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Aiding Subgoal Learning: Effects on Transfer

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Students often memorize a set of steps from examples in domains such as probability and physics, without learning what subgoals those steps achieve. A result of this sort of learning can be that these students fail to solve novel problems that do not permit exactly the same set of steps even though the old goal structure is maintained. Three experiments demonstrated that both labeling and visually isolating a set of steps in examples independently help students learn a subgoal and be more likely to solve novel problems that involve that subgoal but require different steps to achieve it.

What do students learn from worked examples in domains such as probability and physics? Often, students memorize a set of steps to which they attach relatively little meaning. As a result, when faced with a test problem that has the same goal structure but requires different steps for achieving the goals, students may rigidly and incorrectly apply the old steps or fail to produce a solution at all (e.g., Reed, Dempster, & Ettinger, 1985; Ross, 1987, 1989).

Learners are more successful solving novel problems when they learn the goal structure of the problems in that domain (e.g., Anzai & Simon, 1979; Eylon & Reif, 1984). Researchers use the term *subgoal* (and *goal*) in two different ways. The first defines a subgoal to be something people—or computer programs—form when they are working on a problem and reach a point where they do not simply recognize what to do next because they have no options, too many options, conflicting options, etc. A subgoal is formed at this impasse (e.g., Newell, 1990, Chapter 4; VanLehn, 1988). The second considers subgoals to represent the task structure to be learned for solving problems in a particular domain and assumes that these subgoals can be taught to learners (e.g., Catrambone, 1994a; Catrambone & Holyoak, 1990; Dixon, 1987; Eylon & Reif, 1984). From the second point of view, a subgoal groups a set of steps under a meaningful task or purpose (e.g., Anzai & Simon, 1979; Chi & VanLehn, 1991). For instance, in the probability materials used in the current experiments, a set of multiplication and addition steps can be grouped under the subgoal "find the total frequency of the event." It is the second, task-analysis-driven view of subgoals that I followed in the present study.

An instructor might help students learn the goal structure through worked examples. The particular subgoals taught might represent an instructor's judgment about how students should decompose problems into subproblems to solve novel problems effectively. Novel problems are taken to be problems that share the same goal or task structure with training examples but that require a change in the steps for achieving at least one of the subgoals.

Much research indicates that the way learners encode examples influences how likely they are to access examples and how likely they are to apply or adapt examples successfully in new situations (e.g., Brown, Kane, & Echols, 1986; Gentner & Gentner, 1983; Gick & Holyoak, 1983). For instance, Brown et al. found that young children were more likely to use a prior story to help them solve an analogous problem if the children had either spontaneously, or through prompting, previously induced the goal structure of the story.

My aim in this study was to examine whether learners' transfer to novel problems is improved if they study examples designed to convey a solution procedure organized by subgoals versus examples designed to convey only the steps of the solution procedure.

Learning From Examples

Learners, particularly novices, typically prefer to study or refer to examples, as opposed to instructions or descriptions of principles of a domain, when working on problems (e.g., Pirolli & Anderson, 1985). Learners explicitly mention examples when solving a problem (Lancaster & Kolodner, 1988; Ross, 1984), and they follow examples rather than instructions when the two conflict (LeFevre & Dixon, 1986).

Unfortunately, people frequently do not learn from examples what is needed to solve novel problems. Rather, people tend to memorize a set of steps. Attempts to improve this situation have usually found little improvement in transfer. For instance, Reed et al. (1985) provided elaboration designed to help students understand the principles illustrated

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in algebra examples. In general, these elaborations did not improve performance on nonisomorphic problems. Similar transfer difficulties after learning from examples were documented by Gick and Holyoak (1983), Ross (1987, 1989), and others. However, several researchers have demonstrated some success in using examples to help learners transfer to novel problems (e.g., Ward & Sweller, 1990; Zhu & Simon, 1987). It is unclear why conflicting transfer results are found across these studies. The subgoal framework offered below may help to identify situations in which transfer is more or less likely to occur.

Value of Learning Subgoals

Prior work suggests that students can learn subgoals from examples. Chi and her colleagues (Chi & Bassok, 1989; Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Chi & VanLehn, 1991) examined the self-explanations learners produce when studying examples. Chi et al. (1989) divided their students into "good and poor students" (p. 158) as a function of test problem performance that followed an example-studying phase. "Good" students demonstrated superior transfer to novel problems; their self-explanations of the steps from example solutions in physics mechanics problems contained more goals, descriptions of the preconditions for actions, and explications of the consequences of actions, than did the self-explanations of "poor" students (Chi & VanLehn, 1991).

Although students who learn easily and students who have difficulty learning may differ on various dimensions in addition to the self-explanations they produce, one inference that might be drawn from Chi and her colleagues' work is that learning will be improved if examples provide the types of explanations that students who learn easily produce on their own. One of these explanation types are subgoals.

Subgoals are useful because they group a set of steps and in some sense explain what the steps accomplish. If the learner can recognize which subgoals are relevant for solving a novel problem, then those subgoals can guide the learner to the steps from the old solution procedure that need to be modified to achieve the subgoals in the new problem.[1] In contrast, a learner who had simply memorized a string of steps for solving a particular problem type, without grouping sets of steps under the subgoals they achieve, will have fewer cues to direct him or her to the steps that need to be modified for the novel problem.

A subgoal framework can be useful for explaining transfer. The framework can be used to determine the subgoals learners need to know to solve problems in a domain, and it can be used to guide the construction of examples to increase the likelihood that learners learn these subgoals. Researchers conducting studies within this framework can examine issues such as finding the most effective ways to determine the subgoals in a domain and finding effective ways of conveying those subgoals through examples.

Improving Examples

In the current study, a set of steps, which was part of the overall solution, was labeled in training examples as a way of conveying the subgoal those steps achieved. I hypothesized that example solutions that label a set of steps will make a person more likely to learn the subgoal achieved by the steps rather than merely memorizing the steps themselves. Furthermore, I hypothesized that by learning the subgoal, the learner will be more likely to successfully achieve it in a novel problem that requires a different set of steps to achieve that subgoal.

