**4.42.** (a) The Nyquist criterion states that $x_c(t)$ can be recovered as long as

$$\frac{2\pi}{T} \geq 2 \times 2\pi(250) \Longrightarrow T \leq \frac{1}{500}.$$

In this case, $T = 1/500$, so the Nyquist criterion is satisfied, and $x_c(t)$ can be recovered.

(b) Yes. A delay in time does not change the bandwidth of the signal. Hence, $y_c(t)$ has the same bandwidth and same Nyquist sampling rate as $x_c(t)$.

(c) Consider first the following expressions for $X(e^{j\omega})$ and $Y(e^{j\omega})$:

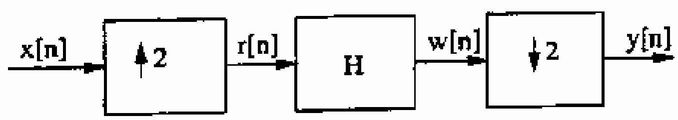
$$\begin{aligned} X(e^{j\omega}) &= \frac{1}{T}X_c(j\Omega)\,|_{\Omega=\frac{\omega}{T}} = \frac{1}{500}X_c(j500\omega) \\ Y(e^{j\omega}) &= \frac{1}{T}Y_c(j\Omega)\,|_{\Omega=\frac{\omega}{T}} = \frac{1}{T}e^{-j\Omega/1000}X_c(j\Omega)\,|_{\Omega=\frac{\omega}{T}} \\ &= \frac{1}{500}e^{-j\omega/2}X_c(j500\omega) \\ &= e^{-j\omega/2}X(e^{j\omega}) \end{aligned}$$

Hence, we let

$$H(e^{j\omega}) = \begin{cases} 2e^{-j\omega}, & |\omega| < \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases}$$

Then, in the following figure,

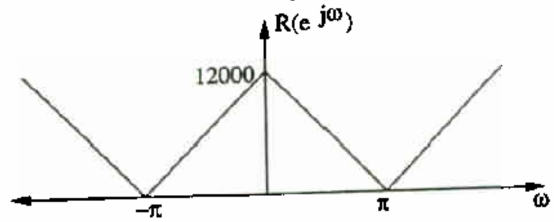
$$\begin{aligned} R(e^{j\omega}) &= X(e^{j2\omega}) \\ W(e^{j\omega}) &= \begin{cases} 2e^{-j\omega}X(e^{j2\omega}), & |\omega| < \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} \\ Y(e^{j\omega}) &= e^{-j\omega/2}X(e^{j\omega}) \end{aligned}$$

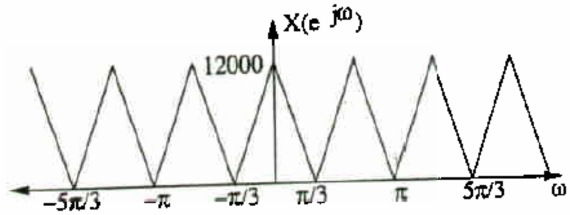



(d) Yes, from our analysis above,

$$H_2(e^{j\omega}) = e^{-j\omega/2}$$

4.44. (a) See the following figure:

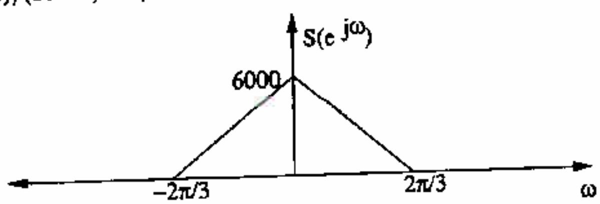



(b) For this to be true, $H(e^{j\omega})$ needs to filter out $X(e^{j\omega})$ for $\pi/3 \leq |\omega| \leq \pi$. Hence let $\omega_0 = \pi/3$. Furthermore, we want

$$\frac{\pi/2}{T_2} = 2\pi(1000) \Longrightarrow T_2 = 1/6000$$

(c) Matching the following figure of $S(e^{j\omega})$ with the figure for $R_c(j\Omega)$, and remembering that $\Omega = \omega/T$, we get $T_3 = (2\pi/3)/(2000\pi) = 1/3000$.



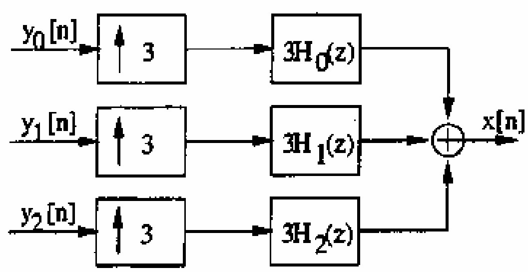

4.46. (a) Notice that

$$\begin{aligned} y_0[n] &= x[3n] \\ y_1[n] &= x[3n+1] \\ y_2[n] &= x[3n+2], \end{aligned}$$

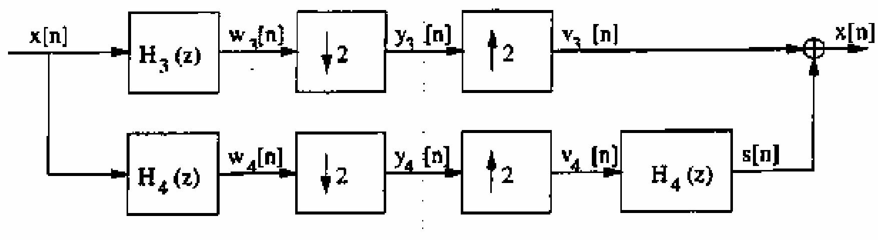
and therefore,

$$x[n] = \begin{cases} y_0[n/3], & n = 3k \\ y_1[(n-1)/3], & n = 3k+1 \\ y_2[(n-2)/3], & n = 3k+2 \end{cases}$$

