

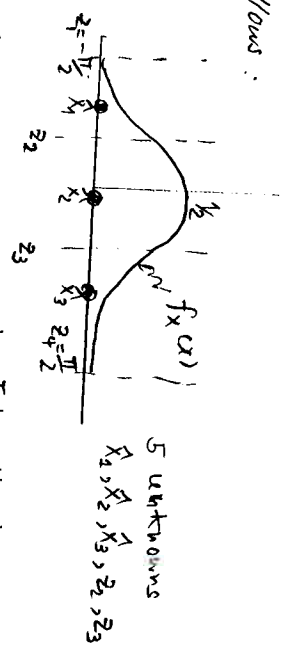
① Formulate the optimal quantizer that minimizes the mean square error as follows.

$$\min_{\hat{x}_i, z_i} D = \sum_{i=1}^3 \int_{z_i}^{z_{i+1}} (x - \hat{x}_i)^2 f_X(x) dx$$

Solve by taking derivative w.r.t. each parameter and set it to zero.

$$0 = \frac{\partial D}{\partial \hat{x}_i} = \int_{z_i}^{z_{i+1}} -2(x - \hat{x}_i) f_X(x) dx$$

$$\hat{x}_i = \frac{\int_{z_i}^{z_{i+1}} x f_X(x) dx}{\int_{z_i}^{z_{i+1}} f_X(x) dx} \quad i=1, 2, 3 \quad \text{evaluate} \Rightarrow$$



Using Integration by parts

$$\hat{x}_1 = \frac{\int_{-\pi/2}^{z_1} x \cos(x) dx}{\int_{-\pi/2}^{z_1} \frac{1}{2} \cos(x) dx} = \frac{(z_1 \sin z_1 - \cos z_1) + \cos(z_1)}{\sin(z_1) + 1} \quad \text{--- ①}$$

$$\hat{x}_2 = \frac{\int_{z_1}^{z_2} x (\frac{1}{2} \cos(x)) dx}{\int_{z_1}^{z_2} \frac{1}{2} \cos(x) dx} = \frac{(z_2 \sin z_2 - z_1 \sin z_1) + (\cos z_1 - \cos z_2)}{\sin z_2 - \sin z_1} \quad \text{--- ②}$$

$$\hat{x}_3 = \frac{\int_{z_2}^{\pi/2} x (\frac{1}{2} \cos(x)) dx}{\int_{z_2}^{\pi/2} \frac{1}{2} \cos(x) dx} = \frac{(\pi/2 - z_2 \sin z_2) - \cos z_2}{1 - \sin z_2} \quad \text{--- ③}$$

$$0 = \frac{\partial D}{\partial z_{i+1}} = (z_{i+1} - \hat{x}_i)^2 f_X(z_{i+1}) - (z_{i+1} - \hat{x}_{i+1})^2 f_X(z_{i+1})$$

$$0 = z_{i+1} - 2z_{i+1} \hat{x}_i + \hat{x}_i^2 - z_{i+1} + 2z_{i+1} \hat{x}_{i+1} - \hat{x}_{i+1}^2$$

$$z_{i+1} = \frac{\hat{x}_{i+1}^2 - \hat{x}_i^2}{2(\hat{x}_{i+1} - \hat{x}_i)} = \frac{\hat{x}_{i+1} + \hat{x}_i}{2} \quad i=1, 2, 3 \quad \text{evaluate} \Rightarrow$$

$$z_2 = \frac{\hat{x}_2 + \hat{x}_1}{2} \quad \text{--- ④}$$

$$z_3 = \frac{\hat{x}_3 + \hat{x}_2}{2} \quad \text{--- ⑤}$$

Note that we have 5 unknowns and 5 equations, but the equations are not linearly independent so we cannot solve them algebraically. I use Lloyd-max iterative method II to obtain the solution.

