ECE-4270 Quiz #2

June 30, 2004

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Student Name: ____________________

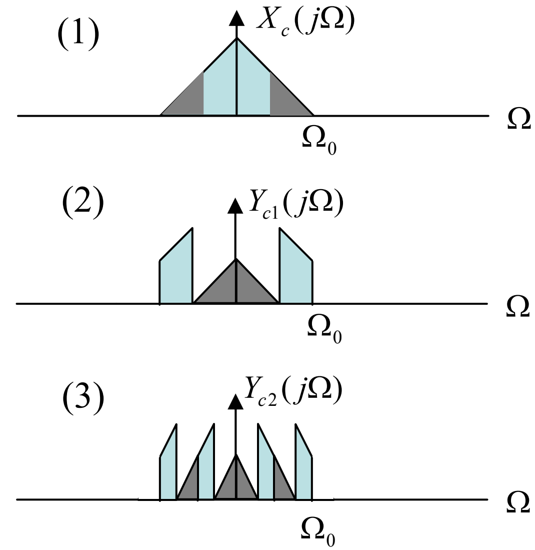
Problem 1 (15 pts. ea., 30 pts total, c) is bonus)

Figure 1 shows a typical spectrum (magnitude) of a continuous time signal $x_c(t)$. (Notice that shade is used to differentiate the high from the low frequency components.) Let $x[n]$ be a discrete-time version of $x_c(t)$, sampled at the rate of Ω_s.

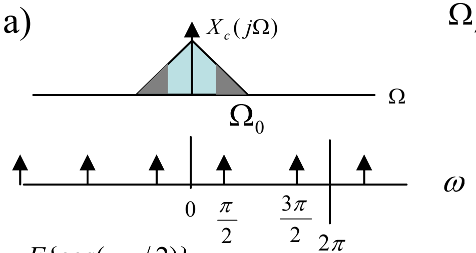
a) Sketch the spectrum of $y[n]$, if $y[n] = x[n]\cos(n\omega_0)$ where $\omega_0 = \pi/2$. Your sketches MUST include following scenarios: $\Omega_s = 2\Omega_0$, $\Omega_s = 3\Omega_0$, and $\Omega_s = 4\Omega_0$

b) Design a discrete-time processing scheme that produces a continuous time output $y_{c1}(t)$ that has a spectrum as shown in figure 2;

c) Design a discrete-time processing scheme that produces a continuous time output $y_{c2}(t)$ that has a spectrum as shown in figure 3.

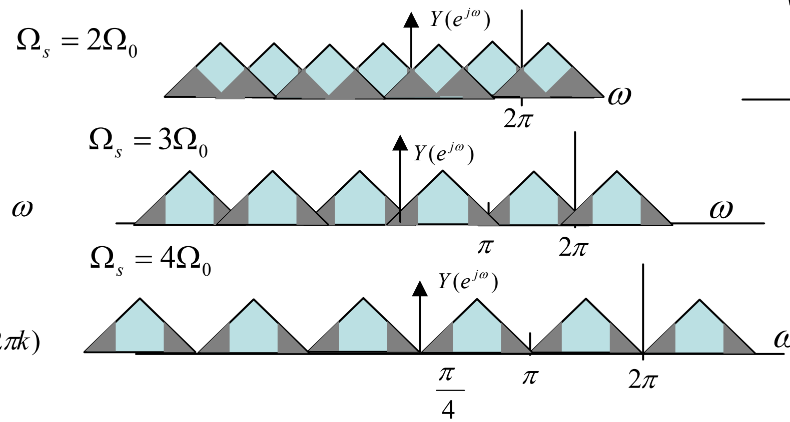


a)

