Investigating Multiple Alternating Cooperative Broadcasts to Enhance Network Longevity

Aravind Kailas
School of Electrical and Computer Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250, USA
Email: aravindk@ieee.org

Mary Ann Ingram
School of Electrical and Computer Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250, USA
Email: mai@ece.gatech.edu

Abstract*—We propose a broadcast protocol that is based on a form of cooperative transmission called the Opportunistic Large Array (OLA). Multiple SNR (or transmission) thresholds are used to define mutually exclusive sets of OLAs, such that the union of the sets includes all the nodes in the network. The new protocol, termed Alternating OLA with Transmission Threshold (A-OLA-T), exercises a different set of OLAs on each consecutive broadcast from the same sink until all sets have transmitted once. Then the sequence repeats. Thus, broadcasts consume energy efficiently and uniformly over the network, and A-OLA-T is especially well suited for static networks. The transmission thresholds are optimized to maximize the network life if broadcasts were the only transmissions. In this paper, we first optimize triples of broadcasts, and then extend the optimization for a higher number of broadcasts.

I. Introduction

In this paper, we propose and analyze a Medium Access Control (MAC)-free "alternating sets" broadcast strategy to increase the network longevity for multihop wireless networks. The 'Alternating OLA with a Transmission Threshold' (A-OLA-T) algorithm introduced in this paper uses a simple form of Cooperative Transmission (CT) called the *Opportunistic Large Array* (OLA) [1], and is an extension of a previous non-alternating OLA with a Transmission Threshold (OLA-T) algorithm [2].

By having two or more nodes cooperate to transmit the same message, CT-based strategies offer the spatial diversity benefits of an array transmitter enabling a dramatic signal-to-noise ratio (SNR) advantage in a multipath fading environment [3], [4]. This advantage can be used to save transmit energy [3], [4]. The Opportunistic Large Array (OLA), a simple form of CT, is a group of nodes that behave without coordination between each other, but naturally fire at approximately the same time in response to energy received from a single source or another OLA [1]. All the transmissions within an OLA are repeats of the same waveform; therefore the signal received from an OLA has the same model as a multipath channel. Small time offsets (because of different distances and computation times) and small frequency offsets (because each node has a different oscillator frequency) are like excess delays and Doppler shifts, respectively. As long as the receiver, such as a RAKE receiver, can tolerate the effective delay and Doppler spreads of the received signal and extract the diversity, decoding can proceed normally. Even though many nodes may participate in an OLA transmission, total transmission energy can still be saved because all nodes can reduce their transmit powers dramatically and large fade margins are not needed.

When used for broadcasting, nodes repeat if they haven't repeated the message before, and the resulting OLAs will "propagate," forming concentric ring shaped OLAs that will eventually include all nodes, under a condition on relay power and receiver sensitivity [1]; we refer to this broadcast scheme as "Basic OLA." OLA with Transmission Threshold (OLA-T) applies an SNR threshold to limit the relaying nodes to those at the edge of the decoding range [2], [5]. Even though these "border nodes" must transmit at a higher power than for Basic OLA to sustain propagation [2], total transmit energy is still saved because only a fraction of the nodes relay. Basic OLA and OLA-T share the important feature that no individual nodes are addressed. Given that the node density is sufficient to sustain OLA transmission, the complexity of these broadcast protocols is absolutely independent of node density, making OLA-based broadcasting very attractive for extremely high density wireless networks.

However, OLA-T has a problem for a fixed source in a static network, because the same nodes relay in every broadcast. Therefore, the OLA nodes die (drain their batteries) first and the network loses sensitivity in the OLA areas. Alternating OLA-T (A-OLA-T) was proposed in [7] to remedy this problem. In A-OLA-T, the transmission threshold is used to divide all the nodes in the network into two mutually exclusive sets of OLAs. The first set is exercised in the initial OLA-T broadcast, while the second set is exercised in the second broadcast. Each succeeding broadcast alternates between the two sets, thereby saving transmit energy via OLA-T, but also draining the batteries uniformly across the network. Conditions were derived in [7] to ensure sustained propagation of both sets of OLAs.

The contribution of the present paper is to extend 2-set A-OLA-T to m-set A-OLA-T where $m > 2$. The condition for sustained propagation for m-set A-OLA-T is derived and the energy savings for m-set A-OLA-T relative to Basic OLA is shown.

