ECE 4270

Fundamentals of Digital Signal Processing

Lecture 13: Frequency Response of Rational LTI Systems

School of Electrical and Computer Engineering
Georgia Institute of Technology
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Ideal Lowpass with Delay

- Frequency response:

$$H(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| \le \pi \end{cases}$$

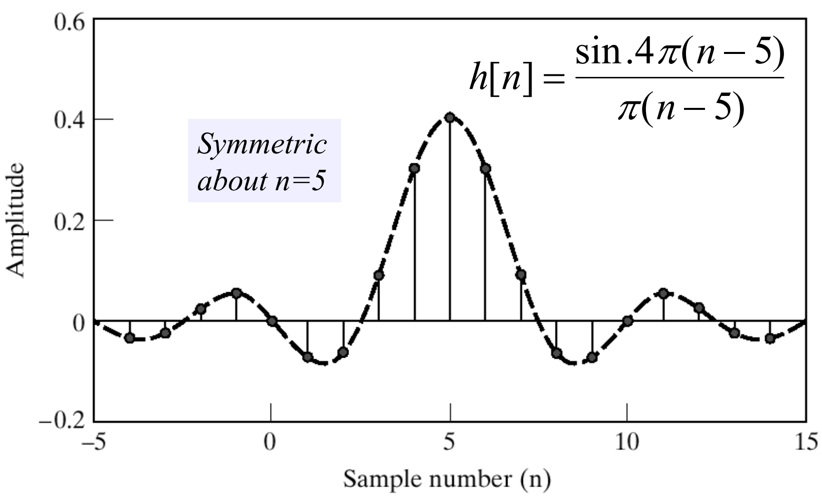
- Magnitude and phase:

$$\left|H(e^{j\omega})\right| = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| \le \pi \end{cases} \qquad \angle H(e^{j\omega}) = -\omega\alpha$$

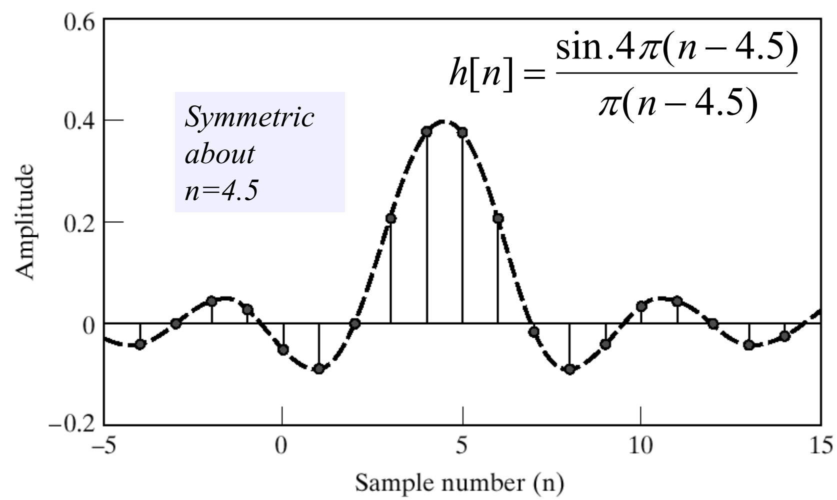
- Impulse response:

$$h[n] = \frac{\sin \omega_c (n-\alpha)}{\pi (n-\alpha)} \qquad -\infty < n < \infty$$