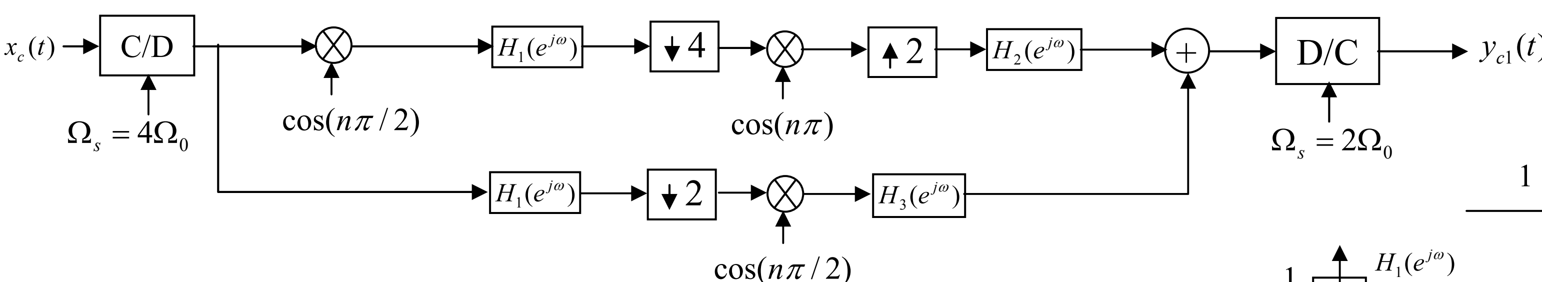


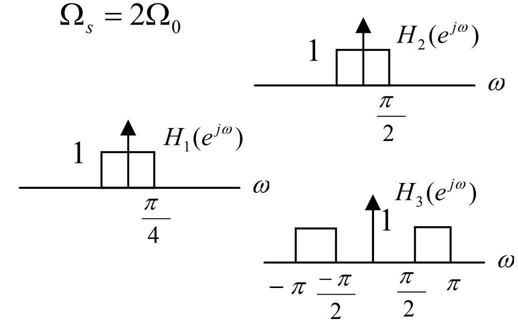
$F\{\cos(n\pi/2)\}$

$$= \sum_k \pi\delta(\omega - \frac{\pi}{2} + 2\pi k) + \pi\delta(\omega + \frac{\pi}{2} + 2\pi k)$$



b)





Problem 2 (6 pts ea. 30 pts total)

A continuous time signal $x_c(t) = \sin\Omega_0 t + \cos\Omega_0 t$, where Ω_0 is 2000 Hz or 4000π in radian per second is being sampled at 8000 Hz (or samples per second) with sampling period $T = 0.125$ ms.

a) Write an expression for the sampled discrete-time sequence $x[n]$ corresponding to this signal and sketch such a sequence as a function of the integer index n.

We'd like to change the sampling rate to 16000 Hz (i.e., sampling period $T' = 0.0625$ ms) through the up-sampling process depicted in Figure 4.24 with $L = 2$.

b) Sketch $x_e[n]$;

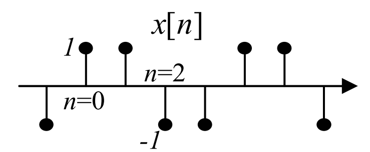
c) Find $X_e(e^{j\omega})$;

d) Find the up-sampled result (give numeric value) $x_i[n]$ for, $n = 0, 1, 2, 3, 4, 5$ through linear interpolation (Eq. 4.92) and sketch the result on top of the result of b above;

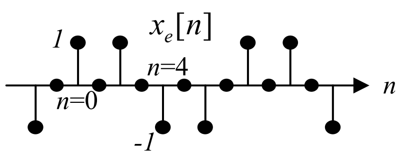
e) Find the distortion $d[n]$ for, $n = 0, 1, 2, 3, 4, 5$, where $d[n] = x_c(nT') - x_i[n]$.

