Alternating Opportunistic Large Arrays in Broadcasting for Network Lifetime Extension

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***Abstract*—We propose a protocol for broadcasting in wireless multihop networks that is based on a form of cooperative transmission called the Opportunistic Large Array (OLA). An SNR ("transmission") threshold is used to define two mutually exclusive sets of OLAs, such that the union of the sets includes all the nodes in the network. The broadcast protocol then alternates between the sets for each broadcast and is called Alternating OLA with Transmission Threshold (A-OLA-T). Under A-OLA-T, all participating nodes transmit with the same low power, therefore the energies of the nodes in the network drain efficiently and uniformly, extending the network life relative to broadcasts that use simple OLA or non-alternating OLAs with a transmission threshold. In this paper, we optimize the A-OLA-T protocol under the continuum assumption (very high node density).**

***Index Terms*—Broadcast, cooperative transmission, opportunistic large arrays, wireless sensor networks.**

I. Introduction

COOPERATIVE Transmission (CT) is an effective way to achieve the benefits of an array transmitter (diversity and/or array gain) by having two or more nodes cooperate to transmit the same message to enhance the energy-efficiency of the wireless system [1], [2]. There are a few works that present the benefits of multi-node cooperation [3]-[6], and a few others that specifically address the energy-efficiency of cooperative *broadcasts* in wireless multi-hop networks [7]-[11]. In this paper, we present a new "alternating sets" broadcast strategy that uses a simple form of cooperative transmission called the *Opportunistic Large Array* (OLA) [7]. The 'Alternating OLA with a Transmission Threshold' (A-OLA-T) algorithm introduced in this paper is an extension of a previous non-alternating OLA with a Transmission Threshold (OLA-T) algorithm [8]. A-OLA-T ensures that all nodes in a network contribute efficiently and equally to broadcasts, thereby increasing the network longevity for multihop wireless networks.

An OLA is a group of nodes that behave without coordination between each other, but naturally fire at approximately the same time in response to energy received from a single source or another OLA [7]. So in OLA-based schemes, each node receives a superposition of signals transmitted by multiple nodes. This is in stark contrast to non-cooperative schemes where ideally each node receives a message from just one transmitter. All the transmissions within an OLA are repeats of the same waveform; therefore the signal received from an OLA has the same model as a multipath channel. As long as the receiver, such as a RAKE receiver, can tolerate the effective delay and Doppler spreads of the received signal and extract the diversity, decoding can proceed normally [11]. Even though many nodes may participate in an OLA transmission, energy can still be saved because all nodes can reduce their transmit powers dramatically and large fade margins are not needed.

In the original OLA-based broadcasting scheme [7], or 'Basic OLA,' the first OLA comprises all the nodes that can decode the transmission from the originating node; then the first OLA transmits and all nodes that can decode that transmission and that haven't decoded that message before, form the second OLA, and so forth. In [10], the authors compared the power efficiency of OLA-based cooperative broadcasting relative to non-cooperative broadcasting, both with optimal power allocation, and showed that the former saved at least 60% of the radiated power. One can control the node participation in each 'hop' or OLA by using an explicit power "transmission" threshold in the receiver, and this is the OLA-T algorithm [8]. We note that the OLA-T concept was proposed as the "Dual Threshold Cooperative Broadcast (DTBC)," but not analyzed in [10]. Compared to Basic OLA, OLA-T was shown in [8] to save up to a maximum of 32% of the transmitted energy by limiting the number of nodes in each OLA.

Unlike the OLA-based schemes above, A-OLA-T optimizes groups of broadcasts instead of a single broadcast. The transmission threshold is used to minimize the OLA sizes while maintaining mutually exclusive sets of OLAs on consecutive broadcasts. An important feature that all the OLA-based schemes share is that no individual nodes are addressed. This makes this protocol scalable with node density.