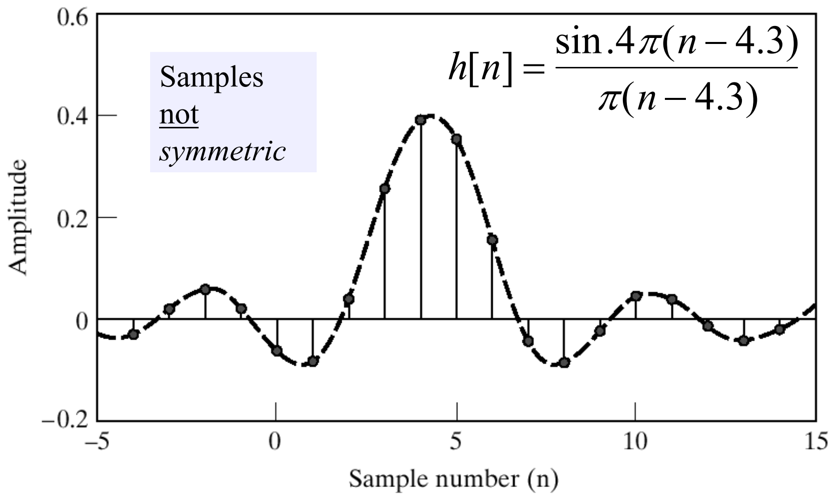
Ideal Lowpass with Linear Phase



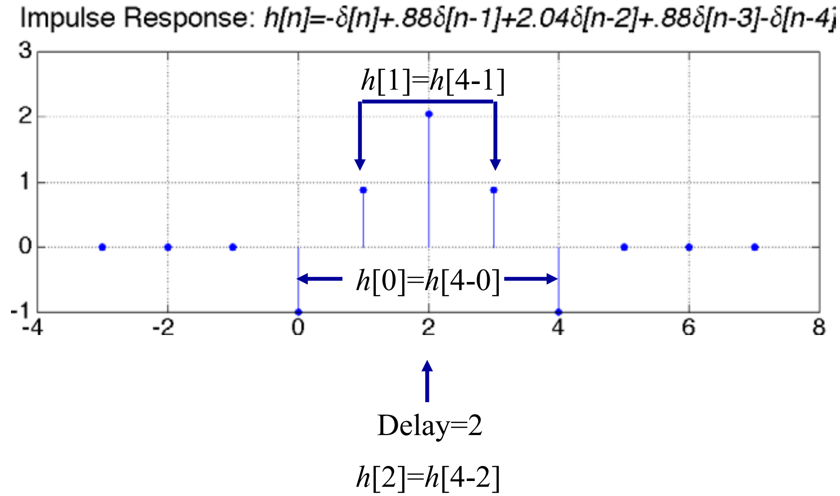
Ideal Lowpass with Linear Phase



Ideal Lowpass with Linear Phase



Impulse Response



Zeroes with a Symmetric Impulse Response

$$H(z) = -1 + .88z^{-1} + 2.04z^{-2} + .88z^{-3} - z^{-4}$$

$$= \frac{-z^4 + .88z^3 + 2.04z^2 + .88z - 1}{z^4}$$

$$H(z^{-1}) = -1 + .88z + 2.04z^2 + .88z^3 - z^4$$

$$= -z^4 + .88z^3 + 2.04z^2 + .88z - 1$$

$$H(z^{-1}) = z^4 H(z)$$

If z_0 is a zero of $H(z)$, then z_0^{-1} is also a zero

$$H(z_0) = 0 \quad \Leftrightarrow \quad H(z_0^{-1}) = 0$$

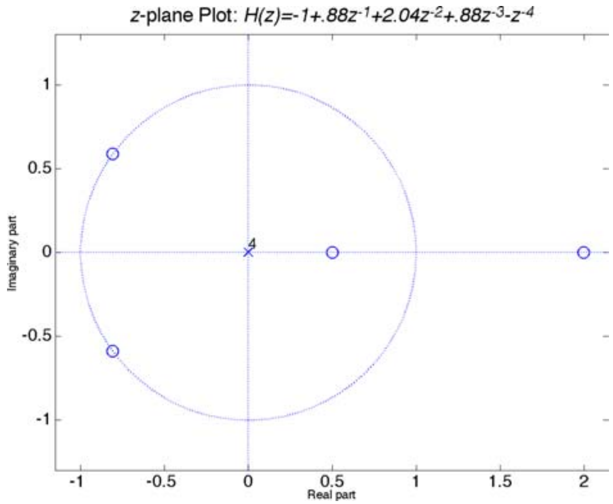
Zero Symmetry

- If z_0 is a zero of $H(z)$, then
  - If the impulse response is (anti)symmetric ($h[n] = +/- h[M-n]$), z_0^{-1} is also a zero
  - If the impulse response is real-valued ($h[n] = h^*[n]$), then $H^*(z^*) = H(z)$, so z_0^* is also a zero

$$z_0 = re^{+j\theta} \Rightarrow z_0^{-1} = r^{-1}e^{-j\theta}, \quad z_0^* = re^{-j\theta}$$

- The z-transform of real, symmetric impulse responses has zeroes that occur in reciprocal, conjugate "quads"
  - Zeroes on the real axis occur in reciprocal pairs
  - Zeroes on the unit circle occur in conjugate pairs
  - Zeroes at z=1 or –1 occur singly

