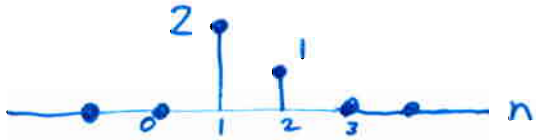
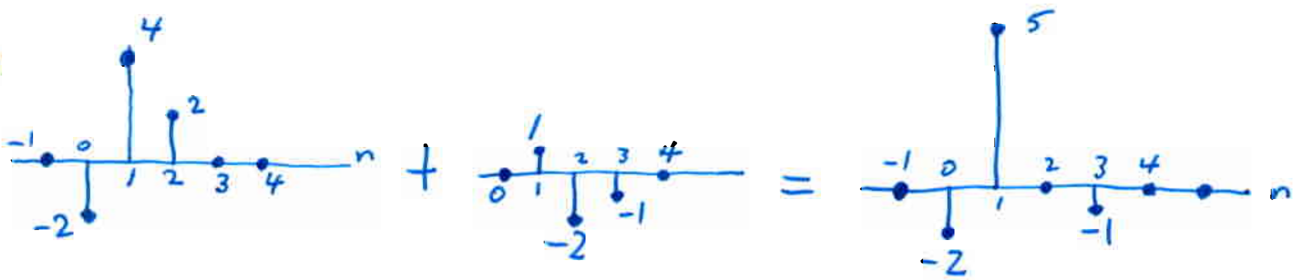
2.22

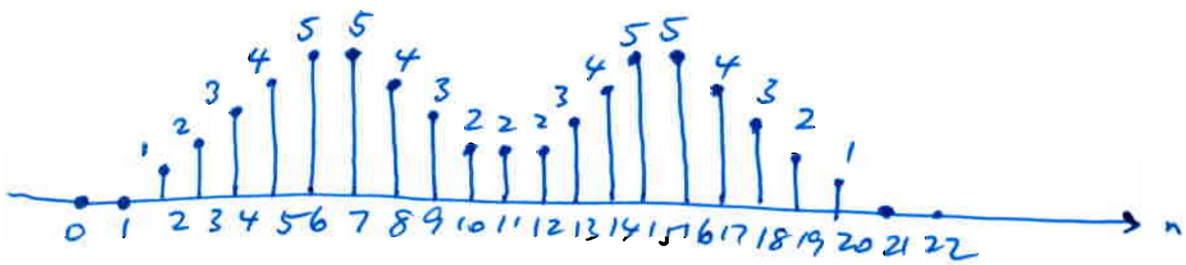
(a) $y[n] = \delta[n-1] * h[n] = h[n-1]$



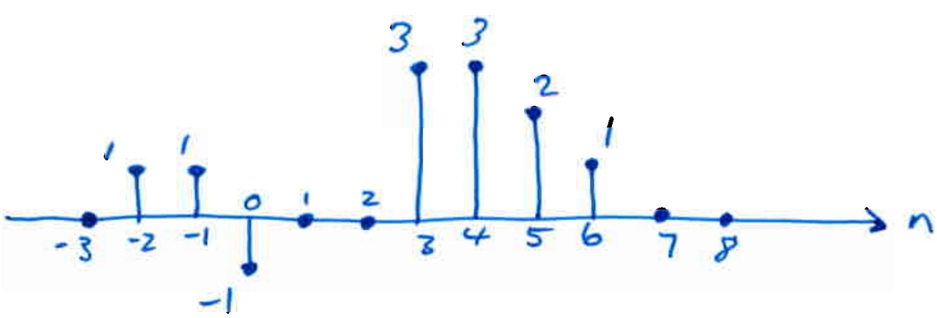
(b)



(c)



(d)



2.25

The output signal will be defined in 5 different regions:

- For $n < 0$:

$$y[n] = \sum_{k=-\infty}^{0} x[k] \cdot h[n-k]$$

Since x[n] = 0 for n < 0, the output will be equal to 0

$$y[n] = 0$$

- For $0 \le n \le N_1$:

$y[n] = \sum_{k=0}^{n} a^k$. Making use of the geometric series rule, we obtain

the following result

$$y[n] = \frac{1-a^{(n+1)}}{1-a}$$

- For $N_1 < n < N_2$:

$$y[n] = \sum_{k=0}^{N_1} a^k$$

$$y[n] = \frac{1-a^{(N_1+1)}}{1-a}$$

- For $N_2 \le n \le N_2 + N_1$

$$y[n] = \frac{1-a^{(N_1+1)}}{1-a} + \sum_{k=N_2}^{n} a^{(k-N_2)}$$

$$y[n] = \frac{1-a^{(N_1+1)}}{1-a} + a^{-N_2} \cdot \frac{a^{N_2} - a^{(n+1)}}{1-a}$$

$$y[n] = \frac{1-a^{(N_1+1)}}{1-a} + \frac{1-a^{(n-N_2+1)}}{1-a}$$

- For $N_2 + N_1 < n$

$$y[n] = \frac{1 - a^{(N_1+1)}}{1-a} + \sum_{k=N_2}^{N_2+N_1} a^{(k-N_2)}$$

$$y[n] = \frac{1 - a^{(N_1+1)}}{1-a} + a^{-N_2} \cdot \frac{a^{N_2} - a^{(N_2+N_1+1)}}{1-a}$$

$$y[n] = \frac{1 - a^{(N_1+1)}}{1-a} + \frac{1 - a^{(N_1+1)}}{1-a}$$

$$y[n] = 2 \cdot \frac{1 - a^{(N_1+1)}}{1-a}$$

PROBLEM 2.30

For each of the following systems, determine whether the system is 1) stable, 2) causal, 3.) linear, and 4.) time invariant.

a.) $T(x[n]) = \cos(\pi n)\, x[n]$

b.) $T(x[n]) = x[n^2]$

c.) $T(x[n]) = x[n] \sum_{k=0}^{\infty} \delta(n-k)$

d.) $T(x[n]) = \sum_{k=n-1}^{\infty} x(k)$

The process of determining stability & causality is simplified if the system is linear & time invariant, (LTI). For example, a general system is stable if every bounded input produces a bounded output. If the system is LTI, the test for stability simplifies, such that

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

is the qualifying criteria. The test for causality simplifies from checking that

$y(n_0)$ depends only on $x(n)$, $n \le n_0$, $\forall n_0$,

to checking that

$h(n) = 0$, $n < 0$,

if the system is LTI.

To take advantage of these properties, linearity and time invariance are determined first.

Linear systems are defined by superposition. If the system is linear, then

$$a_1 y_1[n] + a_2 y_2[n] = T(a_1 x_1[n] + a_2 x_2[n]),$$

where $x_1[n]$ and $x_2[n]$ are input functions that produce $y_1[n]$ and $y_2[n]$. Note that a_1 and a_2 are constants.

Systems are considered time invariant if

$$y(n) = T(x(n)),$$

and

$$y(n-n_0) = T(x[n-n_0]).$$

This is represented by the following figure.

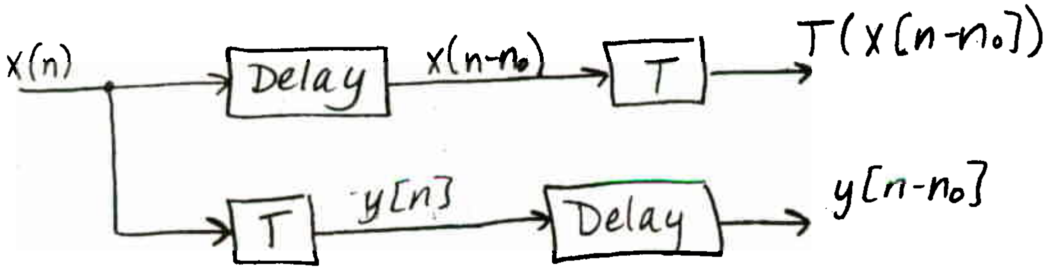


Figure 1: Flow Chart for Time Invariance

Note that $T(x(n-n_0))$ is created when $x(n)$ is delayed, and then transformed, while $y(n-n_0)$ is created when $x(n)$ is transformed and then shifted.

a.) $T(x[n]) = \cos(\pi n)\, x[n]$

LINEARITY:

$y(n) = T(x(n))$

$$T(a_1 x_1(n) + a_2 x_2(n)) = [a_1 x_1(n) + a_2 x_2(n)] \cos(\pi n)$$
$$= a_1 x_1(n) \cos(\pi n) + a_2 x_2(n) \cos(\pi n)$$
$$= a_1 T(x_1(n)) + a_2 T(x_2(n))$$
$$= a_1 y_1(n) + a_2 y_2(n)$$

Since $a_1 y_1(n) + a_2 y_2(n) = T(a_1 x_1(n) + a_2 x_2(n))$, the system is Linear.

