

Worksheet 7.1 Diagonalization of Symmetric Matrices

Worksheet Exercises

1. Construct a spectral decomposition for $A = PDP^T$.

$$A = \begin{pmatrix} 1 & -5 \\ -5 & 1 \end{pmatrix}, \quad P = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 6 & 0 \\ 0 & -4 \end{pmatrix}$$

2. If possible, give an example of:

- (a) a matrix $A \in \mathbb{R}^{2 \times 2}$ that is diagonalizable but not orthogonally diagonalizable
- (b) a matrix $A \in \mathbb{R}^{2 \times 2}$ that is orthogonally diagonalizable but not invertible.

3. Indicate whether the statements are true or false.

- (a) If A is orthogonally diagonalizable, then so is A^2 .
- (b) For any matrix $A \in \mathbb{R}^{m \times n}$, AA^T and $A^T A$ are symmetric matrices.