Name:____________________

Pop Quiz #2 (20 min.)

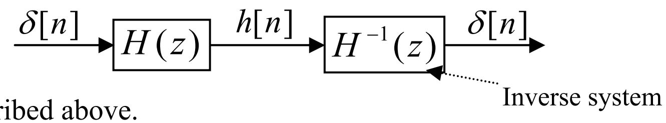
A causal LTI system is defined by the following difference equation $y[n] = \frac{\sqrt{2}}{2} y[n-1] - \frac{1}{4} y[n-2] + x[n]$

a) Find $H(z)$, the system function;

b) Find $h[n]$, the corresponding impulse response;

c) Find the system output $y_u[n]$, for $n \to \infty$, when it is driven by a step function $u[n]$.

An inverse system is one that produces an impulse as output when driven by the impulse response of the system (sometimes up to a certain delay). See Sec. 5.2.2, page 248 for inverse systems.



d) Find the impulse response of the inverse system, $h_{inv}[n]$, for the system described above.

Answer:

a) Take z-transform on both sides,

$$Y(z) = \frac{\sqrt{2}}{2} Y(z) z^{-1} - \frac{1}{4} Y(z) z^{-2} + X(z), \quad \text{that is,} \quad Y(z)\left(1 - \frac{\sqrt{2}}{2} z^{-1} + \frac{1}{4} z^{-2}\right) = X(z)$$

Therefore,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{\sqrt{2}}{2} z^{-1} + \frac{1}{4} z^{-2}} = \frac{1}{\left(1 - 0.5e^{j\pi/4} z^{-1}\right)\left(1 - 0.5e^{-j\pi/4} z^{-1}\right)}$$

b)

$$H(z) = \frac{1}{\left(1 - 0.5e^{j\pi/4} z^{-1}\right)\left(1 - 0.5e^{-j\pi/4} z^{-1}\right)} = \frac{A}{1 - 0.5e^{j\pi/4} z^{-1}} + \frac{B}{1 - 0.5e^{-j\pi/4} z^{-1}} \quad \text{where } A = (1 - e^{-j\pi/2})^{-1} \text{ and } B = (1 - e^{j\pi/2})^{-1}$$

$$h[n] = \frac{(0.5)^n \sin[\pi(n+1)/4]}{\sin[\pi/4]} u[n]$$

c)

$$y_u[n] = \sum_{k=-\infty}^{\infty} u[n-k] h[k] = \sum_{k=-\infty}^{n} h[k] \quad \text{and} \quad y_u[n]\Big|_{n\to\infty} = \sum_{k=-\infty}^{\infty} h[k] = \sum_{k=0}^{\infty} h[k]$$

$$h[n] = 1, \frac{\sqrt{2}}{2}, \frac{1}{4}, 0, -\frac{1}{16}, -\frac{\sqrt{2}}{32}, -\frac{1}{64}, 0, \ldots \quad \text{for } n = 0, 1, 2, 3, .., \text{respectively}$$

$$\sum_{k=0}^{\infty} h[k] = \left(1 + \frac{\sqrt{2}}{2} + \frac{1}{4}\right)\left[\sum_{k=0}^{\infty}\left(-\frac{1}{16}\right)^k\right] = \frac{5 + 2\sqrt{2}}{4} \frac{1}{1 + (1/16)} = \frac{20 + 8\sqrt{2}}{17}, \quad \therefore y_u[n]\Big|_{n\to\infty} = \frac{20 + 8\sqrt{2}}{17}$$

d)

$$H^{-1}(z) = 1 - \frac{\sqrt{2}}{2} z^{-1} + \frac{1}{4} z^{-2} \quad \text{Therefore,} \quad h_{inv}[n] = \delta[n] - \frac{\sqrt{2}}{2} \delta[n-1] + \frac{1}{4} \delta[n-2]$$