II. System Model

For our analysis, we adopt the notation and assumptions of [8], most of which were used earlier in [11]. Half-duplex nodes are assumed to be distributed uniformly and randomly over a continuous area with average node density ρ. The originating node is assumed to be a point source at the center of the given network area. We assume a node can *decode and forward* (DF) a message without error when its received signal-to-noise ratio (SNR) is greater than or equal to a modulation-dependent threshold [11]. Assumption of unit noise variance transforms the SNR threshold to a received power criterion, which is denoted as the decoding or 'lower' threshold, τ_l. We note that the decoding threshold τ_l is not

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explicitly used in real receiver operations. A real receiver always just tries to decode a message. If the message was decoded properly, then it is assumed that the receiver power must have exceeded τ_l. In contrast, the 'Transmission' or 'upper' threshold, τ_u is used explicitly in the receiver to compare against the received SNR. This additional criterion for relaying limits the number of nodes in each hop because a node would relay only if it's received SNR is *less* than τ_u. So the thresholds, τ_l and τ_u, define a range of received powers that correspond to the "significant" boundary nodes, which form the OLA. While each boundary node in OLA-T must transmit a somewhat higher power, compared to Basic OLA, there is still an overall transmit energy savings with OLA-T because of the favorable location of the boundary nodes. We define the Relative Transmission Threshold (RTT) as $\mathcal{R} = \frac{\tau_u}{\tau_l}$.

For simplicity, the *deterministic model* [11] is assumed, which means that the power received at a node is the sum of the powers from each of the node transmissions. This implies that signals received from different nodes are orthogonal. The orthogonality can be approximated, for example, with Direct Sequence Spread Spectrum (DSSS) modulation, RAKE receivers and by allowing transmitting nodes to delay their transmission by a random number of chips [12].

Continuing to follow [11], we assume a non-fading environment and a path-loss exponent of 2. The path loss function in Cartesian coordinates is given by $l(x, y) = (x^2 + y^2)^{-1}$, where (x, y) are the normalized coordinates at the receiver. As in [11], distance d is normalized by a reference distance, d_0. Let power P_0 be the received power at d_0. As in [11], the aggregate path-loss from a circular disc of radius r_0 at an arbitrary distance $p > 1$ from the source is given by

$$\begin{aligned} f(r_0, p) &= \int_0^{r_0} \int_0^{2\pi} l(p - r\cos\theta, r\sin\theta) r dr d\theta \\ &= \pi \ln \frac{p^2}{|p^2 - r_0^2|}. \end{aligned} \quad (1)$$

Let the normalized source and relay transmit powers be denoted by P_s and P_r, respectively, and the relay transmit power per unit area be denoted by $\overline{P_r} = \rho P_r$. The normalization is such that P_s and P_r are actually the SNRs at a receiver d_0 away from the transmitter [8]. We assume a continuum of nodes in the network, which means that we let the node density ρ become very large ($\rho \to \infty$) while $\overline{P_r}$ is kept fixed. Using (1), the received power at a distance p from the source, P_p is given by $P_p = \overline{P_r} \pi \ln \frac{p^2}{|p^2 - r_0^2|}$. We note that non-orthogonal transmissions in fading channels produce similarly shaped OLAs [11], therefore the A-OLA-T concept should work for them as well, although the theoretical results would have to be modified.

Lastly, we define Decoding Ratio (DR) as $\mathcal{D} = \tau_l / \overline{P_r}$, named as such because it can be shown to be the ratio of the receiver sensitivity (i.e. minimum power for decoding at a given data rate) to the power received from a single relay at the 'distance to the nearest neighbor,' $d_{nn} = 1/\sqrt{\rho}$. If ρ is a perfect square, then the d_{nn} would be the distance between the nearest neighbors if the nodes were arranged in a uniform square grid. We note that $\mathcal{D}$ relates to node degree, $\mathcal{K}$, [13] according to $\mathcal{K} = \pi/\mathcal{D}$.