TIME INVARIANCE:

$T(x(n))$ simply multiplies $x(n)$ by $\cos(\pi n)$. The multiplier, $\cos(\pi n)$, is shown in Figure 2

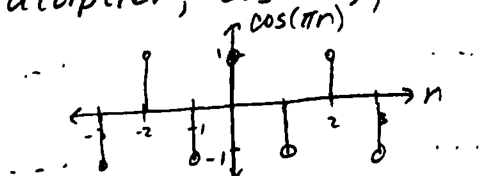


Figure 2: Plot of $\cos(\pi n)$

Thus, $\cos(\pi n) = \begin{cases} 1, & n \text{ is even} \\ -1, & n \text{ is odd} \end{cases}$.

This means that

$$T(x(n)) = \begin{cases} x(n), & n \text{ even} \\ -x(n), & n \text{ odd} \end{cases}.$$

It follows that

$$T(x(n-n_0)) = \begin{cases} x(n-n_0), & n-n_0 = \text{even} \\ -x(n-n_0), & n-n_0 = \text{odd} \end{cases}.$$

Using the method described in Figure 1,

$$y(n) = \begin{cases} x(n), & n \text{ even} \\ -x(n), & n \text{ odd} \end{cases},$$

and

$$y(n-n_0) = \begin{cases} x(n-n_0), & n-n_0 = \text{even} \\ -x(n-n_0), & n-n_0 = \text{odd} \end{cases}.$$

Since $y(n-n_0) = T(x(n-n_0))$, the system is time invariant. This system is LTI, which is intuitive since T performs amplitude modulation on $x(n)$.

STABILITY:

Since the system is LTI, it is stable if

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty .$$

The impulse response is found when $x(n) = \delta(n)$. Thus,

$$h(n) = \begin{cases} \delta(n), & n \text{ even} \\ -\delta(n), & n \text{ odd} \end{cases},$$

but since $\delta(n)$ is only non-zero at $n=0$, it simplifies to

$$h(n) = \delta(n).$$

The system IS stable since

$$\sum_{k=-\infty}^{\infty} \delta(k) < \infty .$$

CAUSAL:

Since the system is LTI, it is causal if

$$h(n) = 0, \quad n<0.$$

The impulse response, $h(n)$, was determined to be

$$h(n) = \delta(n),$$

which only has a non-zero value at $n=0$.

Thus $h(n) = 0$ for $n<0$ holds, & the system IS Causal.

b.) $T(x(n)) = x(n^2)$

LINEARITY:

$y(n) = T(x(n))$

$$T(a_1 x_1(n) + a_2 x_2(n)) = a_1 x_1(n^2) + a_2 x_2(n^2)$$
$$= a_1 y_1(n) + a_2 y_2(n)$$

Since $a_1 y_1(n) + a_2 y_2(n) = T(a_1 x_1(n) + a_2 x_2(n))$, the system IS Linear.

TIME INVARIANCE:

$y(n) = T(x(n))$

Using the method described in Figure 1,

$$T(x(n-n_0)) = x((n-n_0)^2) = x(n^2 - 2nn_0 + n_0^2),$$

while

$$y(n-n_0) = x(n^2 - n_0).$$

Since

$$y(n-n_0) \neq T(x(n-n_0)),$$

the system is NOT time invariant. Thus, the system is NOT LTI.

This answer is intuitive, since T impacts n, rather than $x(n)$.

STABILITY:

The system is stable if bounded inputs, denoted by

$$|x(n)| \leq B_x < \infty, \quad \forall n,$$

produce bounded outputs, i.e.

$$|y(n)| \leq B_y < \infty, \quad \forall n.$$

Since the transform T only acts on n, not $x(n)$, all values of $y(n)$ are also values of $x(n)$. If all $x(n)$ are bounded, it follows that $y(n)$ is bounded for all n. Thus, it IS stable.