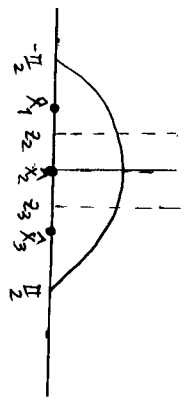
Iterative Procedure:

- ① Start from equation ①, make a guess on \hat{x}_1 and numerically find the value for z_2
- ② Since \hat{x}_1 and z_2 are known, use equation ② to solve for \hat{x}_3 (i.e. $\hat{x}_3 = 2z_2 - \hat{x}_1$)
- ③ Now with \hat{x}_2 known, use equation ③ to solve for \hat{x}_2 (i.e. $\hat{x}_2 = 2z_2 - \hat{x}_1$)
- ④ Use \hat{x}_2 and z_3 to find \hat{x}_3 (i.e. $\hat{x}_3 = 2z_3 - \hat{x}_2$)
- ⑤ Compute another value of z_3 based on the integral in equation ③ call this \hat{x}_3^*
- ⑥ IF $(\hat{x}_3^* - \hat{x}_3) < \text{Threshold}$ we stop, otherwise adjust the guess on \hat{x}_1 in the direction of $(\hat{x}_3^* - \hat{x}_3)$ then repeat the procedure from step 1 again.

② Lloyd-max algorithm yields the following values:

$z_1 = -\pi/2$
$z_2 = -0.4065$
$z_3 = 0.4065$
$z_k = \pi/2$

$x_1 = -0.8130$
$x_2 = 0$
$x_3 = 0.8130$



③
$$D = \sum_{i=1}^3 \int_{z_i}^{z_{i+1}} (x - x_i^*)^2 (\frac{1}{2} \cos x) dx$$

Substitute the values of z_i and x_i^* obtained from part ②, we get

$$D = 0.067753$$

④ Probability of $x_1 = P_n \{ -\pi/2 \leq x < z_2 \}$

Probability of $x_2 = P_n \{ z_2 \leq x < z_3 \} = \int_{-0.4065}^{-\pi/2} \frac{1}{2} \cos x dx = \frac{1}{2} [\sin(-0.4065) - \sin(\frac{\pi}{2})] = \boxed{0.3023}$

Probability of $x_3 = P_n \{ z_3 \leq x < \pi/2 \} = \int_{0.4065}^{\pi/2} \frac{1}{2} \cos x dx = \frac{1}{2} [\sin(0.4065) - \sin(\frac{\pi}{2})] = \boxed{0.3954}$

⑤ Mean Reconstruction value = $E[X^*] = \sum_{j=1}^3 x_j^* P(x_j^*) = \sum_{j=1}^3 x_j^* P\{z_j \leq x < z_{j+1}\} = \boxed{0.3023}$

$$= (-0.8130)(0.3023) + 0 + (0.8130)(0.3023) = \boxed{0}$$

This is to be expected because the property of Lloyd-max quantizer $E[X] = E[X^*] = 0$
 2nd moment of the reconstruction value = $E[X^{*2}] = \sum_{j=1}^3 x_j^{*2} P(x_j^*) = (-0.8130)^2(0.3023) + 0 + (0.8130)^2(0.3023) = \boxed{0.3996}$

Note $E[X^*] = 0.3023$ which is bigger than $E[X] = 0$ as expected

f) The problem for maximizing the quantizer output entropy can be formulated as

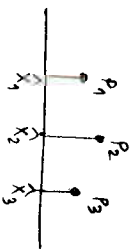
$$\max H(Q) = \sum_{i=1}^3 P_i \log P_i$$

where P_i is the probability of each reconstruction value, defined to be:

$$P_1 = \int_{-\frac{\pi}{2}}^{z_2} \frac{1}{2} \cos x \, dx = \frac{1}{2} (\sin z_2 + 1)$$

$$P_2 = \int_{z_2}^{z_3} \frac{1}{2} \cos x \, dx = \frac{1}{2} (\sin z_3 - \sin z_2)$$

$$P_3 = \int_{z_3}^{\frac{\pi}{2}} \frac{1}{2} \cos x \, dx = \frac{1}{2} (1 - \sin z_3)$$



In fact, the entropy is not a function of the reconstructed values at all. We only need to solve for the unknown boundary points (namely z_2 and z_3) and assign x_i to be anything inside $[z_i, z_{i+1})$.