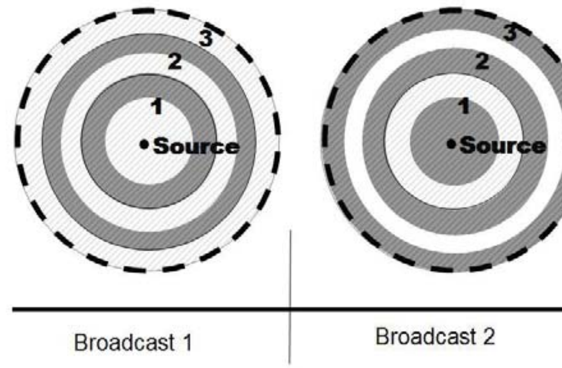

Fig. 1. *The grey strips represent the transmitting nodes (that form the OLAs) which alternate during each broadcast.*

III. ALTERNATING OLA-T (A-OLA-T) FOR TWO SETS

For a fixed source and a static network, OLA-T causes the same subset of nodes to participate in all broadcasts. Let "network lifetime" be defined to be the length of time before the first node dies ("death" happens when the batteries die). If we assume that broadcasts are the only transmissions, then we observe that OLA-T has no advantage over Basic OLA in terms of network lifetime, even though OLA-T consumes less total energy in a single broadcast. In the next section, we show how Alternating OLA-T (A-OLA-T) improves the network lifetime compared to Basic OLA and OLA-T.

The idea of A-OLA-T is that the nodes that do not participate in one broadcast make up the OLAs in the next broadcast. Fig. 1 illustrates the concept. The grey areas on the left of Fig. 1, are the OLAs in "Broadcast 1," which is an OLA-T broadcast, while the grey areas on the right are the OLAs in "Broadcast 2." Ideally these two sets of OLAs have no nodes in common and their union includes all nodes. A-OLA-T extends the network life because each node participates only in every other broadcast.

Broadcast 1 fixes the radii for Broadcast 2. From [8], it is learned that a necessary and sufficient condition for Broadcast 1 success with a constant transmission threshold is the inequality,

$$2 \geq \exp\left(\frac{\mathcal{D}}{\pi}\right) + \exp\left(\frac{-\mathcal{D}\mathcal{R}}{\pi}\right). \quad (2)$$

We observe that when $\mathcal{R} \to \infty$, (2) becomes the condition for successful Basic OLA broadcast [11]:

$$2 \geq \exp\left(\frac{\mathcal{D}}{\pi}\right). \quad (3)$$

Inequality (2) can be re-written as a lower bound on $\mathcal{R}$:

$$\mathcal{R}_{\text{lower bound}} = (-1)\left\{\frac{\pi \ln\left[2 - \exp\left(\frac{\mathcal{D}}{\pi}\right)\right]}{\mathcal{D}}\right\}. \quad (4)$$

We note that the A-OLA-T extension of OLA-T will not work for all $\mathcal{R}$ satisfying (4). Next, we will show that a necessary and sufficient condition for Broadcast 2 to also be successful is an upper bound on $\mathcal{R}$.

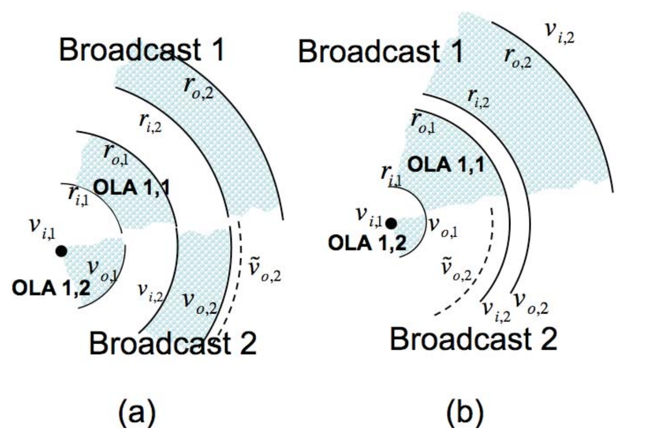

Fig. 2. *Illustration of the A-OLA-T Algorithm with (a) admissible $\mathcal{R}$, (b) inadmissible $\mathcal{R}$.*