CAUSAL:

The system is causal if $y(n_0)$ only depends on $x(n)$, $n \leq n_0$. A simple example shows that this system is not causal.

EX: $y(n) = x(n^2)$

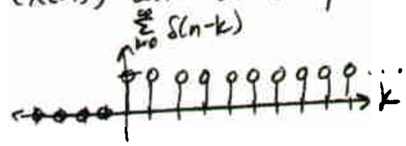
$n = 2$

$y(2) = x(4)$

Thus, $y(n_0)$ depends on future values of $x(n)$. The system is NOT causal.

c. $T(x(n)) = x(n) \sum_{k=0}^{\infty} \delta(n-k)$

The equation for $T(x(n))$ can be simplified.

$\sum_{k=0}^{\infty} \delta(n-k) \Longrightarrow$ 

The plot of $\sum_{k=0}^{\infty} \delta(n-k)$ reveals that it is equivalent to $u(n)$. Thus $T(x(n)) = x(n),\ n \geq 0$, or $x(n)u(n)$.

LINEARITY:

$y(n) = T(x(n))$

$$T(a_1 x_1(n) + a_2 x_2(n)) = a_1 x_1(n) + a_2 x_2(n),\ n>0.$$
$$= a_1 y_1(n) + a_2 y_2(n),\ n>0.$$

Since $T(a_1 x_1(n) + a_2 x_2(n)) = a_1 y_1(n) + a_2 y_2(n)$, the system is Linear.

TIME INVARIANCE:

$y(n) = T(x(n))$

Using the method described in Figure 1,

$$T(x(n-n_0)) = \begin{cases} x(n-n_0), & n-n_0 \geq 0 \\ 0, & n-n_0 < 0 \end{cases},$$

which simplifies to

$$T(x(n-n_0)) = \begin{cases} x(n-n_0), & n \geq n_0 \\ 0, & n < n_0 \end{cases}.$$

Similarly,

$$y(n) = \begin{cases} x(n), & n \geq 0 \\ 0, & n<0 \end{cases},$$

and

$$y(n-n_0) = \begin{cases} x(n-n_0), & n \geq n_0 \\ 0, & n < n_0 \end{cases}.$$

Since $y(n-n_0) = T(x(n-n_0))$, the system IS time invariant. The system is LTI. This is intuitive, since $T(x(n))$ simply samples $x(n)$ for all $n \geq 0$.

STABILITY:

Since the system is LTI, stability is determined with the test

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty.$$

In this case,

$$h(n) = \delta(n) \sum_{k=-\infty}^{\infty} \delta(n-k) = \delta(n)\, u(n) = \delta(n).$$

Testing $h(n)$ reveals that,

$$\sum_{k=-\infty}^{\infty} |\delta(k)| < \infty.$$

Thus, the system is stable.

CAUSAL:

The impulse response, $h(n)$, was found to equal $\delta(n)$. Since $\delta(n) = 0$ for $n < 0$, the system is CAUSAL.

d.) $T(x(n)) = \sum_{k=n-1}^{\infty} x(k)$

LINEARITY:

$y(n) = T(x(n))$

$$T(a_1 x_1(n) + a_2 x_2(n)) = \sum_{k=n-1}^{\infty} (a_1 x_1(k) + a_2 x_2(k))$$
$$= \sum_{k=n-1}^{\infty} a_1 x_1(k) + \sum_{k=n-1}^{\infty} a_2 x_2(k)$$
$$= a_1 \sum_{k=n-1}^{\infty} x_1(k) + a_2 \sum_{k=n-1}^{\infty} x_2(k)$$
$$= a_1 y_1(n) + a_2 y_2(n)$$

Since $T(a_1 x_1(n) + a_2 x_2(n)) = a_1 y_1(n) + a_2 y_2(n)$, the system IS Linear.