This second hypothesis was made for two reasons. First, subgoals were one type of knowledge produced more often by students who learned easily than by students who had difficulty learning in Chi and her colleagues' studies (e.g., Chi et al., 1989). Second, subgoal learning seems to be correlated with better performance at solving problems that involve the same subgoals, even if the problems are novel and require different steps to achieve the subgoals (Anzai & Simon, 1979; Catrambone & Holyoak, 1990; Mawer & Sweller, 1982).

The present study continues a line of research on factors that influence subgoal learning (Catrambone, 1994a; Catrambone & Holyoak, 1990). Catrambone and Holyoak (1990) examined whether learners studying examples that demonstrated multiple ways of achieving a subgoal would be more likely to learn the subgoal than learners studying examples demonstrating a single method. The results suggested that studying multiple methods did not aid subgoal learning, at least within the confines of a 1-hr experiment. However, subgoal learning did appear to be aided when learners studied examples that provided elaborations about the particular methods used for achieving a subgoal. However, these elaborations contained a variety of information including additional domain theory, explanations of the conditions that led to a certain method being chosen, and labels for key groups of steps. Thus, it is not clear which feature(s) of the elaboration promoted subgoal learning.

On the basis of preliminary research (Catrambone, 1994b) and the findings of Chi et al. (1989) suggesting that students who learn easily tend to group steps into subgoals, I examined whether a relatively minimal manipulation, providing a label for a set of steps, would make learners more likely to form a subgoal.

A study by Smith and Goodman (1984) supports the idea that labels can aid subgoal learning and transfer to new situations. The relevant comparison is between a group of students who followed a set of steps for assembling an electric circuit and a group who received a structurally oriented "explanatory schema" (Smith & Goodman, 1984, p. 360) with the steps. This schema consisted of statements that provided a rationale for why sets of steps needed to be

[1] The issue of learning to recognize when a subgoal is appropriate for a particular problem is not addressed here. All training and test problems involved the same subgoals. Rather, the emphasis is on learners' ability to achieve the subgoal in a new way in test problems.

carried out. Each rationale was essentially a statement of a goal that the steps achieved (e.g., "The next thing that you will have to do is to assemble the on–off switch."). When assembling a new circuit, students who had previously received the explanatory schema were more accurate in building the substructures corresponding to the goals even though the required steps were not identical to the ones followed during training.

The current study differs from Smith and Goodman's (1984) work in several ways. First, during training their students carried out a set of instructions, whereas students in the current experiments studied examples. Second, the Smith and Goodman transfer task involved the students' following a new set of instructions, whereas in the present study students had to solve novel problems. Third, the explanatory schema used by Smith and Goodman provided a rationale for why a set of steps were carried out. Conversely, although the labels used in the present experiments presumably helped students group a set of steps, it was left to the students to supply the rationale for why those steps were executed. Thus, in the present study I analyzed transfer in a more demanding situation in which I explicitly examined the role of examples in subgoal learning.

Test Domain: The Poisson Distribution

The Poisson distribution is often used to approximate binomial probabilities for events occurring with some small probability. The Poisson equation is

$$P(X = x) = \frac{[(e^{-\lambda})(\lambda^{x})]}{x!},$$

where λ is the average (the expected value) of the random variable X, e is the mathematical constant 2.718, and x is the number of successes of interest.

Consider an example of the use of the Poisson distribution in which the average number of briefcases owned per lawyer is found so that one can predict the probability of a randomly chosen lawyer owning a certain number of briefcases (see Appendix A). The method for the goal of finding λ, the average frequency of the event (e.g., owning a briefcase), could be represented as:

Goal: Find λ

Method:

1. Multiply each event category (e.g., owning exactly zero briefcases, owning exactly one briefcase, etc.) by its observed frequency.
2. Sum the results.
3. Divide the sum by the total number of trials (number of lawyers) to find the average number of briefcases per lawyer.

Learners are good at memorizing, from examples, the above steps to achieve the subgoal of finding the average frequency of an event for problems with different storylines (Catrambone & Holyoak, 1990). However, they often fail to notice that Steps 1 and 2 in the above method could also be viewed as a method for achieving the subgoal of finding the total frequency of the event (e.g., total number of briefcases owned). As a result, learners often have trouble finding the average frequency of an event when a problem provides the total frequency of that event directly (see Appendix B), rather than requiring that it be derived from the frequencies of various event-categories (Catrambone, 1994b; Catrambone & Holyoak, 1990).

If a person learns the subgoal "find the total frequency of the event," he or she might be better able to find λ in a novel problem that requires a change from the examples in how total frequency is found. For example, in the method discussed earlier for finding the average number of briefcases owned per lawyer, it might be better if the learner's method for finding λ was organized as follows:

Goal: Find λ

Method:

1. Goal: Find total number of briefcases.

   Method:

   a. Multiply each event-category by its observed frequency.
   b. Sum the results to obtain the total number of briefcases.

2. Divide the total number of briefcases by the total number of trials to obtain the average number of briefcases per lawyer.

Experiment 1

In Experiment 1, I examined subgoal learning by manipulating whether a set of steps was labeled. It was assumed that if learners see a label for a set of steps, they are more likely to link those steps to a common subgoal. Subgoal learning was then assessed in two ways. In the first, I analyzed transfer performance—how successfully students found λ—on novel problems by students who were hypothesized to have learned or not to have learned the subgoal of finding the total frequency of an event. In the second, I had these students describe how to solve problems in the Poisson domain. If the descriptions of students in the label condition included statements such as "find the total frequency" and if these were the students who also solved the novel problems more successfully, then this would provide converging evidence that these students learned that subgoal.

In this experiment the no-label group studied examples demonstrating the weighted average method for finding λ (see Appendix A, the no-label solution). The label group's examples differed in that the steps for finding the total frequency were explicitly labeled rather than merged with the overall set of steps for finding λ (see Appendix A, the label solution).

In the test phase students were asked to (a) describe how they would teach someone to solve problems like the ones they had studied and (b) solve problems requiring the use of the weighted average method to find λ or a method in which the total frequency was supplied directly (see Appendix B).