A. Necessary and Sufficient Condition for Broadcast 2 Success

Figs. 2(a) and (b) contain illustrations of successful and unsuccessful A-OLA-T broadcasts, respectively. These figures show how to ensure that both broadcasts are sustaining. The upper parts of both drawings correspond to Broadcast 1, and the outer and inner OLA radii for the k-th OLA ring are labeled $r_{o,k}$ and $r_{i,k}$, respectively. The lower parts of both drawings correspond to Broadcast 2, and the outer and inner OLA radii for the k-th OLA ring are relabeled $v_{o,k}$ and $v_{i,k}$, respectively. The initial conditions for the second broadcast are $v_{i,1} = 0$, and $v_{o,1} = \sqrt{\frac{P_s}{\tau_u}}$, where $v_{o,1}$ was fixed in Broadcast 1. In Fig. 2(a), the first OLA during Broadcast 1 is denoted by *OLA 1,1* and is defined by the radii pair, $r_{i,1}$ and $r_{o,1}$. On the other hand, the first OLA during Broadcast 2 is denoted by by *OLA 1,2* and is the circular disk of radius $v_{o,1}$. Let $\widetilde{v}_{o,2}$ be the decoding range of *OLA 1,2* during Broadcast 2. The key idea is that $\widetilde{v}_{o,2}$ must be greater than $r_{i,2}$. In Fig. 2(a), this inequality is satisfied, while in Fig. 2(b), it is not. More generally, the network designer just needs to check that the decoding range, $\widetilde{v}_{o,k+1}$, of the k-th OLA in Broadcast 2 is always greater than $r_{i,k+1}$, for all k. Alternatively, we can compute the received power at $r_{i,k+1}$ and confirm that it is greater than the minimum. Using $v_{o,k} = r_{i,k}$ and $v_{i,k} = r_{o,k-1}$, we express this as

$$\overline{P_r}\left[f(r_{i,k}, r_{i,k+1}) - f(r_{o,k-1}, r_{i,k+1})\right] \geq \tau_l. \quad (5)$$

Intuitively, we observe that as $\mathcal{R}$ becomes very large, the OLAs during Broadcast 1 become larger and the OLAs of Broadcast 2 become relatively smaller, as shown in Fig. 2(b). As a result, the sets of nodes that did not transmit during Broadcast 1 (or the OLAs during Broadcast 2), eventually become so small that their decoding range (for *OLA 1,2*, this is indicated by the dashed line in Fig. 2(b)) cannot reach the next Broadcast 2 OLA to sustain propagation, i.e., $\widetilde{v}_{o,2} < v_{i,2}$. In other words, for a very high value of $\mathcal{R}$, the k-th OLA in Broadcast 2 may be so weak that no nodes between $v_{i,k+1}$ and $v_{o,k+1}$ can decode the signal. When this happens, OLA formations die off during Broadcast 2 and A-OLA-T fails to achieve network broadcast. Thus, it makes sense for $\mathcal{R}$ to have an upper bound.

Using the initial conditions $r_{o,1} = \sqrt{\frac{P_s}{\tau_l}}$ and $r_{i,1} = \sqrt{\frac{P_s}{\tau_u}}$, recursive formulae for the k-th OLA are given by [8]

$$r_{o,k}^2 = \frac{\beta(\tau_l) r_{o,k-1}^2 - r_{i,k-1}^2}{\beta(\tau_l) - 1}, \; r_{i,k}^2 = \frac{\beta(\tau_u) r_{o,k-1}^2 - r_{i,k-1}^2}{\beta(\tau_u) - 1}. \quad (6)$$

After substituting (6) into (5), and simplifying, we can rewrite the condition in (5) to show the explicit dependence on the Broadcast 1 radii:

$$0 \leq \frac{\beta(\tau_l) r_{i,k}^2 - r_{o,k-1}^2 - \left(\beta(\tau_l) - 1\right) r_{i,k+1}^2}{\beta(\tau_l) - 1}. \quad (7)$$