Now, take derivative w.r.t. unknown parameters (z_2 and z_3) and set it to zero.

$$0 = \frac{\partial H}{\partial z_2} = \sum_{i=1}^3 \frac{\partial H}{\partial P_i} \cdot \frac{\partial P_i}{\partial z_2} = \frac{\partial H}{\partial P_1} \cdot \frac{\partial P_1}{\partial z_2} + \frac{\partial H}{\partial P_2} \cdot \frac{\partial P_2}{\partial z_2} = (\log P_2 + 1) \left(\frac{1}{2} \cos z_2 \right) + (\log P_3 + 1) \left(-\frac{1}{2} \cos z_2 \right)$$

$$0 = \log \left(\frac{P_1}{P_2} \right) \cdot \cos(z_2)$$

$$= \log \left(\frac{\sin z_2 + 1}{\sin z_3 - \sin z_2} \right) \cdot \cos(z_2) \Rightarrow \text{satisfied when}$$

$$\frac{\sin z_2 + 1}{\sin z_3 - \sin z_2} = 1 \Rightarrow \boxed{2 \sin z_2 + 1 = \sin z_3} \quad \text{--- (1)}$$

$$0 = \frac{\partial H}{\partial z_3} = \frac{\partial H}{\partial P_2} \cdot \frac{\partial P_2}{\partial z_3} + \frac{\partial H}{\partial P_3} \cdot \frac{\partial P_3}{\partial z_3} = (\log P_2 + 1) \left(\frac{1}{2} \cos z_3 \right) + (\log P_3 + 1) \left(-\frac{1}{2} \cos z_3 \right)$$

$$0 = \log \left(\frac{P_2}{P_3} \right) \cdot \cos(z_3)$$

$$= \log \left(\frac{\sin z_3 - \sin z_2}{1 - \sin z_3} \right) (\cos z_3) \Rightarrow \text{satisfied when}$$

$$\frac{\sin z_3 - \sin z_2}{1 - \sin z_3} = 1 \Rightarrow$$

$$\boxed{2 \sin z_3 = \sin z_2 + 1} \quad \text{--- (2)}$$

Now, we have two equations and two unknowns, solve for z_2 and z_3 using equation ① + ②

$$z_2 = \arcsin(-\frac{1}{3}) = -0.3398$$

$$z_3 = \arcsin(\frac{1}{3}) = 0.3398$$

You could have come to this a lot earlier.

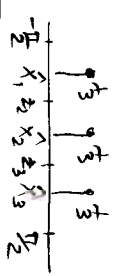
Substitute into equations for P_i we get $P_1 = P_2 = P_3 = \frac{1}{3}$ (uniform distribution)

This is as we expected because from information theory, we know the entropy is maximum with uniform distribution.

Hence,

$$\begin{aligned} z_1 &= -\frac{\pi}{2} \\ z_2 &= -0.3398 \\ z_3 &= 0.3398 \\ z_4 &= \frac{\pi}{2} \end{aligned}$$

$$\begin{aligned} \hat{x}_1 &\in [-\frac{\pi}{2}, -0.3398) \\ \hat{x}_2 &\in [-0.3398, 0.3398) \\ \hat{x}_3 &\in [0.3398, \frac{\pi}{2}) \end{aligned}$$



Note that $\hat{x}_1, \hat{x}_2, \hat{x}_3$ can take on any value in their respective interval without affecting the entropy. However, if distribution is to be minimized, one should assign \hat{x}_i to be the centroid in each respective interval as in Lloyd-Max procedure.

In that case

$$\begin{aligned} \hat{x}_1 &= -0.772 \\ \hat{x}_2 &= 0 \\ \hat{x}_3 &= +0.772 \end{aligned}$$

Prob. 2 Figure A shows a conventional differential coding scheme with the presence of channel noise which is assumed to be independent of the signal and the quantization noise. (We assume that the channel noise is being added to Δ_n rather than the digital bit stream.) An equivalent alternative scheme that permits introduction of quantization noise shaping and post filtering is also drawn in Figure B.