Pole-Zero Plot



Frequency Response

$$H(e^{j\omega}) = -1 + .88e^{-j\omega} + 2.04e^{-j\omega 2} + .88e^{-j\omega 3} - e^{-j\omega 4}$$

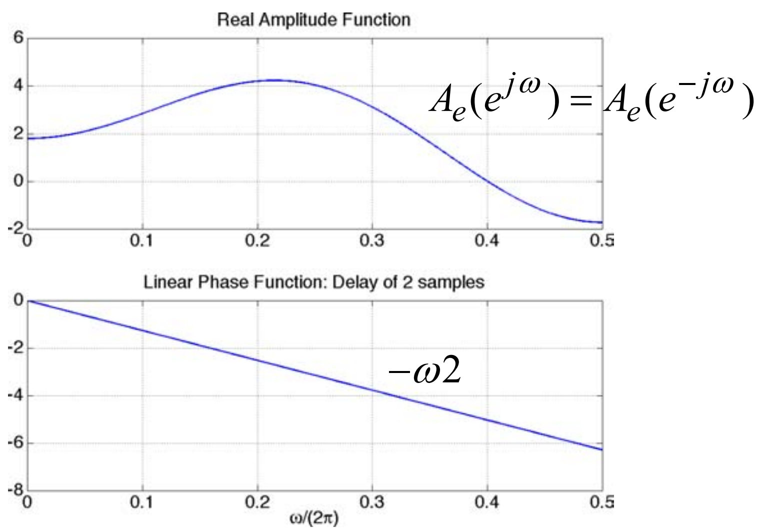
$$H(e^{j\omega}) = \begin{pmatrix} -e^{j\omega 2} + .88e^{j\omega} + 2.04 \\ + .88e^{-j\omega} - e^{-j\omega 2} \end{pmatrix} e^{-j\omega 2}$$

$$H(e^{j\omega}) = \begin{pmatrix} 2.04 + .88(e^{j\omega} + e^{-j\omega}) \\ -(e^{j\omega 2} + e^{-j\omega 2}) \end{pmatrix} e^{-j\omega 2}$$

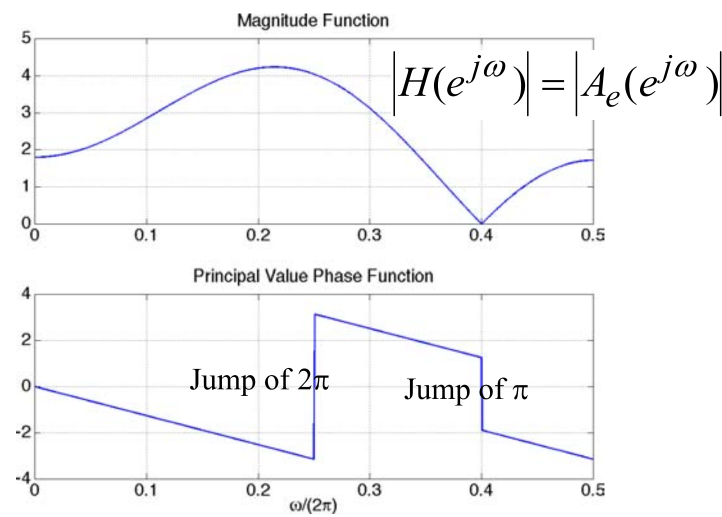
$$H(e^{j\omega}) = [2.04 + 1.76\cos\omega - 2\cos(2\omega)]e^{-j\omega 2}$$

$$H(e^{j\omega}) = A_e(e^{j\omega})e^{-j\omega 2}, \quad A_e(e^{j\omega}) = A_e(e^{-j\omega})$$

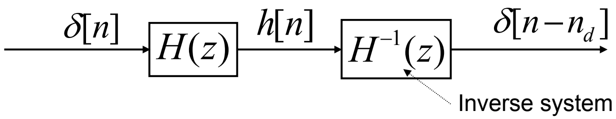
Amplitude and Angle Plot



Magnitude and Phase Plot



Inverse Systems



- An inverse system produces an impulse as output when driven by the impulse response of the original system.
- In practice, the output impulse may have a relative delay with respect to the original impulse.

Example:

$$H(z) = \frac{1}{1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}} \qquad H_{inv}(z) = H^{-1}(z) = 1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}$$

$$h[n] = \frac{r^n \sin[\theta(n+1)]}{\sin\theta} u[n] \qquad h_{inv}[n] = \delta[n] - 2r\cos\theta\,\delta[n-1] + r^2\delta[n-2]$$

