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ECE 3075

HW #12

Solution

Problem 12.1 (6-8.1)

(a) $Y(t) = X(t) + N(t)$

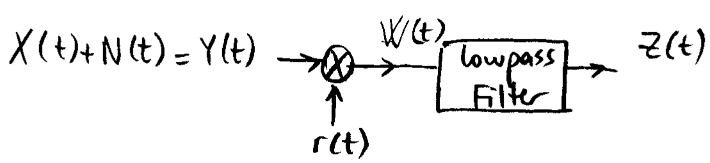
$$R_Y(\tau) = E[Y(t)Y(t+\tau)] = E[(X(t)+N(t))(X(t+\tau)+N(t+\tau))]$$
$$= R_X(\tau) + R_N(\tau) + E[N(t)]\underbrace{E[X(t+\tau)]}_{=0} + \underbrace{E[X(t)]}_{=0}E[N(t+\tau)]$$
$$= R_X(\tau) + R_N(\tau)$$

$$R_X(\tau) = E[X(t)X(t+\tau)] = E[(0.01)^2 \sin(100t+\theta)\sin(100(t+\tau)+\theta)]$$
$$= \frac{(0.01)^2}{2}[\cos 100\tau - \underbrace{E[\cos(200t+100\tau+\theta)]}_{=0}]$$
$$= \frac{(0.01)^2}{2}\cos 100\tau$$

$$\Rightarrow R_Y(0) = R_X(0) + R_N(0) = \boxed{\frac{(0.01)^2}{2} + 10 = 10.00005}$$

(b) $$\frac{10^{-4}}{2} = 10\,(10e^{-100|\tau|}) \Rightarrow \boxed{\tau = 0.145}$$

Problem 12.2 (6-8.2)



(a) $$W(t) = 10\cos(100t+\phi)\,0.01\sin(100t+\theta) + 10\cos(100t+\phi)N(t)$$
$$= 0.05[\sin(\theta-\phi) + \sin(200t+\phi+\theta)] + 10\cos(100t+\phi)N(t)$$

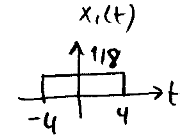
$$E[W(t)] = 0.05\sin(\theta-\phi) + 0.05\sin(200t+\phi+\theta) + 10\cos(100t+\phi)\cancel{E[N(t)]}^{=0}$$

After LP filtering, $\boxed{E[Z(t)] = 0.05\sin(\theta-\phi)}$

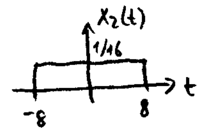
(b) When $\theta - \phi = \frac{\pi}{2} \Rightarrow \boxed{\phi = \theta - \frac{\pi}{2}}$

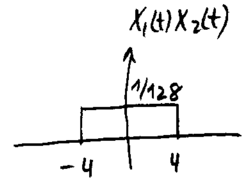
Problem 12.3 (7-2.1)

(a) $\frac{\sin 4w}{4w} \longleftrightarrow \frac{1}{8}\,\mathrm{rect}\left(\frac{t}{8}\right)$:



$\frac{\sin 8w}{8w} \longleftrightarrow \frac{1}{16}\,\mathrm{rect}\left(\frac{t}{16}\right)$:





$$\Rightarrow \int_{-\infty}^{\infty}\left(\frac{\sin 4w}{4w}\right)\left(\frac{\sin 8w}{8w}\right)dw = 2\pi\int_{-\infty}^{\infty} x_1(t)x_2(t)\,dt = 2\pi\int_{-4}^{4}\frac{1}{128}\,dt = \boxed{\frac{\pi}{8} = 0.3927}$$

(b) $\frac{1}{w^4+5w^2+4} = \frac{1}{(w^2+1)(w^2+4)}$, $\quad \frac{1}{w^2+1} \longleftrightarrow \frac{1}{2}e^{-|t|}$

$\frac{1}{w^2+4} \longleftrightarrow \frac{1}{4}e^{-2|t|}$

$$\Rightarrow \int_{-\infty}^{\infty}\frac{1}{(w^2+1)(w^2+4)}dw = 2\pi\int_{-\infty}^{\infty}\frac{1}{2}e^{-|t|}\frac{1}{4}e^{-2|t|}dt = \frac{\pi}{2}\int_{0}^{\infty} e^{-3t}dt = \boxed{\frac{\pi}{6} = 0.5236}$$

Problem 12.4 (7-2.2)

$$\overline{X^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(w)\,dw = \frac{1}{2\pi}\int_{-8\pi}^{8\pi}\left(1-\frac{|w|}{8\pi}\right)dw = \boxed{4}$$

Problem 12.5 (7-3.1)

(a) No , not even

(b) Yes

(c) Yes

(d) No, negative at $w=1$.

(e) Yes

(f) No , not even

Problem 12.6 (7-3.3)

(a) $\overline{X} = \sqrt{\frac{32\pi}{2\pi}} = \boxed{4}$

(b) $\overline{X^2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_X(w)\,dw = 56$

$\Rightarrow \sigma_X^2 = 56 - 16 = \boxed{40}$

(c) $\boxed{0, \pm 6, \pm 12 \text{ rad/s}}$

Problem 12.7 (7-3.4)

(a) From Prob. 6-2.1, $\overline{Y} = 2 \cdot \frac{0.05}{0.1}\cdot\frac{1}{2} = \boxed{0.5}$ (b) $\overline{Y^2} = 1 \Rightarrow \boxed{\sigma_Y^2 = 0.75}$

(c) Using (7-25) with $\overline{Y} = 1$, $t_1 = 0.1$, $\sigma_Y^2 = 1$ (Note that $\overline{Y}$ and σ_Y^2 are for the single pulse amplitude):

$$\boxed{S_X(w) = \left|\frac{\sin w/40}{w/40}\right|^2\left[10 + 200\pi\sum_{n=-\infty}^{\infty}\delta(w-20\pi n)\right]}$$