TIME INVARIANCE:

$y(n) = T(x(n))$

Using the method portrayed in Figure 1,

$$T(x(n-n_0)) = \sum_{k=n-n_0-1}^{\infty} x(k)$$

and

$$y(n) = \sum_{k=n-1}^{\infty} x(k).$$

It follows that

$$y(n-n_0) = \sum_{k=n-n_0-1}^{\infty} x(k).$$

Since $y(n-n_0) = T(x(n-n_0))$, the system is time invariant.

Thus, it is also LTI.

STABILITY:

Since the system is LTI, stability is tested by

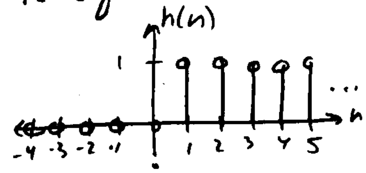
$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty .$$

In this case, the impulse response, $h(n)$, is given by

$$h(n) = \sum_{k=n-1}^{\infty} \delta(k).$$

A plot of $h(n)$ reveals that it is equal to $u(n-1)$.

$$h(n) = u(n-1)$$



Testing for stability shows that

$$\sum_{k=-\infty}^{\infty} |u(k-1)| = \infty .$$

Thus, the system is NOT stable.

CAUSALITY:

Since the system is LTI, causality is determined if

$$h(n) = 0, \quad n < 0.$$

The impulse response was previously determined to be:

$$h(n) = u(n-1),$$

which is 0 for all $n < 0$. Thus, the system IS CAUSAL.

SUMMARY

	LINEAR	TIME INVARIANT	STABLE	CAUSAL
A.)	✓	✓	✓	✓
B.)	✓		✓	
C.)	✓	✓	✓	✓
D.)	✓	✓		✓

OBSERVATIONS

1.) Of the 4 systems, the only one that is unstable has an impulse response that is a shifted unit step function.

2) Of the 4 systems, the only one that is not time invariant is the system whose transform affects n, rather than (x(n)). Since this transform enlarges n, (i.e. n^2), it makes y(n) anticipative. This forces y(n) to be non causal.

- Lisa Ehrman
gt4565a
ECE 4270

2.31) page 76

Consider the difference equation:

$$y[n] + \frac{1}{15} y[n-1] - \frac{2}{5} y[n-2] = x[n]$$

a.) general homogeneous solution

homogeneous form of difference equation

$$y[n] + \frac{1}{15} y[n-1] - \frac{2}{5} y[n-2] = 0$$

characteristic equation:

$$x^2 + \frac{1}{15}x - \frac{2}{5} = 0 \qquad \text{roots: } = -\frac{2}{3} \text{ and } \frac{3}{5},$$

∴ the general solution for the homogeneous difference equation is

A, B = constants

$$\boxed{A_h = A\left(-\frac{2}{3}\right)^n + B\left(\frac{3}{5}\right)^n}$$

b.) impulse response of causal and anti causal versions of difference equation

z-transforms used

$$y[n] + \frac{1}{15} y[n-1] - \frac{2}{5} y[n-2] = x[n]$$

$$Y(z) + \frac{1}{15} Y(z) z^{-1} - \frac{2}{5} Y(z) z^{-2} = X(z)$$

$$Y(z)\left[1 + \frac{1}{15} z^{-1} - \frac{2}{5} z^{-2}\right] = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{\left(1 + \frac{1}{15} z^{-1} - \frac{2}{5} z^{-2}\right)} = \frac{1}{\left(1 + \frac{2}{3} z^{-1}\right)\left(1 - \frac{6}{10} z^{-1}\right)}$$

$$= \frac{0.78947}{(1.5 + z^{-1})} - \frac{0.78947}{\left(-\frac{5}{3} + z^{-1}\right)} = \frac{\left(\frac{2}{3}\right) \cdot 0.78947}{\left(1 + \frac{2}{3} z^{-1}\right)} - \frac{-\left(\frac{3}{5}\right) \cdot 0.78947}{\left(1 + \frac{3}{5} z^{-1}\right)}$$

$$\therefore h_{causal} = \left(-\frac{2}{3}\right)^n u[n] (0.52632) + (0.4737)\left(\frac{3}{5}\right)^n u[n]$$

z-transform rule

$$\frac{1}{1 - az^{-1}} \rightarrow a^n u[n]$$

$$\boxed{h_{causal}[n] = (0.52632)\left(-\frac{2}{3}\right)^n u[n] + (0.4737)\left(\frac{3}{5}\right)^n u[n]}$$

next page

b) Anti-causal:

↳ i.e. $h[n]=0$, $n \geq 0$

$h_{\text{anti causa}}[n] = \sum_{k=1}^{N} (p_k)^n u[-n-1]$ general form

Using homogeneous solution

① $h[n] = [A(-2/3)^n + A_2(3/5)^n]u[-n-1]$ —

$x[n] = \delta[n]$

$h[n] + 1/15\, h[n-1] - 2/5\, h[n-2] = x[n]$ from difference equation

Use formula ① for $h[-1], h[-2]$

$n=0$: $h[0] + 1/5\, h[-1] - 2/5\, h[-2] = \delta[0] = 1$

$1/15\,[A(-3/2) + B(5/3)] - 2/5\,[A(3/2)^2 + B(5/3)^2] = 1$

$A(-3/2) + B(5/3) = 0 \rightarrow -2/5\, h[-1] = 0$

multiply both sides by 15

$A(-3/3) + (5/3)B - 6 \cdot A(3/2)^2 - 6B(5/3)^2 = 15$

$(-3/2 - 6 \cdot 9/4)A + (5/3 - 6 \cdot 25/9)B = 15$

$-15A + (-15)B = 15$

$-A - B = 1$

$A + B = -1 \rightarrow A = -1 - B$

$-3/2\, A + 5/3\, B = 0 \longrightarrow -3/2(-1-B) + 5/3\, B = 0$

$3/2 + 3/2\, B + 5/3\, B = 0$

$B = -9/19$

$\therefore A = -10/19$

Anti-causal
$h_{ac}[n] = [(-10/19)(-2/3)^n + (-9/19)(3/5)^n]u[-n-1]$

c.) prove
causeal h is stable
and anti causal is unstable

causal:

$h_c[n] = \left[\frac{10}{19}\left(-\frac{2}{3}\right)^n + \frac{9}{19}\left(\frac{3}{5}\right)^n\right]u[n]$

$(a)^{\infty} \to 0$ when $a < 1$

$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$

$(-2/3)^n$ and $(3/5)^n$ will decay as $n \to \infty$

$\therefore$ $h_c[n]$ represents a stable system.

anti-causal:

$h_{ac}[n] = \left[-\frac{10}{19}\left(-\frac{2}{3}\right)^n - \frac{9}{19}\left(\frac{3}{5}\right)^n\right]u[-n-1]$

$(a)^{\infty} \to \infty$ when $a > 1$

$\sum_{k=-\infty}^{\infty} |h_{ac}[k]| = \sum_{k=-\infty}^{-1}\left(\frac{-10}{19}\left(\frac{3}{2}\right)^n - \frac{9}{19}\left(\frac{3}{5}\right)^n\right)$

behaves exactly like $\sum_{k=1}^{\infty}\left(\frac{-10}{19}\left(\frac{2}{3}\right)^n - \frac{9}{19}\left(\frac{5}{3}\right)^n\right) \to \infty$

$\therefore$ unstable

d.) Find a particular solution when $x[n] = (3/5)^n u[n]$

$h_{causal}[n] = (0.52632)(-2/3)^n u[n] + (0.4737)(3/5)^n u[n]$

$x[n] \rightarrow X(z) = \dfrac{1}{1 - \frac{3}{5}z^{-1}}$ $\qquad H(z) = \dfrac{(0.52632)}{(1 + \frac{2}{3}z^{-1})} + \dfrac{(0.4737)}{(1 - \frac{3}{5}z^{-1})}$

$$Y(z) = X(z)\,H(z) = \frac{(0.52632)}{(1+\frac{2}{3}z^{-1})(1-\frac{3}{5}z^{-1})} + \frac{(0.4737)}{(1-\frac{3}{5}z^{-1})(1-\frac{3}{5}z^{-1})}$$

$$Y(z) = \frac{0.41552}{1.5 + z^{-1}} - \frac{0.41552}{(-\frac{5}{3} + z^{-1})} + \frac{(0.4737)}{(1-\frac{3}{5}z^{-1})^2}$$