In [8], the closed-form expressions for (6) were found to be

$$r_{o,k}^2 = \frac{\eta_1 A_1^{k-1} - \eta_2 A_2^{k-1}}{A_1 - A_2}, \; r_{i,k}^2 = \frac{\zeta_1 A_1^{k-1} - \zeta_2 A_2^{k-1}}{A_1 - A_2}, \quad (8)$$

where

$$A_1 = \alpha(\tau_l) - \alpha(\tau_u), \quad A_2 = 1, \quad A_1 - A_2 \neq 0, \quad (9)$$

$$\eta_i = \left\{ \left[A_i + \alpha(\tau_u)\right] \frac{P_s}{\tau_l} - \alpha(\tau_l) \frac{P_s}{\tau_u} \right\}, \quad (10)$$

$$\zeta_i = \left\{ \left[1 + \alpha(\tau_u)\right] \frac{P_s}{\tau_l} + \left[A_i - \alpha(\tau_l) - 1\right] \frac{P_s}{\tau_u} \right\}, \quad i \in \{1, 2\}, \quad (11)$$

$$\alpha(\tau) = \left[\beta(\tau) - 1\right]^{-1}, \quad \beta(\tau) = \exp\left[\tau / (\pi \overline{P_r})\right]. \quad (12)$$

Substituting the expressions for $r_{o,k}$ and $r_{i,k}$ from (8)-(11) into (7), and collecting the A_1 and A_2 terms, we get

$$A_1^{k-1} \Omega - A_2^{k-1} \Pi \geq 0. \quad (13)$$

where

$$\Omega = \Big(\alpha(\tau_l) + 1\Big)\zeta_1 - \alpha(\tau_l)\eta_1 A_1^{-1} - \zeta_1 A_1, \text{ and} \quad (14)$$

$$\Pi = \Big(\alpha(\tau_l) + 1\Big)\zeta_2 - \alpha(\tau_l)\eta_2 A_2^{-1} - \zeta_2 A_2.$$

Using $A_2 = 1$ and the expressions for η_2 and ζ_2, we get $\Pi = \zeta_2 - \eta_2 = 0$, which, when applied to (13) along with $A_1 > 0$, the inequality in (13) may be simplified to $\Omega \geq 0$. While not obvious from $\Omega \geq 0$, this inequality implies an upper bound on $\mathcal{R}$. The closed-form expression for the upper bound on $\mathcal{R}$ is derived in the Appendix, and is given by

$$\mathcal{R}_{\text{upper bound}} = \frac{\pi}{\mathcal{D}} \ln \left[\frac{\beta(\tau_l) + 1 + \sqrt{\big(\beta(\tau_l) + 1\big)^2 - 4}}{2} \right], \quad (15)$$

where $\beta(\tau_l)$ is defined in (12). We observe that (15) depends exclusively on τ_l and $\mathcal{D}$, and not on the source power P_s.

We remark that it is not necessary to assume the same $\mathcal{R}$ for both broadcasts or even for different levels within a single broadcast [8]. With the flexibility of level-dependent transmission thresholds (τ_u^k or $\mathcal{R}_k$), a designer may be able to make the decoding ranges in Broadcast 2 match up exactly with the boundaries in Broadcast 1, and thereby save more transmit energy.

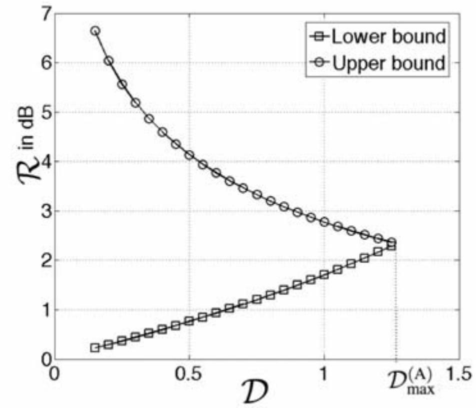

Fig. 3. *Relative Transmission Threshold ($\mathcal{R}$), in dB, Versus Decoding Ratio ($\mathcal{D}$) for A-OLA-T. The $\mathcal{D}$ corresponding to the intersection of the two curves is the $\mathcal{D}_{\max}^{(A)}$.*