Predictions

It was hypothesized that the label group would be more likely than the no-label group to to learn the subgoal to find the total frequency. As a result, students in the label group should be more likely than students in the no-label group to mention the idea of finding a total frequency in their descriptions. It could be argued that this is a trivial prediction because the label group could simply be repeating a label or a generalization of a label from the examples. Nevertheless, if the students who mention the idea of finding total frequency are primarily those in the label group and if these students produce superior transfer performance on problems that provide the total frequency directly, then support would be given to the claims that labels aid subgoal learning and that subgoals aid transfer.

No difference was predicted between the groups in the frequency of mentioning the notion of finding an average (or λ) in their descriptions because all examples mentioned the term *average* in the solutions.

Both groups were predicted to be quite successful solving transfer problems that were isomorphic to the training examples. The label group was expected to perform better than the no-label group on test problems that provided the total frequency directly rather than requiring that it be calculated.

Method

Participants. Participants were 48 students recruited from an introductory psychology class at the Georgia Institute of Technology who received course credit for their participation. None of the students had taken a probability course before participating in the experiment.

Materials and procedure. All students initially studied a cover sheet that briefly described the Poisson distribution and how it could be used as a replacement for more cumbersome techniques for calculating probabilities involving events that can be categorized as successes and failures. The Poisson equation was presented, and a simplified notion of a random variable was also presented.

Students were randomly assigned to one of two groups. The label group ($N = 25$) studied six examples demonstrating the weighted average method for finding λ in which the steps for finding the total frequency were explicitly labeled (see the label solution in Appendix A for an example). The no-label group's ($N = 23$) examples differed in that the steps for finding the total frequency were not labeled (see the no-label solution in Appendix A). The labels seen by the students in the label group were specific to the context of the problem (e.g., "total number of briefcases owned") and were not phrased at a general level (e.g., "total frequency") so that I could minimize additional explicit general domain instruction.

After studying the examples, students were asked to describe how to solve problems in the domain. The instructions were:

> Suppose you were going to teach someone how to solve Poisson distribution problems of the types you have just studied. Please describe the procedure or procedures you would give someone to solve these problems. Please be as complete as possible. Please do not look back at the examples.

After writing their descriptions, students solved three test problems that required the use of the weighted average method for finding λ (isomorphic to the example in Appendix A), and then they solved three test problems in which total frequency was given directly (and thus λ could be found by simply dividing the given number by the total number of trials). The latter type included the problem in Appendix B and two problems isomorphic to it. Students were told not to look back at the examples when solving the test problems.

Students' written solutions were scored for whether they found λ correctly. In addition, students' descriptions of how to solve problems in the domain were scored for two primary features: an explicit mention of trying to find the total frequency and an explicit mention of trying to find an average. Two raters independently scored the descriptions and problem solutions and agreed on scoring 94% of the time. Disagreements were resolved by discussion.

Results

Students were given a score of 1 for a given problem if they found λ correctly and a score of 0 otherwise. The scores for the three problems that were isomorphic to the training examples, Problems 1–3, were summed, thus giving students a score ranging from 0 to 3 for their performance on those problems. Similarly, the scores for the three novel problems, Problems 4–6, were summed, thus giving students a score ranging from 0 to 3 for their performance on those problems.

Transfer as a function of group. As expected, both groups did quite well at finding λ on test problems that were isomorphs to the training examples (Problems 1–3), and there was no significant difference in performance, $F(1, 46) = 0.28$, $p = .60$, $MSE = .40$. The average for the label group was 2.9, and the average for the no-label group was 2.8.

As predicted, however, the label group found λ more successfully than did the no-label group on the three test problems that involved the new method for finding λ (Problems 4–6), $F(1, 46) = 5.32$, $p = .03$, $MSE = 1.82$. The average for the label group was 2.2, and the average for the no-label group was 1.3. The most frequent mistake that students made on these problems was to write in the solution area that not enough information was given to solve the problem.

Descriptions as a function of group. Students' descriptions were scored for whether they explicitly mentioned finding the total frequency and the subgoal of finding the average or λ. As expected, the groups were equally likely to mention finding the average (label, 60% and no-label, 65%), $\chi^2(1) = 0.14$, $p = .71$. As predicted, the label group mentioned finding total frequency more often than did the no-label group (52% vs. 13%), $\chi^2(1) = 8.18$, $p = .004$.

Transfer as a function of descriptions. Students who mentioned finding the total frequency in their descriptions ($N = 16$) tended to perform about the same on the isomorphic test problems when compared with students who did not mention it ($N = 32$), $F(1, 46) = 1.70$, $p = .20$, $MSE =$

0.39 (Ms = 3.0 and 2.75, respectively). As predicted, students who mentioned finding the total frequency did better than did the other students at finding λ on test problems that required the new method for finding λ, $F(1, 46) = 5.52$, $p = .02$, $MSE = 1.81$ (Ms = 2.38 and 1.41, respectively).

Students who mentioned finding the average in their descriptions ($N = 30$) did not perform significantly differently on the isomorphic test problems when compared with students who did not mention it ($N = 18$), $F(1, 46) = 2.06$, $p = .16$, $MSE = 0.39$, (Ms = 2.93 and 2.67, respectively). There was also no significant difference between these groups at finding λ for the problems that required the new method for finding λ, $F(1, 46) = 0.20$, $p = .66$, $MSE = 2.02$ (Ms = 1.80 and 1.61, respectively).

Discussion

Students in the no-label group were not able to find λ as successfully in the novel problems (Problems 4–6) as were the students in the label group. This result suggests that students in the label group were more likely to learn the subgoal of finding the total frequency as part of the solution structure for solving Poisson distribution problems. As a result, the students in the label group were able to find λ in problems that could not be solved with the steps from the training examples.

Converging evidence that the students in the label group had learned the subgoal to find the total frequency comes from students' descriptions of how to solve problems. Students in the label group were four times more likely than students in the no-label group to mention the goal of finding the total frequency. Both groups mentioned the goal of finding λ equally often which was consistent with the fact that λ was labeled in the training examples for both groups.