The authors gratefully acknowledge support for this research from the National Science Foundation under grant CNS-0721296.

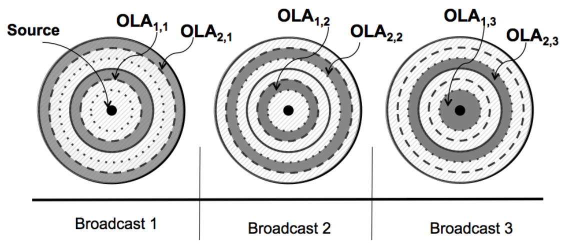

Fig. 1. *A-OLA-T with 3 alternating mutually exclusive sets of OLAs.*

II. System Model

For our analysis, we adopt the notation and assumptions of [2], most of which were used earlier in [8]. Half-duplex nodes are assumed to be distributed uniformly and randomly over a continuous area with average node density ρ. The originating node is assumed to be a point source at the center of the given network area. We assume a node can Decode and Forward (DF) a message without error when its received SNR is greater than or equal to a modulation-dependent threshold [8]. Assumption of unit noise variance transforms the SNR threshold to a received power criterion, which is denoted as the decoding threshold τ_d. The 'Transmission Threshold,' τ_b is used explicitly in the receiver to compare against the received SNR. The thresholds, τ_d and τ_b, define a range of received powers that correspond to the "significant" boundary nodes, which form the OLA. Further, we define the Relative Transmission Threshold (RTT) as $\mathcal{R} = \frac{\tau_b}{\tau_d}$. Continuing to follow [8], we assume a non-fading environment. The normalized path loss function in Cartesian coordinates is given by $l(x, y) = (x^2 + y^2)^{-1}$, where (x, y) are the normalized coordinates at the receiver.

For simplicity, the "deterministic model" [8] is assumed, which means that the power received at a node is the sum of the powers from each of the node transmissions. This implies that signals received from different nodes are orthogonal. The orthogonality can be approximated, for example, with Direct Sequence Spread Spectrum (DSSS) modulation, RAKE receivers and by allowing transmitting nodes to delay their transmission by a random number of chips [9], [10].

Let the normalized source and relay powers be denoted by P_s and P_r, respectively. Let the relay transmit power per unit area be denoted by $\overline{P_r} = \rho P_r$. We assume a continuum of nodes in the network, which means that the we let the node density ρ become very large ($\rho \to \infty$) while $\overline{P_r}$ is kept fixed. We remark that the normalized relay transmit power, P_r, is actually the SNR received by a node at the reference distance away from a single relay node. We note that non-orthogonal transmissions in fading channels produce similarly shaped OLAs [8], therefore the A-OLA-T concept should work for them as well, although the theoretical results would have to be modified.

Lastly, we define Decoding Ratio (DR) as $\mathcal{D} = \tau_d/\overline{P_r}$, because it can be shown to be the ratio of the receiver sensitivity (i.e. minimum power for decoding at a given data rate) to the power received from a single relay at the 'distance to the nearest neighbor,' $d_{nn} = 1/\sqrt{\rho}$. If ρ is a perfect square, then the d_{nn} would be the minimum distance between the nearest neighbors if the nodes were arranged in a uniform square grid. The respective conditions for successful broadcast for both Basic OLA and OLA-T are upper bounds on $\mathcal{D}$, and the transmit energy is optimized for each protocol when $\mathcal{D}$ meets it upper bound.

The strategy in this paper is to assume a certain A-OLA-T property, namely the "equal area property," which is true for the optimized $m = 2$ case, to also be true for the optimized $m > 2$ case. Then, this property is used to derive the upper bound on $\mathcal{D}$ for the $m > 2$ case.