(b) Yes. Since the bandwidth of the filters are $2\pi/3$, there is no aliasing introduced by downsampling. Hence to reconstruct $x[n]$, we need the system shown in the following figure:



(c) Yes, $x[n]$ can be reconstructed from $y_3[n]$ and $y_4[n]$ as demonstrated by the following figure:




In the following discussion, let $x_e[n]$ denote the even samples of $x[n]$, and $x_o[n]$ denote the odd samples of $x[n]$:

$$x_e[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

$$x_o[n] = \begin{cases} 0, & n \text{ even} \\ x[n], & n \text{ odd} \end{cases}$$

In the figure, $y_3[n] = x[2n]$, and hence,

$$\begin{aligned} v_3[n] &= \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases} \\ &= x_e[n] \end{aligned}$$

Furthermore, it can be verified using the IDFT that the impulse response $h_4[n]$ corresponding to $H_4(e^{j\omega})$ is

$$h_4[n] = \begin{cases} -2/(j\pi n), & n \text{ odd} \\ 0, & \text{otherwise} \end{cases}$$

Notice in particular that every other sample of the impulse response $h_4[n]$ is zero. Also, from the form of $H_4(e^{j\omega})$, it is clear that $H_4(e^{j\omega})H_4(e^{j\omega}) = 1$, and hence $h_4[n] * h_4[n] = \delta[n]$.

Therefore,

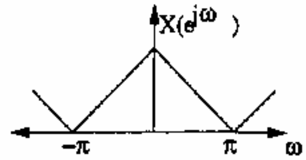
$$\begin{aligned} v_4[n] &= \begin{cases} y_4[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases} \\ &= \begin{cases} w_4[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases} \\ &= \begin{cases} (x * h_4)[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases} \\ &= x_o[n] * h_4[n] \end{aligned}$$

where the last equality follows from the fact that $h_4[n]$ is non-zero only in the odd samples.

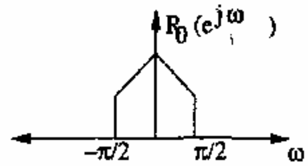
Now, $s[n] = v_4[n] * h_4[n] = x_o[n] * h_4[n] * h_4[n] = x_o[n]$, and since $x[n] = x_e[n] + x_o[n]$, $s[n] + v_3[n] = x[n]$.

**4.53.** Sketches appear below.

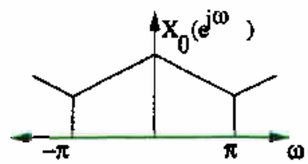
(a) First, $X(e^{j\omega})$ is plotted.



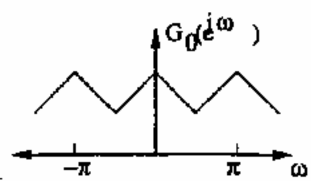

The lowpass filter cuts off at $\frac{\pi}{2}$.




The downsampler expands the frequency axis. Since $R_0(e^{j\omega})$ is bandlimited to $\frac{\pi}{M}$, no aliasing occurs.




The upsampler compresses the frequency axis by a factor of 2.




The lowpass filter cuts off at $\frac{\pi}{2} \Rightarrow Y_0(e^{j\omega}) = R_0(e^{j\omega})$ as sketched above.

(b) $G_0(e^{j\omega}) = \frac{1}{2}\left(X(e^{j\omega})H_0(e^{j\omega}) + X(e^{j(\omega+\pi)})H_0(e^{j(\omega+\pi)})\right)$

(c)

$$\begin{aligned}
Y_0(e^{j\omega}) &= \frac{1}{2}H_0(e^{j\omega})\left(X(e^{j\omega})H_0(e^{j\omega}) + X(e^{j(\omega+\pi)})H_0(e^{j(\omega+\pi)})\right)\\
Y_1(e^{j\omega}) &= \frac{1}{2}H_1(e^{j\omega})\left(X(e^{j\omega})H_1(e^{j\omega}) + X(e^{j(\omega+\pi)})H_1(e^{j(\omega+\pi)})\right)\\
Y(e^{j\omega}) &= Y_0(e^{j\omega}) - Y_1(e^{j\omega})\\
&= \frac{1}{2}X(e^{j\omega})\left[H_0^2(e^{j\omega}) - H_1^2(e^{j\omega})\right]\\
&\quad + \frac{1}{2}X(e^{j(\omega+\pi)})\underbrace{\left[H_0(e^{j\omega})H_0(e^{j(\omega+\pi)}) - H_1(e^{j\omega})H_1(e^{j(\omega+\pi)})\right]}_{=0}
\end{aligned}$$

The aliasing terms always cancel. $Y(e^{j\omega})$ is proportional to $X(e^{j\omega})$ if $[H_0^2(e^{j\omega}) - H_1^2(e^{j\omega})]$ is a constant.

$X(e^{j\omega}) = 0, \pi/3 \le |\omega| \le \pi$. $x[n]$ can be thought of as an oversampled signal. The approach is to determine whether $n_0$ is odd or even, then sample so that $n_0$ is avoided, upsampled and lowpass filter. This recovers $x[n_0]$.