It could be argued that those students who wrote descriptions in which they mentioned the subgoal of finding the total frequency were also people who were simply better at transfer. However, the finding that the label and no-label groups had differential far transfer success (i.e., success at solving problems requiring new or modified steps compared to the examples) suggests that the training manipulation affected subgoal learning in addition to any effects that were due to individual differences.

The results from this experiment are consistent with the claim that manipulations to examples that promote attention to subgoals can help students learn those subgoals. Students in the label group learned the subgoal to find the total frequency, and this subgoal helped their performance on test problems that required a new way to find the total frequency. However, this new way was to recognize that the total frequency was given in the problem. Perhaps a more stringent test of whether learning a subgoal helps a learner to achieve it when a novel method is required would be to give students test problems that require a new method of calculating total frequency. Experiment 2 was designed to do this.

Experiment 2

In Experiment 2, I examined students' ability to modify an old method for calculating the total frequency to find λ. As in Experiment 1, the steps for finding the total frequency were either labeled or unlabeled in the training examples.

The new method for finding total frequency was to add the frequencies of a number of events rather than to multiply event- categories by their frequencies and then add the products (such as the method used in Appendix A). One of the two test problems requiring this modified method involved children finding seashells on the beach (see Appendix C). The numbers of shells found by a child on Day 1, Day 2, Day 3, Day 4, and Day 5 are given. The problem then asks for the probability of a randomly chosen child finding a certain number of shells. If the label group is more likely than the no-label group to learn the subgoal of finding the total frequency, then the students in the label group should be more successful than the students in the no-label group at modifying their approach for finding total frequency and thus, finding λ in this problem.

The solutions presented for the training examples were the same as in Experiment 1 except that the steps for finding total frequency and finding λ were circled, either separately or together (see Figure 1). Students saw both presentations for each example and were asked to pick the one they felt circled the steps that "go together." It was hypothesized that if a student learned the subgoal to find the total frequency, then he or she would be more likely to prefer that the steps for finding the total frequency be separated from the step for finding λ. (This is the solution in Figure 1A for the label students and Figure 1C for the no-label students.)

For each test problem, students were asked to circle the parts of their solutions that went together. Again, because students in the label group were predicted to be more likely than students in the no-label group to have learned the subgoal to find the total frequency, it was predicted that students in the label group would be more likely than students in the no-label group to circle the steps for finding the total frequency separately from the step for finding λ. Students' circling performance could provide converging evidence, along with their transfer performance, for subgoal learning.

Method

Participants. Participants were 52 students from an introductory psychology class at the Georgia Institute of Technology who participated for course credit. None of the students had taken a probability course before participating in the experiment. Students were randomly assigned to the label ($N = 26$) and no-label ($N = 26$) groups.

Materials and procedure. Both groups studied the same cover sheet used in Experiment 1 and then studied three examples illustrating the weighed average method of finding λ. For the label group, the subgoal of finding the total frequency of the event was labeled. This subgoal was not labeled for the no-label group. The examples were a subset of those used in Experiment 1.

Students saw two solutions to each example. The solutions were identical except for how the steps were circled. For instance, for

A. Label Students, Circled Separately

Total number of briefcases owned = [1(180) + 2(17) + 3(13) + 4(9)] = 289

$E(X) = \frac{289}{219} = 1.32 = \lambda$ = average number of briefcases owned per lawyer

$P(X=x) = \frac{[(e^{-\lambda})(\lambda^{x})]}{x!}$, so $P(X=2) = \frac{[(2.718^{-1.32})(1.32^{2})]}{2!} = \frac{(.27)(1.74)}{2} = .235$

B. Label Students, Circled Together

Total number of briefcases owned = [1(180) + 2(17) + 3(13) + 4(9)] = 289
$E(X) = \frac{289}{219} = 1.32 = \lambda$ = average number of briefcases owned per lawyer

$P(X=x) = \frac{[(e^{-\lambda})(\lambda^{x})]}{x!}$, so $P(X=2) = \frac{[(2.718^{-1.32})(1.32^{2})]}{2!} = \frac{(.27)(1.74)}{2} = .235$

C. No-Label Students, Circled Separately

$E(X) = \frac{1(180) + 2(17) + 3(13) + 4(9)}{219} = \frac{289}{219} = 1.32 = \lambda$ = average number of briefcases owned per lawyer

$P(X=x) = \frac{[(e^{-\lambda})(\lambda^{x})]}{x!}$, so $P(X=2) = \frac{[(2.718^{-1.32})(1.32^{2})]}{2!} = \frac{(.27)(1.74)}{2} = .235$

D. No-Label Students, Circled Together

$E(X) = \frac{1(180) + 2(17) + 3(13) + 4(9)}{219} = \frac{289}{219} = 1.32 = \lambda$ = average number of briefcases owned per lawyer

$P(X=x) = \frac{[(e^{-\lambda})(\lambda^{x})]}{x!}$, so $P(X=2) = \frac{[(2.718^{-1.32})(1.32^{2})]}{2!} = \frac{(.27)(1.74)}{2} = .235$

Figure 1. Example solutions for the problem in Appendix A in which the steps for finding the total frequency and λ are circled either separately or together in Experiment 2.

the training example in Appendix A, students in the label group saw the solutions in Figures 1A and 1B, whereas students in the no-label group saw the solutions in Figures 1C and 1D. Students were given the following instructions:

> For each example, please note that the solution is presented twice. The two presentations of each solution have the steps circled. The circles are used to indicate steps that might "go together." For instance, suppose you were following a recipe for cooking something. Perhaps the first three steps of the recipe involved putting various ingredients into a bowl and the fourth step involved stirring the ingredients with a spoon and the fifth step involved using a blender to finishing the mixing. You might draw a circle around the first three steps because they involve "adding ingredients," and you might draw a circle around the the fourth and fifth steps because they involve "blending the ingredients." The two solutions for each example are identical except that different steps are in the circles. For each example, indicate whether Presentation 1 or Presentation 2 is the one with which you most agree.

As a counterbalancing measure, the solutions that had the steps for finding the total frequency and λ in the same circle came first for half of the students in each group, whereas for the rest of the students the solution that had the steps for finding the total frequency and finding λ circled separately came first. The solution order had no effect on students' preferences for solutions or on transfer performance, and thus, was collapsed over for all analyses.