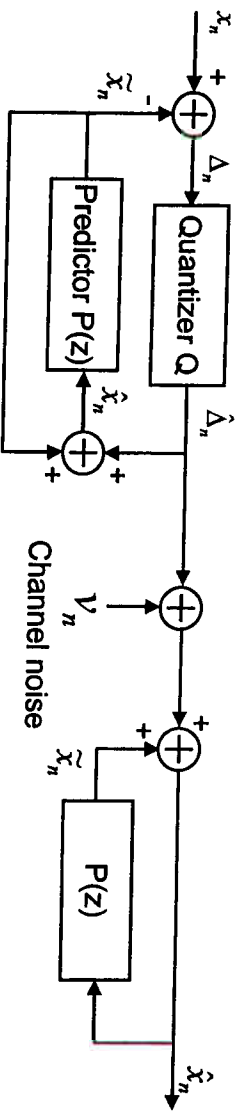


Figure A

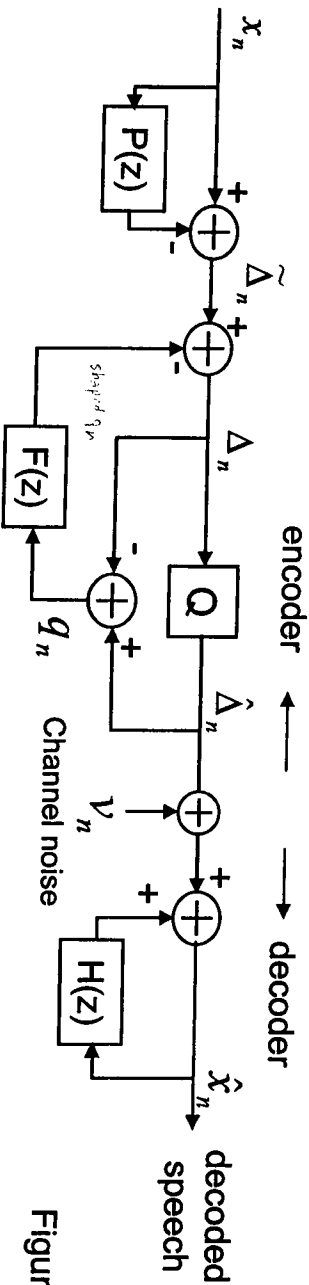


Figure B

- Derive the conditions (on P , F and H) under which these two schemes are equivalent;
- Let us assume that the quantization noise and the channel noise are both white. Also assume that in the absence of channel noise, $F_o(z)$ and $H_o(z)$ lead to the "optimal" noise shaping and post-filtering for a given quantizer Q such that the quantization noise is just below the audible threshold. What would you do to keep the overall noise as unnoticeable as possible without resorting to increasing the bit rate? Address the question from both the SONR and the noise spectral shaping points of view.

Problem 2

(2)

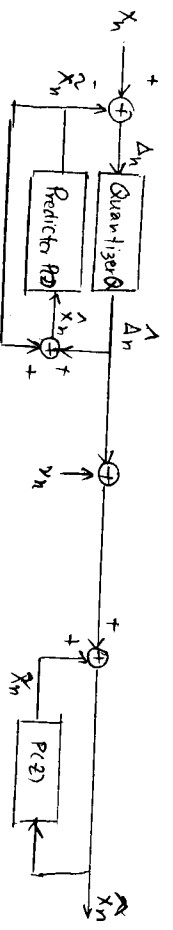


Fig A

Quantization block is replaced by an additive noise model:



In z-domain:

Encoder

$$\hat{\Delta}(z) = \Delta(z) + Q(z) \quad \text{--- (1) (quantization stage)}$$

$$\Delta(z) = X(z) - \hat{X}(z) \quad \text{--- (2)}$$

$$\hat{X}(z) = \hat{X}(z) \cdot P(z) \quad \text{--- (3)}$$

$$\hat{X}(z) = \hat{\Delta}(z) + \hat{X}(z) \quad \text{--- (4)}$$

substitute (4) into (3), we get

$$\hat{X}(z) = \frac{\hat{\Delta}(z) P(z)}{1 - P(z)} \quad \text{--- (5)}$$

substitute (5) into (2), we get

$$\Delta(z) = X(z) - \left[\frac{P(z)}{1 - P(z)} \right] \hat{\Delta}(z) \quad \text{--- (6)}$$

substitute (6) into (1), we get

$$\hat{\Delta}(z) = X(z) - \left[\frac{P(z)}{1 - P(z)} \right] \hat{\Delta}(z) + Q(z)$$

$$\left[\frac{1}{1 - P(z)} \right] \hat{\Delta}(z) = X(z) + Q(z)$$

$$\hat{\Delta}(z) = \underbrace{[1 - P(z)]}_{\text{Residual from linear prediction}} X(z) + \underbrace{[1 - P(z)]}_{\text{shaped quantization noise}} Q(z) \quad \text{--- (*)}$$