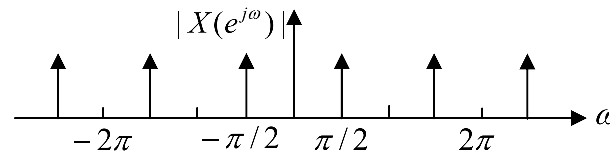
Answer:

a) $\Omega_0 T = 2\pi \bullet 0.25 = \pi/2, \quad \therefore x[n] = x_c(nT) = \sin\Omega_0 nT + \cos\Omega_0 nT = \sin\frac{n\pi}{2} + \cos\frac{n\pi}{2}$



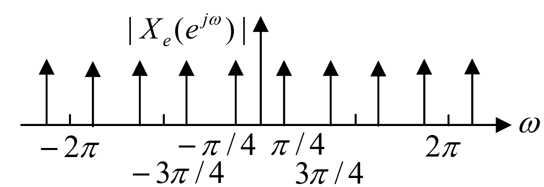
b)





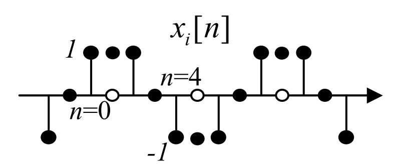
c) $x[n] = \sin\frac{n\pi}{2} + \cos\frac{n\pi}{2} = \frac{e^{jn\pi/2} - e^{-jn\pi/2}}{2j} + \frac{e^{jn\pi/2} + e^{-jn\pi/2}}{2} = \left(\frac{1-j}{2}\right)e^{jn\pi/2} + \left(\frac{1+j}{2}\right)e^{-jn\pi/2}$

$$X(e^{j\omega}) = \pi\sum_{k=-\infty}^{\infty}\left\{(1-j)\delta(\omega - \frac{\pi}{2} + 2k\pi) + (1+j)\delta(\omega + \frac{\pi}{2} + 2k\pi)\right\} \text{ and } X_e(e^{j\omega}) = X(e^{j2\omega})$$



d) For L=2, linear interpolation involves 3 points, one in the center, one to the left and one to the right; off-center ones have to be averaged first.

$x_i[0] = 1, x_i[1] = 1, x_i[2] = 1, x_i[3] = 0, x_i[4] = -1, x_i[5] = -1, x_i[6] = -1, x_i[7] = 0$



e) $x_c(0T') = 1,\ x_c(T') = \sqrt{2},\ x_c(2T') = 1,\ x_c(3T') = 0,\ x_c(4T') = -1,\ x_c(5T') = -\sqrt{2},\ x_c(6T') = -1,\ x_c(7T') = 0$

$d[0] = 0,\ d[1] = \sqrt{2} - 1,\ d[2] = 0,\ d[3] = 0,\ d[4] = 0,\ d[5] = 1 - \sqrt{2},\ d[6] = 0,\ d[7] = 0$

Problem 3 (7 pts ea. 42 pts total)

A causal LTI system has a pole-pair at $z = 0.8e^{\pm j\pi/3}$ and a zero at $z = -0.68$ with gain equal to 1 at DC (i.e. $\omega = 0$).

a) Sketch the pole-zero plot on the z-plane together with the unit circle;

b) Write the corresponding system function $H(z)$;

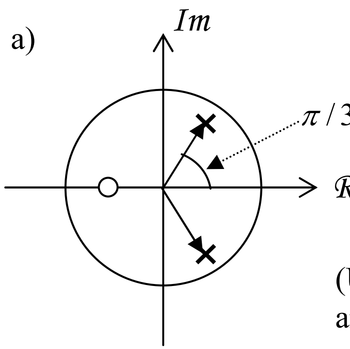
c) Find the impulse response $h[n]$ of the system;

d) Find approximately the difference in dB between the maximum and the minimum of the log-magnitude of the frequency response $20\log_{10}|H(e^{j\omega})|$;

e) Find the slope of the log-magnitude of the frequency response $20\log_{10}|H(e^{j\omega})|$ at $\omega = 0$ and π;

f) A new system has an impulse response $h'[n]$ that is obtained by multiplying $h[n]$ with $g[n] = (-1)^n$, i.e., $h'[n] = (-1)^n h[n]$. Find the poles and zeros of the new system.

