

Problem 1.3

(a) The probability space (i.e. the space of outcomes) is defined for this experiment as:

$$S = \{HH, HT, TH, TT\}$$

The outcomes are equally likely.

(b) The probability space is:

$$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

We can think of nothing better than assume equally likely outcomes.

(c) $S = \{0, 1, 2, \dots, 18\}$

The outcomes are not equally likely because 0 can only happen if both digits are 0, on the other hand, 6 can happen with several different combinations of the last two digits.

Problem 1.4

(a) 4 bad out of 25 \Rightarrow $\boxed{4/25}$

(b) One bad is out \Rightarrow 3 bad out of 24 \Rightarrow $\boxed{3/24}$

(c) One good is out \Rightarrow 4 bad out of 24 \Rightarrow $\boxed{4/24}$

Problem 1.5

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

\uparrow $P\{\text{king is drawn}\}$ \uparrow $P\{\text{spade is drawn}\}$ \uparrow $P\{\text{spade king is drawn}\}$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \boxed{\frac{4}{13}}$$

(b) $P(A \cap B) = \boxed{\frac{1}{52}}$

(c) $P(A \cup \bar{B}) = P(A) + P(\bar{B}) - P(A \cap \bar{B})$

\uparrow $P\{\text{king is drawn}\}$ \uparrow $P\{\text{spade is not drawn}\}$ \uparrow $P\{\text{any king other than spade is drawn}\}$

$$= \frac{4}{52} + \frac{39}{52} - \frac{3}{52} = \boxed{\frac{10}{13}}$$

$$(d) P(A \cup C) = P(A) + P(C) - P(A \cap C) = \frac{4}{52} + \frac{1}{52} = \boxed{\frac{5}{52}}$$

$$(e) P(B \cup C) = P(B) + P(C) - P(B \cap C) = \frac{13}{52} + \frac{1}{52} - \frac{1}{52} = \boxed{\frac{1}{4}}$$

$$(f) P(A \cap C) = \boxed{0} \text{ (cannot draw both a king and ten of spades)}$$

$$(g) P(B \cap C) = \boxed{\frac{1}{52}}$$

$$(h) P((A \cap B) \cup \bar{C}) = P(A \cap B) + P(\bar{C}) - P(A \cap B \cap \bar{C}) \\ = \frac{1}{52} + \frac{51}{52} - 0 = \boxed{1}$$

$$(i) P(A \cap B \cap C) = P((A \cap C) \cap B) = P(\emptyset \cap B) = P(\emptyset) = \boxed{0}$$