After studying the examples, students solved five target problems. The first was a weighted average problem isomorphic to the examples. The second also involved finding λ as a weighted average; however, the divisor (total number of trials) was not given directly in the problem. Rather, it had to be found by adding the number of members in each category (see Appendix D). The third provided the value of the total frequency directly (see Appendix B). The fourth and fifth involved a modification of the old method for finding the total frequency: instead of the total frequency being found by multiplying the event-categories by their frequencies and then summing, it was found by adding a set of frequencies (see Appendix C for an example).

After they solved each test problem students were asked to circle the steps of their solution that went together. Students were told not to look back at the examples when working on the test problems.

Predictions

Training examples solution preferences. Students in the label group were predicted to be more likely than students in the no-label group to prefer solutions in which the steps for finding the total frequency were circled separately from the steps for finding λ. However, it should be noted that the "circled separately" presentation for the students in the no-label group (see Figure 1C) looks a bit odd because a set of steps that are part of a fraction are circled, and the denominator is left out of the circle. Thus, students could be predisposed not to choose this presentation.

Transfer performance. It was predicted that both groups would do well on the first test problem, an isomorph to the training examples. For the second problem both groups were expected to perform similarly. Given that the subgoal of finding the number of trials was not emphasized in the training examples for either group, performance was expected to be poor. As in Experiment 1, the label group was predicted to do better at finding λ on the third test problem, which involved the recognition that the total frequency was given directly. The label group was predicted to do better on the fourth and fifth test problems, which involved finding the total frequency by adding a set of frequencies. The subgoal of finding the total frequency should have aided students in the label group in figuring out which steps they needed to modify to find the total frequency.

Segmenting of solutions to test problems. Students in the label group were predicted to be more likely than students in the no-label group to circle the steps for finding the total frequency separately from the step for finding λ in their solutions to the test problems that involve calculating the total frequency (Problems 1, 2, 4, and 5). No circling difference was predicted between the groups for the problem in which the total frequency was given directly (Problem 3) because it would have been rather odd to circle the given total frequency as a step.

Two raters independently scored the example solution preferences and problem solutions, and they agreed 88% of the time. Disagreements were resolved by discussion.

Results and Discussion

Training examples solution preferences. A given student invariably chose the same circling scheme across the three examples (i.e., either the one in which the steps for finding total frequency and the step for finding λ were circled separately or the one in which they were circled together). This was not surprising because the training examples were isomorphs. Thus, students were categorized into one of two groups: those who preferred separate circles for total frequency and λ and those who preferred that total frequency and λ be in the same circle. As predicted, students in the label group were more likely than students in the no-label group to choose solutions in which the steps for finding the total frequency were circled separately from the step for finding λ (50% vs. 19%), $\chi^2(1) = 5.44$, $p = .02$.

Transfer performance. All but one student, from the label group, correctly solved the first test problem, a weighted average problem isomorphic to the training examples.

The second test problem was also a weighted average problem, but the total number of trials, the value that would be placed in the denominator when calculating λ, was not given directly. Rather, the student had to calculate the total number of trials by adding the number of workers. As expected, the proportion of students who found λ correctly in each group did not differ significantly (label, 85% and no label, 73%), $\chi^2(1) = 1.04$, $p = .31$, although the overall performance was higher than expected.

The third test problem provided the value for the total frequency directly. As predicted, the label group correctly found λ significantly more often than did the no-label group (88% vs. 54%), $\chi^2(1) = 7.59$, $p = .006$. This replicates the finding from Experiment 1.

The fourth and fifth test problems required the students to calculate the total frequency by adding the simple frequencies (e.g., the number of shells found each day). Students were given a score of 1 for a given problem if they found λ correctly and a score of 0 otherwise. The scores for the two problems were summed, thus giving students a score ranging from 0 to 2 for their performance on those problems.

As predicted, the label group found λ more successfully than did the no-label group, $F(1, 50) = 4.15$, $p = .047$, $MSE = 0.46$ (Ms = 1.50 and 1.12, respectively). Students who solved these problems incorrectly tended, for example, to multiply day by number of shells (Problem 4) or write that not enough information was given. This result suggests that learning a subgoal does aid a person in achieving it in a novel problem that requires a modification of an old method.

Segmenting of solutions to test problems. For each test problem that required λ to be calculated (Problems 1, 2, 4, and 5), students were given a score of 1 if they circled the steps for finding the total frequency separately from the step for finding λ in their solution and a score of 0 otherwise. The scores for the problems were summed, thus giving students a score ranging from 0 to 4.

As expected, students in the label group were more likely to circle the steps for finding the total frequency separately from the step for finding λ in their solutions than were the students in the no-label group, $F(1, 50) = 8.39$, $p = .006$, $MSE = 1.32$ (Ms = 1.15 and 0.23, respectively).

Also as expected, there was no difference between the groups on Problem 3 in which the total frequently was provided directly, $\chi^2(1) = 2.08$, $p = .15$. Only two students, both in the label group, circled the total frequency. The rest of the students simply circled the entire set of calculations that they used for finding λ (if they found a value for λ).

Experiment 3

The results from the first two experiments are consistent with the interpretation that the design of examples that promote attention to subgoals can be successful in helping students to learn those subgoals and to transfer more successfully. In both experiments a labeling manipulation was used to promote attention to the subgoals. However, the examples containing a label for total frequency differed from the no-label examples in an additional way: The steps for finding total frequency were on a separate line from the rest of the steps for finding λ. Thus, it is possible that this visual isolation either by itself or in combination with the label, produced the superior transfer. Perhaps manipulations that lead learners to group a set of steps will make those learners more likely to form a subgoal to relate those steps. In Experiment 3, I examined this possibility with four groups of students.

