillem, *Probabilistic Methods of Signal & System Analysis*, 3rd edition
- Homework: assignment given every week; due one week after date of assignment at class time
- Other information, announcements, regular update: http://users.ece.gatech.edu/~juang

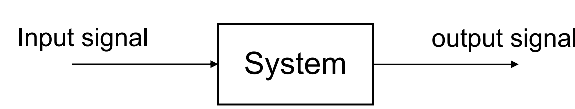
Exams and Grading Schedule

- Three quizzes and one final exam
- Open book tests
- Grading schedule

Quiz #1	May 28	15%
Quiz #2	June 18	15%
Quiz #3	July 9	15%
Homework	Every week, unless noted otherwise	20%
Final	July 28, 1130-1420	35%

System & Signal

- Concept of signals and systems
- Description or characterization of signals & systems
- Analysis of signal and system
- Deterministic description versus characterization of random systems and signals
- Foundation of analysis: probability theory, random variables and stochastic processes



Signal

- An expression that carries “useful” information.
- Origin of signal
  - Measurements of phenomenon or behavior
    e.g., temperature in a room, path of a storm, stock price
  - Man-made expressions
    e.g., language in conversation, flag waving, a wink
- A signal may represent the occurrence of an event, in a discrete manner (e.g., the result of an election), or as a continuous-time function (e.g., a music note or a sequence of music notes realized by an instrument).
- A signal can be parametric (e.g., the frequency of a pure tone) or non-parametric (e.g., thumbs-up).

System

- A system is a function that maps or transforms an input domain to an output domain. For example,

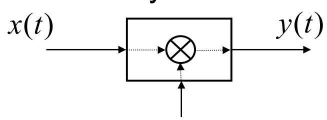
  $y = f(x;\lambda)$

  where x is input and λ is the parameter of the system.
- A system that generates a signal is often called a **source**; other system behave as a processor that processes the input signal to produce the output signal.
- A system is defined by a set of system parameters, which determine how the input-output mapping is to be conducted.

Deterministic vs. Random Signals/Systems An Example

- Deterministic signal or system

$$x(t;\omega,\theta) = A\cos(\omega t + \theta)$$
$$y(t) = x(t)\sin(2\omega t)$$



- Random signal or system

$$\mathbf{x}(t;\boldsymbol{\omega},\boldsymbol{\theta}) = \mathbf{A}\cos(\boldsymbol{\omega} t + \boldsymbol{\theta}) + \mathbf{n}(t)$$
$$\mathbf{y}(t) = \mathbf{x}(t)\sin(2\boldsymbol{\omega} t) + \mathbf{v}(t)$$

The values of $\boldsymbol{\omega}, \boldsymbol{\theta}$ are uncertain.

$\mathbf{n}$(t) and $\mathbf{v}$(t) are noise/contamination.

Why Use Probabilistic Methods for Analysis

- Many sources produce signal that cannot be easily described in a deterministic manner. E.g., speech.
- Observation of real signal is almost always contaminated by noise or disturbance which may be random. For example, a returned radar signal is always mixed with noise. A speech signal recorded in an office has background noise and some echo.
- Parameters of a system may fluctuate unpredictably, resulting in uncertainty in input-output relationship.
- When uncertainty is present, a deterministic characterization of the signal or the system becomes inadequate.

Probability Theory

- Characterization of random events
- Approaches:
  - Relative frequency approach: event counting
  - Axiomatic approach: set theory
- Elementary set theory
- Probability space:
  - Observation or event space
  - Probability assignment
  - Fundamental axioms

Some Notions

- An **experiment** is an action that produces **outcome**; for example, tossing a coin with H(ead) and T(ail) as possible outcomes.
- A **random experiment** is one in which the outcome is uncertain until the experiment is performed.
- A single performance in an experiment is called a **trial**.
- A trial results in an observed outcome, which is a **realization** of the trial.
- An **event** refers to a description of outcome; we say an event has occurred if the description of the event is observed in the realization of a trial.

Approaches to the Theory of Probability

- Relative frequency approach
  - An event that occurs more frequently has higher probability and vice versa
  - Empirical reasoning; readily understandable
  - Difficult to generalize
- Axiomatic approach
  - Probability is a real number between 0 and 1 assigned to events in the observation space
  - The number called probability satisfies a set of postulates so as to form a structure for further generalization and applications

Elementary Set Theory

- A set is a collection of objects known as elements.
- A space contains all the elements of interest, including the null element or empty set, ϕ , and the space, S itself.
- All sets formed by elements of the space are subsets of the space, S.

$a \in A \quad \Rightarrow \quad a$ belongs to (or is an element of) A;

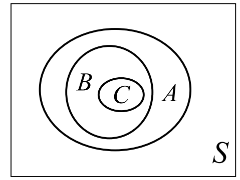
$A \subset B \quad \Rightarrow$ Set A is a subset of set B; or, B contains A

$A = B \quad iff \quad A \subset B$ and $B \subset A$.

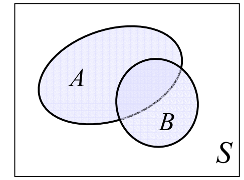
Set Operations

- Sum or Union: $A \cup B$ is a set that contains all the elements that are elements of A or of B **or** of both.
- Product or Intersection: $A \cap B$ is a set that contains all the elements that are common to both A **and** B.
- Complement: The complement of A, $\overline{A}$, is a set that contains all the elements of S that are not in A.
- Difference: The difference of two sets, $A - B$, is a set consisting of the elements of A that are not in B.

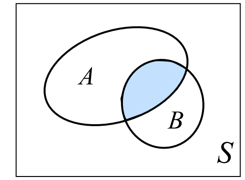
Venn Diagram



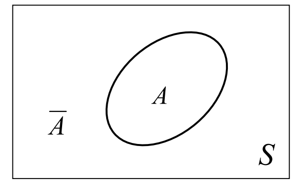
$C \subset B \subset A \subset S$



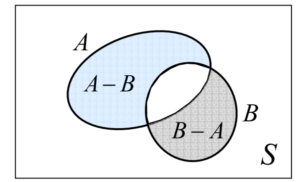
$A \cup B$



$A \cap B$



$\overline{A}$



$A - B$ and $B - A$

Axiomatic Approach

- Built upon set theory
- A **Probability Space** consists of three components: the observation space, S, the probability measure, Pr(), and the assignment of probability, Pr(event), that satisfies a set of axioms.
- **Observation Space** is a space whose elements are all the outcomes of an experiment.
- **Events** are subsets of the observation space.
- **Three axioms** to satisfy:

Non-negativity: $\Pr(A) \geq 0$

Sure event & total probability: $\Pr(S) = 1$

Exclusivity: If $A \cap B = \phi$, then $\Pr(A \cup B) = \Pr(A) + \Pr(B)$