Decoder

$$\hat{X}(z) = \hat{X}(z) \cdot P(z)$$

$$\hat{X}(z) = \hat{X}(z) + \hat{\Delta}(z) + \underbrace{v(z)}_{\text{Channel Noise}}$$

$$= \hat{X}(z) \cdot P(z) + \hat{\Delta}(z) + v(z)$$

$$\hat{X}(z) = \frac{\hat{\Delta}(z) + v(z)}{(1 - P(z))} \quad \text{--- (*)}$$

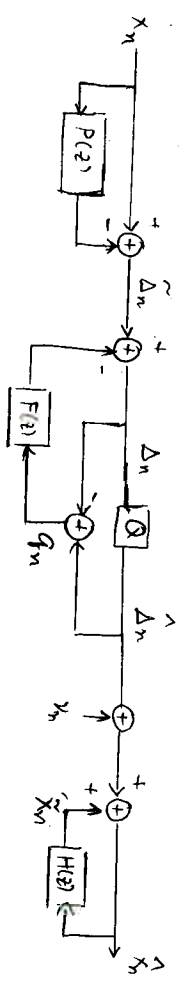


Fig. B.

In z-domain:

Encoder:

$$\hat{\Delta}(z) = \Delta(z) + Q(z) \quad \text{--- (1) (Quantization error)}$$

$$\Delta(z) = \hat{\Delta}(z) - Q(z) \cdot F(z) \quad \text{--- (2)}$$

$$\hat{\Delta}(z) = X(z) - X(z)P(z) = (1 - P(z)) \cdot X(z) \quad \text{--- (3)}$$

Substitute (3) into (2) we get

$$\Delta(z) = [1 - P(z)] \cdot X(z) - Q(z) \cdot F(z) \quad \text{--- (4)}$$

Substitute (4) into (1) we get

$$\hat{\Delta}(z) = \{ [1 - P(z)] \cdot X(z) - Q(z) \cdot F(z) \} + Q(z)$$

$$\hat{\Delta}(z) = \underbrace{[1 - P(z)] \cdot X(z)}_{\text{Residual from linear prediction}} + \underbrace{[1 - F(z)] \cdot Q(z)}_{\text{shaped quantization noise}}$$

Comparing this equation to equation (8) we observe that the two encoders are equivalent if $P = F$. The noise shaping mechanism will be the same for both schemes.

must assign

$$P = F = H$$

Therefore, combining constraints from decoder side and encoder side, we in order for both schemes to be equivalent.

Decoder

channel noise.

$$\hat{X}(z) = \tilde{X}(z) + \hat{\Delta}(z) + V(z)$$

$$\hat{X}(z) = \hat{X}(z) \cdot H(z)$$

That is:

$$\hat{X}(z) = \{ \hat{X}(z) \cdot H(z) \} + \hat{\Delta}(z) + V(z)$$

$$\hat{X}(z) = \frac{\hat{\Delta}(z) + V(z)}{[1 - H(z)]}$$

Comparing this equation to equation (8) we observe that the decoders are equivalent if $P = H$.

Case 1

b. Without the channel noise, $F_0(z)$ and $H_0(z)$ leads to "optimal" code for quantizer Q .

Recall the decoded output has the form:

$$\hat{X}(z) = \frac{\hat{A}(z) + V(z)}{[1 - H_0(z)]}$$

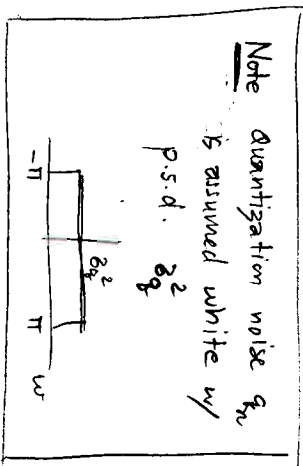
↑ assumed no channel noise

While the encoded signal has the form

$$\hat{A}(z) = [1 - P(z)]X(z) + [1 - F_0(z)]Q(z)$$

Thus,

$$\hat{X}(z) = \underbrace{\left[\frac{1 - P(z)}{1 - H_0(z)} \right]}_{\text{signal portion}} X(z) + \underbrace{\left[\frac{1 - F_0(z)}{1 - H_0(z)} \right]}_{\text{noise portion}} Q(z)$$



Hence, in terms of overall SQNR (encoder+decoder), F_0 and H_0 provided the largest value.