The visual-isolation group studied example solutions such as the one in Appendix E (this is a solution to the problem in Appendix A) in which the total frequency steps were on their own line without a label. The separate-line label group studied example solutions isomorphic to those seen by the label group in Experiment 1 (see Appendix A). Note, though, that this solution style, besides labeling the steps for finding the total frequency, places those steps on a separate line from the rest of the steps for finding λ. Thus, students in this group saw solutions that involved both labeling and visual isolation. To examine whether there was any interaction between labeling and visual isolation, I had the same-line label group study examples that had the steps for finding the total frequency labeled but had those steps located on the same line as the rest of the steps for finding λ (see Appendix E). Finally, a no-label group analogous to the group from Experiment 1 (see Appendix A) was included as a baseline.

According to the label view, the label is primarily responsible for subgoal learning. Thus, students who study examples in which the steps for finding total frequency are labeled—the separate-line label and the same-line label groups—should outperform the visual-isolation and no-label groups on transfer problems because the steps for finding total frequency were not labeled in the examples studied by the latter two groups.

According to the grouping view, grouping—and thus subgoal learning—is promoted by labeling or visual isolation. Under this view, the label groups and the visual-isolation group should perform similarly, and all these groups should outperform the no-label group. Neither view explicitly predicted an interaction between labeling and visual isolation. However, if a label and visual isolation are required for grouping and subgoal formation, then the separate-line label group should perform better than all other groups on the transfer problems.

Method

Participants. Participants were 118 students from an introductory psychology class at the Georgia Institute of Technology who participated for course credit. None of the students had taken a probability course before participating in the experiment.

Materials and procedure. All students studied the same cover sheet used in the prior experiments and then studied two isomorphic examples illustrating the weighted average method for finding λ. The examples were a subset of those used in Experiment 1. Fewer study examples were used in this experiment because pilot testing with students receiving six, three, or two examples showed no effect of number of examples on performance.

Students were randomly assigned to one of four groups. Students in the same-line label group ($N = 30$) had the steps for finding the total frequency labeled, but the steps were on the same line as the rest of the steps for finding λ. The solution to the problem in Appendix A studied by the same-line label group is shown in Appendix E. The separate-line-label group ($N = 30$) also had the steps for finding total frequency labeled, but they were on a separate line from the rest of the steps for finding λ. The solution to the problem in Appendix A that was studied by the separate-line label group was identical to the one studied by the label group in Experiment 1 (see Appendix A). The visual-isolation group ($N = 29$) studied examples in which the steps for finding total frequency were unlabeled but on their own line (see Appendix E). The no-label group ($N = 29$) saw solution types identical to those studied by the no-label group in Experiment 1 (see Appendix A).

As in Experiment 1, students were asked to describe how to solve problems in the domain after they finished studying the examples. After writing their descriptions, students solved three test problems. The first problem required the use of the weighted average method for finding λ (isomorphic to the example in Appendix A). The second problem provided the total frequency directly (see Appendix B). The third problem involved the addition of simple frequencies to get the total frequency (see Appendix C).

Students were told not to look back at the examples when solving the test problems.

Students' written solutions were scored for whether they found λ correctly. Students' descriptions of how to solve problems in the domain were scored for whether the students mentioned trying to find the total frequency and trying to find an average. Two raters independently scored the descriptions and problem solutions and agreed on scoring 90% of the time. Disagreements were resolved by discussion.

Results and Discussion

Transfer as a function of group. As expected, all groups did quite well at finding λ on the test problem that was isomorphic to the training examples. Only two students, one in the same-line label group and one in the separate-line label group, solved this problem incorrectly.

For the two far transfer problems, students were given a score of 1 for a given problem if they found λ correctly and a score of 0 otherwise. The scores for the two problems were summed, thus giving students a score ranging from 0 to 2 for their performance on those problems.

There were significant differences among the four groups with respect to finding λ in the novel test problems, $F(3, 114) = 5.53$, $p = .0014$, $MSE = 0.52$, with means of 1.7, 1.6, 1.5, and 1.0 for same-line label, separate-line label, visual-isolation, and no-label groups, respectively. As predicted by the grouping view, Shaffer (1986) sequential Bonferroni pairwise comparisons (familywise $\alpha = .05$) indicated that both label groups and the visual-isolation group outperformed the no-label group, all $ps < .0167$. Also consistent with the grouping view, but not with the label view, was the finding that no reliable performance differences were found between the two label groups or between either label group and the visual-isolation group (all $ps > .05$).

Descriptions as a function of group. Students' descriptions were scored for whether they explicitly mentioned the subgoal of finding the total frequency and the subgoal of finding the average or λ. As expected, the groups were equally likely to mention finding λ (same-line label, 87%; separate-line label, 87%; visual isolation, 76%; and no label, 83%), $\chi^2(3) = 1.62$, $p = .65$.

There were significant differences among the four groups with respect to the frequency of mentioning finding total frequency, $\chi^2(3) = 17.43$, $p = .0006$, with percentages of 43%, 57%, 31%, and 7% for same-line label, separate-line label, visual-isolation, and no-label groups, respectively. Consistent with the grouping view and the transfer results, pairwise comparisons (familywise $\alpha = .05$) indicated that the two label groups each mentioned finding total frequency more often than did the no-label group (both $ps < .0167$, as did the visual-isolation group ($p < .0167$, one-tailed). Differences among the separate-line, same-line, and visual-isolation conditions were not statistically significant according to the Shaffer (1986) sequential Bonferroni procedure, all $ps > .025$.

Transfer as a function of descriptions. As expected, there were no significant performance differences on the novel test problems between students who mentioned finding λ ($N = 98$) and those who did not ($N = 20$), $F(1, 116) = 1.04$, $p = .31$, $MSE = 0.57$ ($Ms = 1.5$ and 1.3, respectively).

Students who mentioned finding the total frequency ($N = 41$) were more successful than other participants ($N = 77$) at finding λ on the novel test problems, $F(1, 116) = 5.77$, $p = .018$, $MSE = .55$ ($Ms = 1.7$ and 1.3, respectively).

The results of Experiment 3—similar transfer performance by the label groups and visual-isolation groups, and better performance than the no-label group—support the grouping view of subgoal formation and also suggest that there is no apparent interaction between labeling and visual isolation on subgoal formation. The tendency of the label and visual-isolation groups to mention finding total frequency in their descriptions more often than the no-label group is also consistent with the claim that these students were more likely to form a subgoal for finding total frequency.