$$Y(z) = \frac{\frac{2}{3}(0.4155)}{\frac{2}{3}(1.5 + z^{-1})} - \frac{(\frac{-3}{5})(0.41552)}{(\frac{-3}{5})(-\frac{5}{3} + z^{-1})} + \frac{(0.4737)}{(1-\frac{3}{5}z^{-1})^2}$$

$$Y(z) = \frac{0.277}{1 + \frac{2}{3}z^{-1}} + \frac{0.249312}{(1-\frac{3}{5}z^{-1})} + \frac{(0.4737)}{(1-\frac{3}{5}z^{-1})^2}$$

$$Y(z) \xrightarrow{z\text{-transform}} y[n] = 0.277 \cdot (-2/3)^n u[n] + (0.249312)\cdot(3/5)^n u[n] + (0.4737)(n)(3/5)^n u[n]$$

$$\boxed{y[n] = \left[(0.277)\left(\tfrac{-2}{3}\right)^n + (0.249)\left(\tfrac{3}{5}\right)^n + (0.4737)(n)\left(\tfrac{3}{5}\right)^n\right]\cdot u[n]}$$

chapter 2, problem 40. P80.

Wen Jiang

Solution:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$$= \sum_{k=-\infty}^{\infty} (\tfrac{3}{2})^k u[k] \cos((n-k)\pi)\, u[n-k]$$

clearly $y[n]=0$ for all $n<0$.

for $n \geq 0$, $y[n] = \sum_{k=0}^{n} (\tfrac{3}{2})^k \cos[(n-k)\pi]$

$$= \sum_{k=0}^{n} (\tfrac{3}{2})^k (-1)^{n-k} = (-1)^n \sum_{k=0}^{n} (-\tfrac{3}{2})^k$$

$$= (-1)^n \cdot \frac{1-(-\frac{3}{2})^{n+1}}{1+\frac{3}{2}}$$

Hence, the steady-state response is

if n is even, $y[n] = \dfrac{1-(-\frac{3}{2})^{n+1}}{1+\frac{3}{2}} \to \dfrac{1}{1+\frac{3}{2}} = 0.8 - 0.4j$

if n is odd, $y[n] = -\dfrac{1-(-\frac{3}{2})^{n+1}}{1+\frac{3}{2}} \to -\dfrac{1}{1+\frac{3}{2}} = -0.8+0.4j$.

Also, I used program to compute the steady-state response. and I got the same answer. Here I prefer to theoretical derivation.

(2.42)

Problem Statement

Consider the system in Figure P2.42-1.

(a) Find the impulse response $h[n]$ of the overall system.

(b) Find the frequency response of the overall system.

(c) Specify a difference equation that relates the output $y[n]$ to the input $x[n]$.

(d) Is this system causal? Under what condition would the system be stable?

