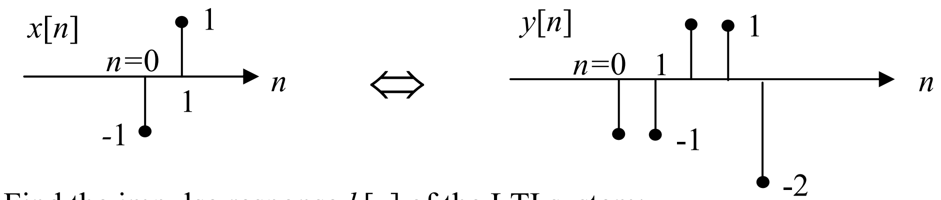
Pop Quiz 1 Name: ____________________

Signals $x[n]$ and $y[n]$ below form an input-output pair of a causal LTI system.



a) Find the impulse response $h[n]$ of the LTI system;

b) Find the frequency response of the LTI system.

c) Analysis (covered in class already).

Answer: a) $y[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$, $h[n] = 0$ for $n < 0$ due to causality.

$$y[0] = -1 = x[0]h[0] = -h[0], \qquad \therefore h[0] = 1$$

$$y[1] = -1 = x[1]h[0] + x[0]h[1] = 1 - h[1] \qquad \therefore h[1] = 2$$

$$y[2] = 1 = x[1]h[1] + x[0]h[2] = 2 - h[2] \qquad \therefore h[2] = 1$$

$$y[3] = 1 = x[1]h[2] + x[0]h[3] = 1 - h[3] \qquad \therefore h[3] = 0$$

$$y[4] = -2 = x[1]h[3] + x[0]h[4] = -h[4] \qquad \therefore h[4] = 2$$

$$y[5] = 0 = x[1]h[4] + x[0]h[5] = 2 - h[5] \qquad \therefore h[5] = 2$$

$$y[6] = 0 = x[1]h[5] + x[0]h[6] = 2 - h[6] \qquad \therefore h[1] = 2$$

$$\therefore\ h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2] + 2u[n-4]$$

b) $h_1[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$

$$H_1(e^{j\omega}) = 1 + 2e^{-j\omega} + e^{-j2\omega} = (1 + e^{-j\omega})^2 = \left[2e^{-j\frac{\omega}{2}} \cos\frac{\omega}{2}\right]^2 = 4e^{-j\omega}\cos^2\frac{\omega}{2}$$

$$F\{u[n-4]\} = e^{-j4\omega}\left\{\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)\right\}$$

Therefore, $H(e^{j\omega}) = H_1(e^{j\omega}) + F\{u[n-4]\} = 4e^{-j\omega}\cos^2\frac{\omega}{2} + e^{-j4\omega}\left\{\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)\right\}$