Finally, students who mentioned finding total frequency in their descriptions showed better transfer on the novel transfer problems. Students in Experiment 1 who mentioned total frequency also transferred better on the novel problems. In addition, in a prior study (Catrambone, 1994b) I also found this relationship between descriptions and transfer performance. Together, these studies suggest that students' descriptions provide a potentially useful measure of subgoal learning. The transfer and description results provide converging evidence for the hypothesis that manipulations that encourage grouping enhance subgoal learning and thus, transfer.

General Discussion

Learners' problem-solving knowledge in domains such as probability often seems to be focused on the mathematical steps illustrated in training examples. This translates into poor performance on transfer problems. If subgoals could be conveyed to learners, then learners should be more successful on novel problems. One way these subgoals can aid learners is by guiding them to the steps in the old solution procedure that need to be changed to achieve the subgoals in a novel problem.

All three experiments presented here suggest that if people are led to learn a subgoal, in this case either by studying examples that label the steps that achieve the subgoal or by visually isolating those steps, then they are more likely to successfully achieve it in a novel problem. It is impressive to find a performance difference between the label and visual-isolation groups compared with the no-label groups given that the new methods for finding the total frequency in the test problems were seemingly straightforward: adding a set of frequencies or recognizing that the total frequency was given directly. This may partly be an effect that is due to *Einstellung* or mechanization (Luchins, 1942): Students may have been biased against altering a memorized set of steps.

Besides transfer differences, the experiments provide additional evidence that manipulations that encourage grouping can help people learn subgoals. Across Experiments 1 and 3, descriptions given by students in the label and visual-isolation groups of how to solve Poisson problems mentioned the goal to find the total frequency more often than did descriptions given by students in the no-label group. In Experiment 2, students in the label group preferred, more than did students in the no-label group, example solutions that separated the steps for finding total frequency from the step for finding λ. Students in the label group were also more likely than were students in the no-label group to circle the total frequency steps as a unit in their solutions to the test problems.

The superior transfer performance of the label and visual-isolation groups across the experiments is particularly impressive in light of the extensive elaboration provided to students in the study by Catrambone and Holyoak (1990). In that study, the overall transfer advantage for students receiving elaboration, although not reliable across all far transfer problems, was in line with performance by students in the label and visual-isolation groups in the present study who received a much more minimal manipulation. Thus, it appears that the label and visual isolation manipulations may be distillations of the information that provided the most benefit in the earlier study.

Related problem solving research by Reed et al. (1985) and Ross (1987, 1989) demonstrates that learners become attached to superficial details and mathematical procedures of examples in lieu of acquiring more generalized knowledge about how to solve problems in a particular domain. Although Reed et al. attempted to provide elaborations to help students go beyond the mathematical details of the training problems, they found poor performance on far transfer problems. The elaborations, though, may have failed to provide support for subgoal learning. The results of the present study suggest that a large amount of elaboration is certainly not the key to improving transfer from examples (see also Kieras & Bovair, 1984); rather, the additional information can be quite minimal if it focuses on the right kind of knowledge. This knowledge can be fruitfully conceptualized as subgoals.

Implications

The results strongly suggest that learners are often unable on their own to make a set of mathematical steps meaningful. Rather, students need help from either a teacher or textbook. This is an important finding because some educators may have a tendency to assume that the meaning of a set of steps is obvious and that the students will surely recognize their overall purpose. This assumption may be wrong much more often than educators would like to believe.

Even when educators recognize the value of subgoals, they might not be skilled at identifying the best ones for a particular problem-solving domain, especially in areas such as math and physics. For instance, Chi et al. (1989, p. 149) discussed a problematic mechanics example from a textbook. In the example, a block was suspended from a ceiling by two pieces of rope joined at a knot and by a third piece of rope going from the knot to the block. The task was to find the magnitude of two of the forces given the third force. The solution states that the knot where the three strings are joined should be considered the body. However, no explanation was given as to why this decision was made. The decision was made because to find a force in terms of other forces, the student must determine that the forces act on a common point. In this problem the only place where all three forces act was the knot. This critical subgoal of finding a common point where the forces were acting was information that would have been useful for students to have when solving future problems. However, instead of conveying this subgoal, the example was more likely to convey a series of steps that may or may not have been useful for other problems.

There may not exist a correct set of subgoals to be learned for a particular domain. Researchers might show one set to be more effective than another by looking at the problem-solving performance of students taught one set or the other. Thus, educators who differ in opinion about the usefulness of certain subgoals can explicitly compare their subgoal sets because the educators are at least using a common cognitive language.

The present study provides some information on this issue. Across the studies most students formed the subgoal to find λ. However, this subgoal was not all that useful to the students for solving novel transfer problems. If it were, then students in the no-label group should have successfully found λ in those problems as often as did the other students. That the no-label group did not suggests that learning the lower level subgoal of finding total frequency was crucial for transfer success. It is not clear whether an appropriate level can be proven or derived with a logical analysis or a cognitive architecture, but it is an interesting issue for future research.

Besides differing on which subgoals they believe are the most useful to teach, educators and researchers might also be unclear about the best ways to aid subgoal learning. The current results indicate that labeling and visual isolation can be effective techniques.

Extensions

Whereas the current results suggest that examples can be improved to aid problem-solving transfer, Chi and her colleagues (e.g., Chi & Bassok, 1989; Chi & VanLehn, 1991) suggested that educators can improve learning by teaching students to produce better self-explanations. Chi showed that individuals vary in terms of what kinds of information they extract from examples. In fact, students in the present study may have varied in what they learned from examples regardless of the experimental manipulations. That is, students who learn easily could have learned the goal to find the total frequency on their own when studying the examples regardless of the manipulation, whereas the students who have difficulty learning might have been less likely to do so (e.g., Chi et al., 1989). Unfortunately, no a priori information was collected by which to classify students in the experiments presented here. As a result, it is possible that the effects of the manipulations might have been clouded. In future experiments similar to the ones presented here, researchers could segregate students into those who learn easily and those who do not (by means of SAT scores or performance on some prior task) and examine whether the manipulations affect both groups in the same way.