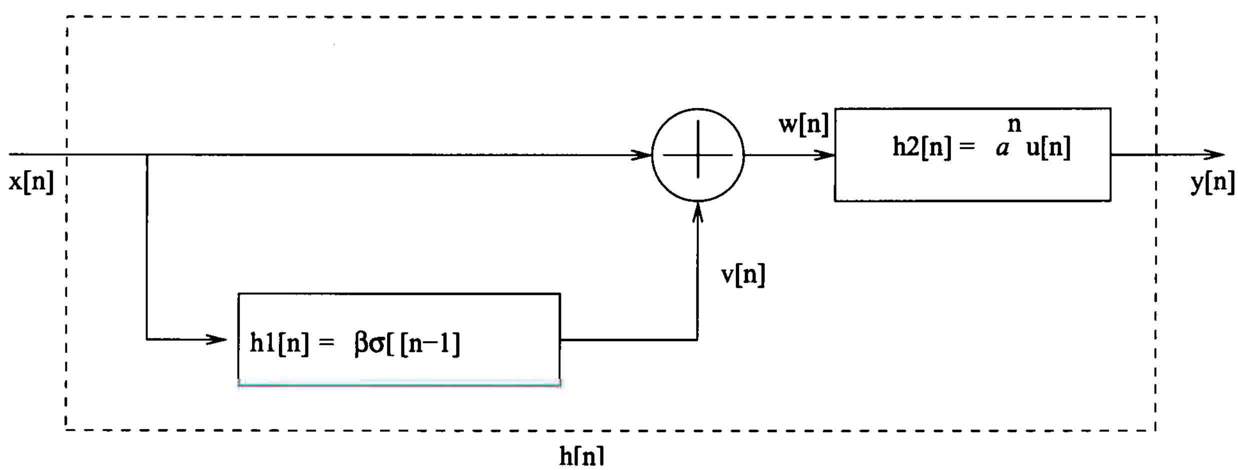


Figure 1: Figure P2.42-1

Detailed Solution

$v[n]$ and $w[n]$ are shown in Figure 1. We have:

$$v[n] = x[n] * h_1[n]$$

$$v[n] = \sum_{k=-\infty}^{k=+\infty} \beta\delta[k-1]x[n-k]$$

$$v[n] = \beta x[n-1]$$

and for $w[n]$ we have:

$$w[n] = x[n] + \beta x[n-1]$$

It's easier to find the frequency response of the system first and then use DTFT^{-1} to find the impulse response.

part(b):

$$y[n] = w[n] * h_2[n]$$

$$Y(e^{j\omega}) = W(e^{j\omega})H_2(e^{j\omega})$$

$$Y(e^{j\omega}) = (X(e^{j\omega}) + \beta e^{-j\omega}X(e^{j\omega}))H_2(e^{j\omega})$$

$$Y(e^{j\omega}) = X(e^{j\omega})(1 + \beta e^{-j\omega}).H_2(e^{j\omega})$$

Therefore the frequency response ($H(e^{j\omega})$) of the system is:

$$H(e^{j\omega}) = \frac{1+\beta e^{-j\omega}}{1-\alpha e^{-j\omega}}$$

part(a):

$$H(e^{j\omega}) = \frac{1}{1-\alpha e^{-j\omega}} + \beta\frac{e^{-j\omega}}{1-\alpha e^{-j\omega}}$$

using the inverse of F.T. we have :

$$h[n] = \alpha^n u[n] + \beta\alpha^{n-1}u[n-1]$$

part(c):

$$y[n] = h[n] * x[n]$$

$$y[n] = \sum_{k=-\infty}^{k=+\infty} h[k]x[n-k]$$

$$y[n] = \sum_{k=-\infty}^{k=+\infty} x[n-k](\alpha^k u[k] + \beta\alpha^{k-1}u[k-1])$$

$$y[n] = \sum_{k=0}^{k=+\infty} \alpha^k x[n-k] + \sum_{k=1}^{k=+\infty} \beta\alpha^{k-1}x[n-k]$$

$$y[n-1] = \sum_{k=0}^{k=+\infty} \alpha^k x[n-1-k] + \sum_{k=1}^{k=+\infty} \beta\alpha^{k-1} x[n-1-k]$$

Substitute $l + k = k'$:

$$y[n-1] = \sum_{k'=1}^{k'=+\infty} \alpha^{k'-1} x[n-k'] + \sum_{k'=2}^{k'=+\infty} \beta\alpha^{k'-2} x[n-k']$$

We can rewrite $y[n]$ as follows:

$$y[n] = x[n] + \sum_{k=1}^{k=+\infty} \alpha^k x[n-k] + \beta x[n-1] + \sum_{k=2}^{k=+\infty} \beta\alpha^{k-1} x[n-k]$$

$$y[n] = x[n] + \beta x[n-1] + \alpha\Big(\sum_{k=1}^{k=+\infty} \alpha^{k-1} x[n-k] + \sum_{k=2}^{k=+\infty} \beta\alpha^{k-2} x[n-k]\Big)$$

$$y[n] = x[n] + \beta x[n-1] + \alpha y[n-1]$$

$$y[n] - \alpha y[n-1] = x[n] + \beta x[n-1]$$

part(d): Causality requires that $h[n] = 0$ for $n < 0$. It is clear that the condition holds for h[n] of the overall system obtained in part a.

Stability requires that $\sum_{k=-\infty}^{+\infty} |h[k]| < \infty$. To satisfy the stability condition for $h[n]$ (from part (a)), the condition $|\alpha| < 1$ must be true.