B. Discussion

Fig. 3 is a plot of the upper and lower bounds for relative transmission threshold, $\mathcal{R}$, in dB for A-OLA-T, as a function of the decoding ratio, $\mathcal{D}$. First, we observe that as $\mathcal{D}$ decreases, the difference between the upper and lower bounds increases. As an example, for a small decrease in $\mathcal{D}$ from 1.2 to 1, the range of $\mathcal{R}$ increases from $[2.1, 2.4]$ to $[1.7, 2.8]$. This has two reasons. Decreasing $\mathcal{D}$ could be done by increasing the $\overline{P_r}$, which enables Broadcast 1 to be successful with more slender OLAs. This corresponds to a decrease of the lower bound. Fatter Broadcast 2 OLAs more easily reach across the next pair of boundaries and so this increases the upper bound. Next, decreasing τ_l also decreases $\mathcal{D}$. Decreasing τ_l decreases the lower bound, because a lower value of τ_l corresponds to a lower SNR requirement at the receiving node, and so in order to meet this power requirement, the OLAs can afford to have fewer nodes during Broadcast 1. OLAs during Broadcast 1 become thinner but more powerful, and the OLAs during Broadcast 2 grow thicker. This is implied by an increase in the upper bound.

We also observe from Fig. 3 that the upper and lower bounds converge as $\mathcal{D}$ increases. This also implies an upper bound on $\mathcal{D}$ for A-OLA-T, $\mathcal{D}_{\max}^{(A)} = \frac{\tau_l}{\overline{P_r}_{\min}^{(A)}}$, where $\overline{P_r}_{\min}^{(A)}$ is the minimum value of $\overline{P_r}$ for a given τ_l. We were not able to obtain an exact value of $\mathcal{D}_{\max}^{(A)}$, however, using numerical analysis we found $\mathcal{D}_{\max}^{(A)} \approx 1.27$. We note from (3) that $\mathcal{D}$ has a higher upper bound for Basic OLA, $\mathcal{D}_{\max}^{(O)} = \pi \ln(2) \approx 2.18$. For $\mathcal{D} > \mathcal{D}_{\max}^{(A)}$, network broadcast fails for A-OLA-T because the OLAs die out during Broadcast 2. For A-OLA-T, we have from $\mathcal{D}_{\max}^{(A)}$ that $\overline{P_r}_{\min}^{(A)} \simeq 0.78\tau_l$. From (3), the minimum $\overline{P_r}$ for Basic OLA, denoted by $\overline{P_r}_{\min}^{(O)}$, is $\overline{P_r}_{\min}^{(O)} = 0.46\tau_l$. We observe that A-OLA-T requires less than double the power of Basic OLA, because it uses border nodes.

Next, we compute the "broadcast life" extension of A-OLA-T compared to Basic OLA. By broadcast life, we mean the lifetime of the network if only broadcasts were transmitted. If A-OLA-T and Basic OLA use the same $\overline{P_r}$, then A-OLA-T doubles the network life compared to Basic OLA. However, this is not a fair comparison since Basic OLA can achieve successful broadcast at a lower $\overline{P_r}$. Since for a given protocol, all nodes use the same amount of power in broadcasts, we assume the broadcast life of the network is inversely proportional to the time-averaged power transmitted by each node. For Basic OLA, the time-averaged power is $\overline{P_r}^{(O)}$. For A-OLA-T, the time-averaged power is $\frac{\overline{P_r}^{(A)}}{2}$, since each node transmits only every other broadcast. The ratio of broadcast lives of Basic OLA to A-OLA-T is therefore $2\frac{\overline{P_r}^{(O)}}{\overline{P_r}^{(A)}}$, and the 'Fraction of Life Extension' (FLE), may be defined as

$$\text{FLE} = 2\frac{\overline{P_r}^{(O)}}{\overline{P_r}^{(A)}} - 1. \tag{16}$$