Researchers could further test the subgoal approach by teaching learners subgoals that are hypothesized not to match well with the subgoals needed to solve test problems. These learners should perform as poorly as, or perhaps worse than, learners who simply memorized a set of steps. That is, if subgoals are used to guide problem solving performance, then inappropriate subgoals should hinder transfer.

It is suggested here that an important factor for educators interested in promoting transfer is to ensure that somehow the relevant subgoals for a domain are conveyed to learners. This implies that a crucial early step in teaching problem solving in a domain is for teachers to spend time identifying to themselves the useful subgoals from a novice's perspective. How might this be done? One possibility is to first identify a target set of problems that the instructor wants the students to be able to solve. Then the instructor should write out the solutions to these problems and analyze them to determine the subgoals achieved by groups of steps that constitute the solution procedures to the problems. Clearly, it will be important for researchers to find a standardized way of identifying subgoals that are to be taught. This might be a difficult task, but one that could greatly benefit teaching and learning.

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(*Appendixes follow on next page*)

Appendix A

Training Example Used in Experiment 1 With No-Label and Label Solutions

A judge noticed that some of the 219 lawyers at City Hall owned more than one briefcase. She counted the number of briefcases each lawyer owned and found that 180 of the lawyers owned exactly 1 briefcase, 17 owned 2 briefcases, 13 owned 3 briefcases, and 9 owned 4 briefcases. Use the Poisson distribution to determine the probability of a randomly chosen lawyer at City Hall owning exactly two briefcases.

a. No-Label Solution:

$$E(X) = \frac{1(180) + 2(17) + 3(13) + 4(9)}{219} = \frac{289}{219} = 1.32 = \lambda = \text{average number of briefcases owned per lawyer}$$

$$P(X = x) = \frac{[(e^{-\lambda})(\lambda^{x})]}{x!}$$

$$P(X = 2) = \frac{[(2.718^{-1.32})(1.32^{2})]}{2!} = \frac{(.27)(1.74)}{2} = .235$$

b. Label Solution:

Total number of briefcases owned = [1(180) + 2(17) + 3(13) + 4(9)] = 289

$$E(X) = \frac{289}{219} = 1.32 = \lambda = \text{average number of briefcases owned per lawyer}$$

$$P(X = x) = \frac{[(e^{-\lambda})(\lambda^{x})]}{x!}$$

$$P(X = 2) = \frac{[(2.718^{-1.32})(1.32^{2})]}{2!} = \frac{(.27)(1.74)}{2} = .235$$

Appendix B

Test Problem in Which Total Frequency Is Given Directly

A number of celebrities were asked how many commercials they made over the last year. The 20 celebrities made a total of 71 commercials. Use the Poisson distribution to determine the probability that a randomly chosen celebrity made exactly 5 commercials.

Solution (not seen by students):

$$E(X) = \frac{71}{20} = 3.55 = \lambda = \text{average number of commercials per celebrity}$$

$$P(X = 5) = \frac{[(2.718^{-3.55})(3.55^{5})]}{5!} = \frac{(.029)(563.8)}{120} = .135$$

Appendix C

Test Problem in Which Total Frequency Is Calculated by Adding Simple Frequencies (Experiment 2)

Over the course of the summer, a group of 5 kids used to walk along the beach each day collecting seashells. We know that on Day 1 Joe found 4 shells, on Day 2 Sue found 2 shells, on Day 3 Mary found 5 shells, on Day 4 Roger found 3 shells, and on Day 5 Bill found 6 shells. Use the Poisson distribution to determine the probability of a randomly chosen kid finding 3 shells on a particular day.

Solution (not seen by students):

$$E(X) = \frac{4 + 2 + 5 + 3 + 6}{5} = \frac{20}{5} = 4.0 = \lambda = \text{average number of shells per kid}$$

$$P(X = 3) = \frac{[(2.718^{-4.0})(4.0^3)]}{3!} = \frac{(.018)(64)}{6} = .195$$

Appendix D

Test Problem With Weighted Average Method to Find λ but in Which Trials (Denominator) Must Be Found by Adding Number of Members in Each Category (Experiment 2)

A construction crew had a varying number of people who knew how to use a jackhammer, depending on the particular job that was needed. On 10 of the jobs they did, only one person knew how to use a jackhammer, on 13 of the jobs 2 people knew how to use jackhammers, on 6 of the jobs 3 people knew how to use jackhammers, and on 7 of the jobs 4 people knew how to use jackhammers. Use the Poisson distribution to determine the probability of exactly two people in the crew knowing how to use a jackhammer on a randomly chosen job.

Solution (not seen by students):

$$E(X) = \frac{1(10) + 2(13) + 3(6) + 4(7)}{10 + 13 + 6 + 7} = \frac{82}{36} = 2.28 = \lambda = \text{average number of “knowers” per job}$$

$$P(X = 2) = \frac{[(2.718^{-2.28})(2.28^2)]}{2!} = \frac{(.102)(5.2)}{2} = .265$$

Appendix E

Instance of Example Solution Studied by the Visual Isolation and Same Line Groups (Experiment 3)

a. Visual-Isolation Group

$$1(180) + 2(17) + 3(13) + 4(9) = 289$$

$$E(X) = \frac{289}{219} = 1.32 = \lambda = \text{average number of briefcases owned per lawyer}$$

$$P(X = x) = \frac{[(e^{-\lambda})(\lambda^x)]}{x!}$$

$$P(X = 2) = \frac{[(2.718^{-1.32})(1.32^2)]}{2!} = \frac{(.27)(1.74)}{2} = .235$$

b. Same-Line Label Group

$$E(X) = \frac{1(180) + 2(17) + 3(13) + 4(9)}{219} = \frac{\text{total number of briefcases owned}}{219} = \frac{289}{219}$$

$$= 1.32 = \lambda = \text{average number of briefcases owned per lawyer}$$

$$P(X = x) = \frac{[(e^{-\lambda})(\lambda^x)]}{x!}$$

$$P(X = 2) = \frac{[(2.718^{-1.32})(1.32^2)]}{2!} = \frac{(.27)(1.74)}{2} = .235$$

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