III. Alternating OLA with Transmission Threshold (A-OLA-T)

A. A-OLA-T Concept

Fig. 1 illustrates the A-OLA-T concept with 3 alternating sets of OLAs. Each broadcast is an OLA-T broadcast. The grey areas in the left of Fig. 1, are the OLAs in "Broadcast 1," while the grey areas in the center and on the right, are the OLAs in "Broadcast 2," and "Broadcast 3," respectively. Ideally these three sets of OLAs have no nodes in common and their union includes all nodes. In Fig. 1, the sets of OLAs during Broadcasts 1, 2, and 3 comprise $\text{OLA}_{1,1}$ and $\text{OLA}_{2,1}$, $\text{OLA}_{1,2}$ and $\text{OLA}_{2,2}$, and $\text{OLA}_{1,3}$ and $\text{OLA}_{2,3}$, respectively; these sets do not have any common nodes and their union includes all the nodes in the network. This increases the network longevity for broadcast applications because each node participates once in every three broadcasts, and therefore the load is shared equally. For the two-set A-OLA-T, Broadcast 1 fixes the radii for Broadcast 2. The trick then is to choose transmission thresholds to ensure that the detection boundaries in Broadcast 2 exceed (or match up) with transmission threshold boundaries in Broadcast 1. In [7], it was established that there exists a maximum value of $\mathcal{D}$, denoted by $\mathcal{D}_{\text{max}}^{(\text{A})}$, and when $\mathcal{D} > \mathcal{D}_{\text{max}}^{(\text{A})}$, network broadcast fails for A-OLA-T because the OLAs die out during Broadcast 2. $\mathcal{D}_{\text{max}}^{(\text{A})}$ implies a minimum value of $\overline{P_r}$ for a given τ_d, denoted by $\overline{P_r}_{\text{min}}^{(\text{A})}$.

Compared to Basic OLA, A-OLA-T with two sets extends the network longevity by about 17% when both OLA-based protocols operate in their minimum power configuration [7]. This work may be useful for future very large and very fine-grained monitoring applications, of the type that may be enabled by sensor nodes that do energy harvesting.

B. Equal Area Property

Let the Ratio of Areas be the ratio of the total area of the Broadcast 1 OLAs to the total area of the network, and be given by

$$\tilde{\Psi} = \frac{\sum_{k=1}^{L} \left(r_{d,k}^2 - r_{b,k}^2 \right)}{r_{d,L}^2}, \tag{1}$$

where $r_{d,k}$ and $r_{b,k}$ denote the outer and inner boundary radii, respectively, for the k-th OLA ring formed during the

Broadcast 1, and L is the number of OLAs in the OLA-T network. In [7], it was shown that for the $m = 2$ case, $\tilde{\Psi} = 1/2$ when $\mathcal{D} = \mathcal{D}_{\max}^{(A)}$. This implies that the respective accumulated areas of the two sets of OLAs during Broadcasts 1 and 2 are equal.

C. m Alternating Sets of OLAs, $m > 2$

In this section, we show that using m alternating sets of OLAs ($m > 2$) extends the life of the network even more than for $m = 2$. To show this, we conjecture that the Equal Area Property applies to the $m > 2$ case. Assuming that the conjecture is true implies that $\tilde{\Psi} = \frac{1}{m}$ for all broadcast sets, when the system is in its lowest energy configuration, i.e. when $\mathcal{D} = \mathcal{D}_{\max}^{(A)}$. We confirm the assumption numerically in the next section. Based on the assumption, we are able to derive an expression for $\mathcal{D}_{\max}^{(A)}$, which in turn, allows us to quantify the relative transmit energy consumption of m-set A-OLA-T to Basic OLA.

The derivation of $\mathcal{D}_{\max}^{(A)}$ is sketched here and the details are in the appendices. We use the closed-form expressions for OLA-T ring radii from [2] to put $\tilde{\Psi}$ for Broadcast 1 solely in terms of $\mathcal{R}$ and $\mathcal{D}$. Then setting $\tilde{\Psi} = \frac{1}{m}$ allows an expression for $\mathcal{R}$ in terms of $\mathcal{D}$ and m. Next, assuming the lowest energy configuration means that $\mathcal{R}$ must be equal to its lower bound (in [2], the upper and lower bounds on $\mathcal{R}$ meet at the minimum energy configuration for $m = 2$). Solving this equality for $\mathcal{D}$ yields the expression