$$SQNR = 10 \times \log_{10} \left[\frac{\text{signal power}}{\text{noise power}} \right] = 10 \times \log_{10} \left\{ \frac{\int_{-\pi}^{\pi} |1 - F_0(\omega)|^2 \cdot \frac{\sigma_q^2}{2\pi} d\omega}{\int_{-\pi}^{\pi} |1 - H_0(\omega)|^2 \cdot \frac{\sigma_q^2}{2\pi} d\omega} \right\}$$

Case 2

Now, with the channel noise added, there is an extra noise term at the decoder.

$$\hat{X}(z) = \underbrace{\left[\frac{1 - P(z)}{1 - H_0(z)} \right]}_{\text{signal portion (same)}} X(z) + \underbrace{\left[\frac{1 - F_0(z)}{1 - H_0(z)} \right]}_{\text{noise portion (increased)}} Q(z) + \underbrace{\left[\frac{1}{1 - H_0(z)} \right]}_{\text{channel noise term}} V(z)$$

↑ quantization noise

Thus, $SQNR' = 10 \times \log_{10}$

$$\left\{ \frac{\int_{-\pi}^{\pi} |1 - F_0(\omega)|^2 \cdot \frac{\sigma_q^2}{2\pi} d\omega + \int_{-\pi}^{\pi} \left| \frac{1}{1 - H_0(\omega)} \right|^2 \cdot \frac{\sigma_v^2}{2\pi} d\omega}{\int_{-\pi}^{\pi} |1 - H_0(\omega)|^2 \cdot \frac{\sigma_q^2}{2\pi} d\omega} \right\}$$

signal power

(I assume v_1 and g_n independent and both white w/ p.s.d. σ_v^2 and σ_g^2 respectively)

Without changing anything it is clear that the denominator of $SQNR'$ is larger and thus: $SQNR \rightarrow SQNR'$ (more distortion, more audible noise in Case 2)

However, with proper adjustment of F_0 we can achieve $SQNR' = SQNR$ (equal the optimal w/ channel noise)

Case 3: With channel noise and proper adjustment of the noise shaping filter F .

This is the procedure:

- ① Keep P and H_0 unchanged \Rightarrow Hence, the ^{output} signal power remains the same.
- ② Adjust F s.t. the following condition satisfies.

$$\underbrace{\int_{-\pi}^{\pi} \left| \frac{1-F(\omega)}{1-H_0(\omega)} \right|^2 \cdot \frac{\sigma_g^2}{2\pi} d\omega}_{\text{overall noise level by channel noise}} + \underbrace{\int_{-\pi}^{\pi} \left| \frac{1}{1-H_0(\omega)} \right|^2 \cdot \frac{\sigma_v^2}{2\pi} d\omega}_{\text{optimal noise level w/o channel noise}} = \int_{-\pi}^{\pi} \left| \frac{1-F(\omega)}{1-H_0(\omega)} \right|^2 \cdot \frac{\sigma_g^2}{2\pi} d\omega$$

③ In this method we only need to find another noise shaping filter F that will create the overall noise as small as the optimal noise level prior to the channel noise introduction.

④ The solution should exist as the only unknown is F , we do have the remaining information on H_0 , F_0 , σ_g^2 , and σ_v^2 . However, since we're working with the integral of $|1-F(\omega)|^2$ there maybe more than one solution that satisfy the above equation.

We can interpret the new spectral shaping filter F as the system that tries to shape both the quantization noise and the channel noise even before the encoded signal is exposed to the channel. This is possible because ^{yes} we know the exact characteristics of the channel "prior" to building the encoder. So the encoder is able to equalize the channel noise beforehand s.t. the addition of the channel noise does not hurt the performance of the decoded output (same SNR) You can never achieve the same SNR without channel

Prob. 3 Multi-stage vector quantization was discussed a number of times in algorithm development as well as in standards talks. Provide, as complete as possible, a list of references to multi-stage or multiple stage vector quantization in standards documents, with synoptic explanation (why and how it is used) for each of the citation you include in the list. Also, when and where was the technique first proposed in the literature?

Please flip to the next page →

1. BH, Juang and AH Gray, *Multiple Stage Vector Quantization for Speech Coding*, Proceedings of ICASSP, vol. 1, pp. 597-600, Paris, April 1982 [In April of 1982, Paris, France, the Multiple Stage Vector Quantization was first proposed in the literature in ICASSP'82 by Dr. Juang and A.H. Gray. Multistage VQ involves several successive VQ codebooks, each encoding the residual of the previous stage(s). The goal is to reduce the storage as well as the computation in traditional single-stage vector quantization.]