FLE can be evaluated for any powers that satisfy $\overline{P_r}^{(A)} \geq 0.78\tau_l$ and $\overline{P_r}^{(O)} \geq 0.46\tau_l$. However, when the the minimum powers are substituted, then (16) becomes

$$\widehat{\text{FLE}} = 2\frac{\overline{P_r}_{\min}^{(O)}}{\overline{P_r}_{\min}^{(A)}} - 1 = 2\frac{\mathcal{D}_{\max}^{(A)}}{\mathcal{D}_{\max}^{(O)}} - 1 \approx 0.17. \tag{17}$$

This means that A-OLA-T can offer a 17% life extension when both protocols are optimized.

IV. Conclusions

In this paper, we proposed and analyzed a novel same-source broadcast strategy that extends the life of a wireless ad hoc or sensor network by alternating between mutually exclusive sets of opportunistic large arrays (OLAs) in two consecutive broadcasts. In this strategy, all participating nodes transmit with the same power. We showed that A-OLA-T extends the network life by a maximum of 17% relative to Basic OLA when both protocols operate in their minimum energy configuration. Further, when A-OLA-T is compared to OLA-T, the battery-life of the nodes is doubled. The key parameter was the transmission threshold, which was assumed constant for the whole network. Potential extensions of this work include an analysis of A-OLA-T for more than two OLA sets, finite densities of nodes, other path-loss exponents, fading environments, radiated versus non-radiated energy, and for practical synchronization and SNR estimation.

Acknowledgment

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Appendix

The condition for *Broadcast 2* OLA formations to propagate throughout the network is given by $\Omega \geq 0$ and (14). To determine the values that make $\Omega = 0$, substitute the expressions for η_1 and ζ_1 from (10) and (11), respectively, and get

$$0 = \Big(\alpha(\tau_l) + 1\Big)\Big(1 + \alpha(\tau_u)\Big)\left[\frac{P_s}{\tau_l} - \frac{P_s}{\tau_u}\right] - \alpha(\tau_l)\alpha(\tau_l)\left[\frac{P_s}{\tau_l} - \frac{P_s}{\tau_u}\right]A_1^{-1}\Big(1 + \alpha(\tau_u)\Big)\left[\frac{P_s}{\tau_l} - \frac{P_s}{\tau_u}\right]A_1.$$

We assume $\tau_l - \tau_u > 0$, and $P_s \neq 0$; therefore, we can divide out the square bracketed term. Further simplification results in $0 = A_1^2 - \left(\big[\alpha(\tau_u)\big]^2 + 1\right)A_1 + \big[\alpha(\tau_u)\big]^2$. This equation is quadratic in A_1, and the roots are $A_1 = \left[\alpha(\tau_u)\right]^2$ and $A_1 = 1$. Recall that $A_1 - A_2$ is a factor in the denominator of the closed-form expressions for the OLA-T radii as given in (8). So, during the derivation for Ω, we had assumed that $A_1 - A_2 \neq 0$. Since $A_2 = 1$, we must take the root $A_1 = \left[\alpha(\tau_u)\right]^2$. We re-substitute the expression for A_1 in (9) to get $\left[\beta(\tau_u)\right]^2 - \left(\beta(\tau_l) + 1\right)\beta(\tau_u) + 1 = 0$, which is quadratic in $\beta(\tau_u)$, with roots $r_{1,2} = (0.5)\left[\beta(\tau_l) + 1 \pm \sqrt{\left(\beta(\tau_l) + 1\right)^2 - 4}\right]$. Without loss of generality, we assume the larger root is r_1. Each root implies a different relationship between τ_l and τ_u, which leads to two values of $\mathcal{R}$ where $\Omega = 0$. The greater of the two values is the upper bound on $\mathcal{R}$. So, $\beta(\tau_u) = r_1 \Rightarrow \tau_u = \overline{P_r}\pi \ln(r_1)$, and the upper bound is given by $\mathcal{R}_{\text{upper bound}} = \frac{\pi \ln(r_1)}{\mathcal{D}}$.

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