$$\mathcal{D}_{\max}^{(A)} = \pi \ln\left(\frac{m+1}{m}\right). \tag{2}$$

Next, we compute the "broadcast life" extension of A-OLA-T compared to Basic OLA. By broadcast life, we mean the lifetime of the network if only broadcasts were transmitted and if only radiated energy is considered. At a first glance, it might seem that A-OLA-T increases the battery life of the sensors in the network by a factor of m compared to Basic OLA. This is true if A-OLA-T and Basic OLA use the same $\overline{P_r}$. However, this would not be a fair comparison since Basic OLA can achieve successful broadcast at a lower $\overline{P_r}$. Since for a given protocol, all nodes use the same amount of power in broadcasts, we assume the broadcast life of the network is inversely proportional to the time-averaged power transmitted by each node. For Basic OLA, the time-averaged power is $\overline{P_r}^{(O)}$. For A-OLA-T with m sets, the time-averaged power is $\frac{\overline{P_r}^{(A)}}{m}$, since each node transmits only every other broadcast. The ratio of broadcast lives of Basic OLA to A-OLA-T is therefore $m\frac{\overline{P_r}^{(O)}}{\overline{P_r}^{(A)}}$, and the 'Fraction of Life Extension' (FLE), may be defined as

$$\text{FLE} = m\frac{\overline{P_r}^{(O)}}{\overline{P_r}^{(A)}} - 1. \tag{3}$$

FLE can be evaluated for any powers that satisfy

$$\overline{P_r}^{(A)} \geq [\mathcal{D}_{\max}^{(A)}]^{-1}\tau_d, \ \& \ \overline{P_r}^{(O)} \geq [\mathcal{D}_{\max}^{(O)}]^{-1}\tau_d. \tag{4}$$

From [8], $\mathcal{D}_{\max}^{(O)} = \pi \ln 2$. Substituting the values of $\mathcal{D}_{\max}$ in both the inequalities, we have

$$\begin{aligned} \overline{P_r}^{(A)} &\geq \left[\pi \ln\left(\frac{m+1}{m}\right)\right]^{-1}\tau_d, \text{ and} \\ \overline{P_r}^{(O)} &\geq 0.46\tau_d. \end{aligned}$$

So $\overline{P_r}^{(A)}$ increases with m. When the the minimum powers from (4) are substituted, then (3) becomes

$$\widehat{\text{FLE}} = m\frac{\overline{P_r}^{(O)}_{\min}}{\overline{P_r}^{(A)}_{\min}} - 1 = m\frac{\mathcal{D}_{\max}^{(A)}}{\mathcal{D}_{\max}^{(O)}} - 1 \approx 0.44 \text{ as } m \to \infty, \tag{5}$$

where $\widehat{\text{FLE}}$ represents the FLE achieved by A-OLA-T relative to Basic OLA when both protocols operate in their minimum power configurations.

IV. Numerical Results

First, the conjecture on the asymptotic convergence of the ratio of the accumulated areas of the mutually exclusive sets of OLAs to 1 is verified numerically in Fig. 2 for $m = 3$. Fig. 2 is a plot of the ratio of areas versus k for the three successive broadcasts. As seen in the figure, these curves overlap, i.e., the ratio of areas for Broadcasts 1 and 2, Broadcasts 2 and 3, and Broadcasts 3 and 1 grow in the exact same way as a function of the OLA index. Convergence of ratio of areas to ≈ 1 implies that the widths of adjacent OLAs from Broadcast 1, 2, and 3 become equal.

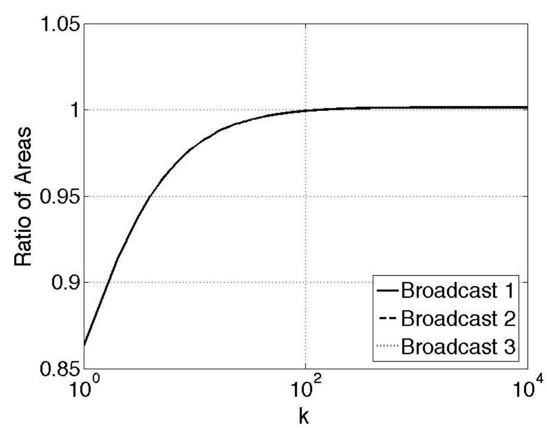

Fig. 2. *Ratio of areas versus k for $m = 3$.*

Next, we establish the network lifetime extensions using m-A-OLA-T. Fig. 3 is a plot of the FLE versus the number of alternating sets, m, on a logarithmic scale. We observe that as m increases, the FLE increases (solid line), and for a large number of alternating sets, it reaches its asymptotic value (shown by dash-dot line) of around 0.45. This means that m-set A-OLA-T can offer a maximum life extension of about 44% when both protocols are optimized. When $m = 2$, $\widehat{\text{FLE}} = 0.17$, which is consistent with the findings in [7].