2. W. LeBlanc, B. Bhattacharya, S. Mahmoud, and V. Cupperman. *Efficient search and design procedures for robust multi-stage VQ of LPC parameters for 4kb/s speech coding*. IEEE Trans. Speech and Audio Proc., 1:373-385, 1993.

[Aimed for coding linear prediction coding (LPC) parameters at low complexity and good robustness using rates as low as 22 b/frame. A joint codebook design strategy for multistage VQ which improves convergence speed and the VQ performance measures is presented. The best performance/complexity tradeoffs are obtained with relatively small size codebooks cascaded in a 3-6 stage configuration.]

3. McCree, A., Truong, K., George, E. B., Barrwell, T. P. & Viswanathan, V. (1996), *A 2.4 Kbits MELP Coder Candidate for the New U.S. Federal Standard*, Proc. ICASSP'96 pp. 200-203.

[MSVQ is utilized as an integral component of the MELP codec in quantization of the short-term LPC parameters, namely using 25-bit MSVQ to code LSFs in each frame.]

4. ITU-T. *Coding of speech at 8 kbits using conjugate-structure algebraic-code-excited linear-prediction(cs-acelp)*. Technical Report G.729, International Telecommunications Union, Geneva, 1996.

[2-stage VQ is used for quantization of the Line Spectral Frequencies in the encoder. 18 out of 80 possible bits/frame are used to encode 10 LSFs]

[MSVQ framework where the quantized vector is formed by adding the transformed outputs of a multistage codebook rather than just adding the outputs of the stages. Experimental results based on speech spectrum quantization show that the proposed VQ techniques outperform MSVQ of same bit rate.]

9. Han, W.J.[Woo-Jin], Kim, E.K.[Eun-Kyoung], Oh, Y.H.[Yung-Hwan],
Multicodetbook split vector quantization of LSF parameters, SPLetters(9), No. 12, December 2002, pp. 418-421.

[Using MSVQ idea but the multiple codebooks having different sizes are trained. The minimal-size codebook satisfying the spectral distortion are determined. Experimental results have shown that the proposed method reduces the number of outliers significantly and achieves a better rate-distortion performance compared with the fixed-bit-rate SVQ, multistage VQ, and variable-bit-rate SVQ.]

10. V. Krishnan, D.V. Anderson, K.K. Truong, *Optimal multistage vector quantization of LPC parameters over noisy channels*. Speech and Audio Proc, IEEE, Vol. 12, Issue 1, pp1-8, Jan. 2004.

[A novel channel-optimized multistage vector quantization (CO-MSVQ) codec is presented, in which the stage codebooks are jointly designed. The mean and the variance of the spectral distortion were shown to be smaller and the perceptual quality of the reconstructed speech was found to be better than that obtained using the sequentially designed CM-MSVQ.]

25
Prob. 4 What is scalable coding? Discuss how video coding can be made scalable. Cite practices commonly found in various video coding standards and suggest your own idea if any.

Scalable coding is the coding scheme that allow the increase of the "accuracy" or "quality" of the reconstruction when bandwidth is available, while permitting a lower-quality reconstruction when the bandwidth is insufficient for enhancement.

In video coding, scalable coding is possible if more than one bitstreams are used to encode the data. The base bitstream is low-rate encoded which provide basic reconstruction bandwidth alone. There are a few practices commonly used.

- ① SNR-scalable: The base layer contain the quantized data of a video frame. The quantization error introduced by the first quantizer is itself quantized and transmitted as the enhancement layer. Side info required by the decoder such as motion vectors is transmitted in the base layer. (MPEG-2)
- ② spatially scalable: The decoded picture or video frame at a higher layer is obtained by an upsampling of the base layer to increase resolution. The lower layer is in a way used as a prediction in a higher layer. (HDTV)
- ③ temporally scalable: The enhancement layer contains the bitstream corresponding to frames that would occur between frames of the base layer. (MPEG-2)

My proposal: spatio-temporal scalable: base layer is the low-resolution, low frame rate (missing some frames) and enhancement layer allow improvement in resolution and frame rate simultaneously

Spring 2004

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Final Exam

Benefits: allow broader dynamic range for transmission rates from very low-resolution, low-frame rate to high-resolution high-frame rate bitstreams