a)



b)
$$H(z) = A\frac{z+0.68}{\left(z-0.8e^{j\pi/3}\right)\left(z-0.8e^{-j\pi/3}\right)} = A\frac{z^{-1}(1+0.68z^{-1})}{\left(1-0.8e^{j\pi/3}z^{-1}\right)\left(1-0.8e^{-j\pi/3}z^{-1}\right)} \qquad |z| > 0.8$$

$$H(e^{j0}) = A\frac{1.68}{(1-0.8e^{j\pi/3})(1-0.8e^{-j\pi/3})} = 2A = 1, \quad \therefore A = 1/2 \qquad r = 0.8, \quad \theta = \frac{\pi}{3}$$

c)
$$H(z) = \frac{0.5z^{-1}}{\left(1-0.8e^{j\pi/3}z^{-1}\right)\left(1-0.8e^{-j\pi/3}z^{-1}\right)} + \frac{0.34z^{-2}}{\left(1-0.8e^{j\pi/3}z^{-1}\right)\left(1-0.8e^{-j\pi/3}z^{-1}\right)}$$

(Use Eqs. 5.72 and 5.73)
$$h[n] = 0.5\frac{(0.8)^{n-1}\sin[(n)\pi/3]}{\sin[\pi/3]}u[n-1] + 0.34\frac{(0.8)^{n-2}\sin[(n-1)\pi/3]}{\sin[\pi/3]}u[n-2]$$

d)
$$20\log_{10}|H(e^{j\omega})| = 20\log_{10}(0.5) + 10\log_{10}(1+0.68^2+1.36\cos\omega) - 10\log_{10}[1-2r\cos(\theta-\omega)+r^2] - 10\log_{10}[1-2r\cos(\theta+\omega)+r^2]$$

$20\log_{10}|H(e^{j\omega})|$ attains maximum and minimum approximately at $\omega = \pi/3$ and π respectively.

$$\left[20\log_{10}|H(e^{j\omega})|\right]_{max} \approx 7.4\text{ dB} \text{ and } \left[20\log_{10}|H(e^{j\omega})|\right]_{min} \approx -23.7\text{ dB}, \quad \left[20\log_{10}|H(e^{j\omega})|\right]_{max} - \left[20\log_{10}|H(e^{j\omega})|\right]_{min} \approx 31.1\text{ dB}$$

e)
$$\frac{d}{d\omega}20\log_{10}|H(e^{j\omega})| = \frac{-1.36\sin\omega}{1.4624+1.36\cos\omega} + \frac{2r\sin(\theta-\omega)}{1-2r\cos(\theta-\omega)+r^2} - \frac{2r\sin(\theta+\omega)}{1-2r\cos(\theta+\omega)+r^2} = D(\omega)$$

$$D(\omega)\big|_{\omega=0} = 0 + \frac{2r\sin(\theta)}{1-2r\cos(\theta)+r^2} - \frac{2r\sin(\theta)}{1-2r\cos(\theta)+r^2} = 0 \qquad D(\omega)\big|_{\omega=\pi} = 0 + \frac{2r\sin(\theta-\pi)}{1-2r\cos(\theta-\pi)+r^2} - \frac{2r\sin(\theta+\pi)}{1-2r\cos(\theta+\pi)+r^2} = 0$$

f)
$$H'(z) = \sum_{n=-\infty}^{\infty} h'[n]z^{-n} = \sum_{n=-\infty}^{\infty}(-1)^n h[n]z^{-n} = \sum_{n=-\infty}^{\infty} h[n](-z)^{-n} = H(-z)$$

$$= \frac{-1}{2}\frac{z-0.68}{\left(z+0.8e^{j\pi/3}\right)\left(z+0.8e^{-j\pi/3}\right)}$$

Poles: $z = -0.8e^{\pm j\pi/3} = 0.8e^{\pm j(\pi/3)+j\pi} = 0.8e^{\pm j(2\pi/3)}$

Zero: $z = 0.68$