Finally, it remains to check if infinite network broadcast can be achieved when the m-set A-OLA-T is operating in

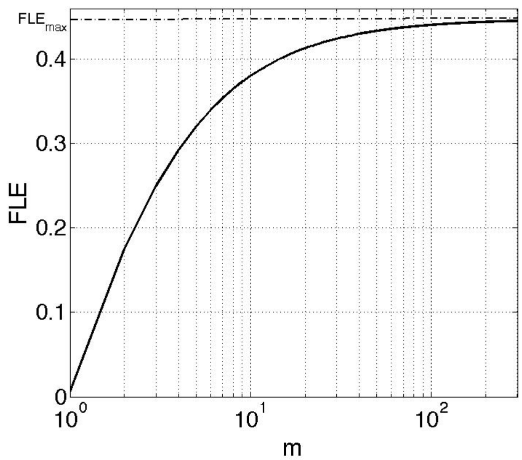

Fig. 3. *FLE as a function of the number of alternating sets, m.*

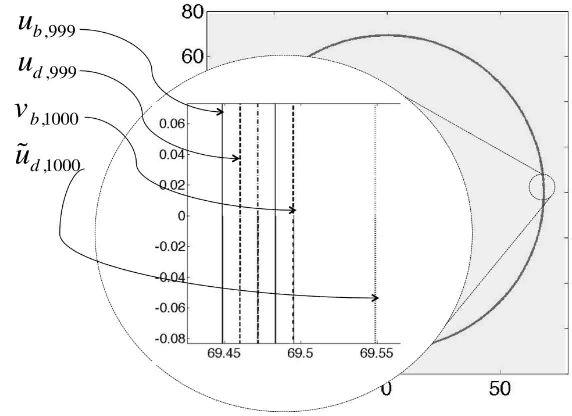

Fig. 4. *3-set A-OLA-T radii growth in the minimum power case. The* 999*-th and* 1000*-th levels are shown in the figure.*

the minimum power configuration, i.e., at $\mathcal{D} = \mathcal{D}_{\max}^{(A)}$, which is given by (2). For our example, we use Matlab simulations and choose $m = 3$. Let $v_{d,k}$ and $v_{b,k}$, denote the outer and inner boundary radii for the k-th OLA ring formed during the Broadcast 2, respectively. If $u_{d,k}$ and $u_{b,k}$, denote the outer and inner boundary radii, respectively, for the k-th OLA ring formed during the Broadcast 3, and if $\tilde{u}_{d,k+1}$ represents the decoding range of the $(k+1)$-st OLA, then $\tilde{u}_{d,k+1} \geq v_{b,k+1}$ must hold to guarantee infinite network broadcast. The inner and outer boundaries have been simulated using the closed form expressions given by (7). It is remarked that even though the continuum assumptions of [2] are used for these simulations, it has been shown in [2] using Monte-Carlo simulations that the continuum and deterministic assumptions can be approximated well by networks of finite density with Rayleigh fading channels. We test infinite network broadcast numerically at $\mathcal{D} = \mathcal{D}_{\max}^{(A)}$. The shaded background in Fig. 4 is a plot of the 3-set A-OLA-T normalized radii at $\mathcal{D}_{\max}^{(A)}$ for the 999-th and 1000-th levels as a function of normalized distance. The white circle in the foreground is a magnified version of the region enclosed by the smaller dotted circle. The normalized Source power, P_s was chosen to be 5 and using (2), $\mathcal{D}_{\max}^{(A)} = 0.9038$. We now explain the plot in the foreground. Continuing to follow the notations from the previous paragraph, Broadcast 3 boundary radii for the 999-th level, $u_{b,999}$ and $u_{d,999}$, are represented by the solid and dashed lines, respectively. The dashed line (second from the right) is the Broadcast 2 inner boundary radii for the 1000-th level, $v_{b,1000}$. The right-most dotted line represents the decoding range of the 1000-th OLA, $\tilde{u}_{d,1000}$. From Fig. 4, we observe that $\tilde{u}_{d,1000} > v_{b,1000}$, and so this is indicative of infinite network broadcast at $\mathcal{D}_{\max}^{(A)}$. It was observed that for $\mathcal{D} > \mathcal{D}_{\max}^{(A)}$, Broadcast 3 OLAs die out (not shown in the paper).

