ECE 3075A

***Random Signals***

**Lecture 1**

# Introduction to Probability Theory & Its Applications in Electrical & Computer Engineering

School of Electrical and Computer Engineering
Georgia Institute of Technology
Fall, 2003



## Syllabus

- Lecturer: Professor B.H. Juang
- GTA: TBA
- Lecture time: MWF 1500-1555 @ Van Leer C341
- Office hours: MF 1400-1455 @ Bunger Henry 310
- Text: Cooper & McGillem, *Probabilistic Methods of Signal & System Analysis*, 3rd edition
- Homework: assignment given every week; due one week after date of assignment at class time
- Other information, announcements, regular update:
  http://users.ece.gatech.edu/~juang



## Exams and Grading Schedule

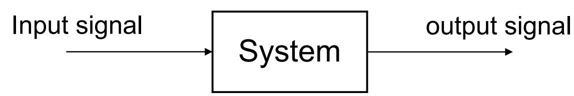
- Three quizzes and one final exam
- Open book tests
- Grading schedule

| | | |
|---|---|---|
| Quiz #1 | September 15 | 15% |
| Quiz #2 | October 10 | 15% |
| Quiz #3 | November 10 | 15% |
| Homework | Every week, unless noted otherwise | 20% |
| Final | July 28, 1130-1420 | 35% |



## System & Signal

- Concept of signals and systems
- Description or characterization of signals & systems
- Deterministic description versus characterization of random systems and signals
- Analysis of signal and system
- Foundation of analysis: probability theory, random variables and stochastic processes



# Signal

- An expression that carries “useful” information.
- Origin of signal
  - Measurements of phenomenon or behavior
    e.g., temperature in a room, path of a storm, stock price
  - Man-made expressions
    e.g., language in conversation, flag waving, a wink
- A signal may represent the occurrence of an event, in a discrete manner (e.g., the result of an election), or as a continuous-time function (e.g., a music note or a sequence of music notes realized by an instrument).
- A signal can be parametric (e.g., the frequency of a pure tone) or non-parametric (e.g., thumbs-up).



# System

- A system is a function that maps or transforms an input domain to an output domain. It behaves as a processor or a transducer that responds to an input signal to produce an output signal.
- A system is defined by a set of system parameters, which determine how the input-output mapping is to be conducted. For example,

  $y = f(x;\lambda)$

  where $x$ is input and $\lambda$ is the parameter of the system.
- A system can also be defined non-parametrically.
- A system that generates a signal is often called a **source.** (Question: How to describe the behavior of a source?)

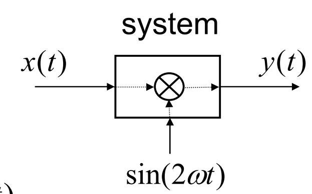


# Deterministic vs. Random Signals/Systems

- Deterministic (or non-random) signal or system
  Examples:

  $x(t;\omega,\theta) = A\cos(\omega t + \theta)$

  $y(t) = x(t)\sin(2\omega t)$




- Random signal or system

  $\mathbf{x}(t;\boldsymbol{\omega},\boldsymbol{\theta}) = \mathbf{A}\cos(\boldsymbol{\omega} t + \boldsymbol{\theta}) + \mathbf{n}(t)$

  $\mathbf{y}(t) = \mathbf{x}(t)\sin(2\boldsymbol{\omega} t) + \mathbf{v}(t)$

  The values of $\boldsymbol{\omega}, \boldsymbol{\theta}$ are **uncertain**.

  **n**(t) and **v**(t) are noise/contamination.



# Why Use Probabilistic Methods for Analysis

- Many sources produce signal that cannot be easily described in a deterministic manner. E.g., speech.
- Observation of real signal is almost always contaminated by noise or disturbance which may be random. For example, a returned radar signal is always mixed with noise. A speech signal recorded in an office has background noise and some echo.
- Parameters of a system may fluctuate unpredictably, resulting in uncertainty in input-output relationship.
- When uncertainty is present, a deterministic characterization of the signal or the system becomes inadequate.

## Preliminary Concept - Example

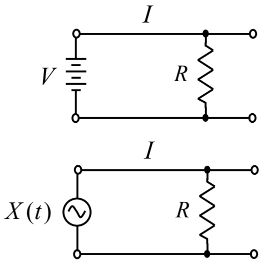
Measurement of power of a signal

$V$ is constant – a DC signal

$$I = \frac{V}{R} \quad \text{and} \quad P = IV = I^2 R$$

$X(t) = V_0 \cos(\omega t + \theta)$ - an AC signal

$$I = \frac{V_0 \cos(\omega t + \theta)}{R} = I_0 \cos(\omega t + \theta)$$




RMS current $= \left\{ \frac{1}{T} \int_0^T I_0^2 \cos^2(\omega t) dt \right\}^{1/2} = \frac{I_0}{\sqrt{2}} = 0.707 I_0 \qquad P = \frac{I_0^2 R}{2}$

What if $X(t)$ is a random signal or noise?



## Review of Calculus & Math Quantities

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$
$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\cos(x) = \frac{1}{2}\left[e^{jx} + e^{-jx}\right] \qquad \sin(x) = \frac{1}{2j}\left[e^{jx} - e^{-jx}\right]$$

$$2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y)$$
$$2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y)$$
$$2\sin(x)\cos(y) = \sin(x-y) + \sin(x+y)$$

$$2\cos^2(x) = 1 + \cos(2x)$$
$$2\sin^2(x) = 1 - \cos(2x)$$



## Review of Calculus

$$\int (a+bx)^n dx = \frac{(a+bx)^{n+1}}{b(n+1)} \qquad n > 0$$

$$\int_{-3}^{2} (1+2x)^4 dx = \frac{(1+2x)^5}{2(4+1)}\Bigg|_{x=-3}^{2} = \frac{5^5 - (-5)^5}{10} = 5^4 = 625$$

$$\int \frac{dx}{a+bx} = \frac{1}{b} \ln|a+bx|$$

$$\int e^{ax} dx = \frac{e^{ax}}{a} \qquad \int_0^{100} 2e^{-3x} dx = \frac{2}{-3} e^{-3x}\Bigg|_{x=0}^{100} \approx \frac{2}{3}$$

$$\int x e^{ax} dx = e^{ax}[x a^{-1} - a^{-2}]$$

$$\int \cos(x) dx = \sin(x) \qquad \int \sin(x) dx = -\cos(x)$$