Practical Issues: The analysis in this paper has assumed a continuum of nodes and that all nodes transmit orthogonal signals, neither of which is true in practice. However, results based on these assumptions have been shown to be closely approximated with high densities and limited orthogonality in fading channels [2], [8]; in [2], several examples of un-normalized variables (i.e. relay powers in dBm, densities in number of nodes per m^2, etc) are given that are consistent with the high density assumption.

Nevertheless, finite density might mean that higher than minimum powers will be needed to ensure successful broadcast for both Basic OLA and A-OLA-T. The additional power needed might be called the "density margin," and is a subject of ongoing research. Finite density and multipath fading will limit the number of sets that could be used by A-OLA-T to some relatively low number.

Another practical issue is that radiated energy is not the only energy consumed by a relay. There is usually "base" of energy required by the electronics [11], and sometimes, the energy required by the receiver electronics exceeds that of the transmitter electronics [11]. Since radiated and circuit-consumed energies are added in a "total energy" model of a node, then, the total energy Fraction of Energy Saved (FES) will be lower in comparison to Basic OLA than what is shown in this paper, since both protocols would have the same circuit-consumed energies.

V. Conclusions

In this paper, we proposed and analyzed a novel same-source broadcast strategy that extends the life of a static wireless ad hoc or sensor network by alternating between mutually exclusive sets of Opportunistic Large Arrays (OLAs) in groups of broadcasts. In this strategy, all participating nodes transmit with the same power. We showed that the Alternating OLA with a Transmission Threshold (A-OLA-T) with m sets, $m >> 1$ extends the network life by a maximum of 44% relative to the Basic OLA when both protocols operate in their minimum energy configuration. Potential extensions of this work include an analysis of A-OLA-T for finite densities of nodes, other path-loss exponents, fading environments, radiated versus non-radiated energy, and a consideration of the limitations of practical synchronization and SNR estimation.

APPENDIX

A. Ratio of Areas

We first derive a simplified expression for the ratio of accumulated OLA areas in a OLA-T broadcast to the total network area, denoted as $\tilde{\Psi}$ in (1). Thus, the expression will apply to Broadcast 1 of A-OLA-T. For simplicity of analysis, consider the term

$$\frac{r_{d,k}^2 - r_{b,k}^2}{r_{d,k}^2 - r_{d,k-1}^2}, \quad (6)$$

which is the ratio of the k-th OLA in OLA-T to the k-th step-size. From [2], the closed-form expressions for OLA-T radii, which apply to Broadcast 1 in A-OLA-T, are given by

$$r_{d,k}^2 = \frac{\eta_1 A_1^{k-1} - \eta_2 A_2^{k-1}}{A_1 - A_2},\ r_{b,k}^2 = \frac{\zeta_1 A_1^{k-1} - \zeta_2 A_2^{k-1}}{A_1 - A_2}, \quad (7)$$

where

$$A_1 = \alpha(\tau_d) - \alpha(\tau_b), \quad A_2 = 1, \quad A_1 - A_2 \neq 0,$$
$$\eta_i = \left\{ [A_i + \alpha(\tau_b)] \frac{P_s}{\tau_d} - \alpha(\tau_d) \frac{P_s}{\tau_b} \right\},$$
$$\zeta_i = \left\{ [1 + \alpha(\tau_b)] \frac{P_s}{\tau_d} + [A_i - \alpha(\tau_d) - 1] \frac{P_s}{\tau_b} \right\},$$
$$i \in \{1,2\},\ \alpha(\tau) = [\beta(\tau) - 1]^{-1},\ \beta(\tau) = \exp\left[\tau/(\pi \overline{P_r})\right].$$

Substituting the closed-form expressions from (7) into (6), we get

$$\frac{r_{d,k}^2 - r_{b,k}^2}{r_{d,k}^2 - r_{d,k-1}^2} = 1 - \frac{\alpha(\tau_b)}{\alpha(\tau_d)}. \quad (8)$$

We observe that this ratio is independent of OLA index k. Solving (8) for $r_{d,k}^2 - r_{b,k}^2$ and substituting into (1), and noting that $\sum_{k=1}^{L} \left(r_{d,k}^2 - r_{d,k-1}^2\right) = r_{d,L}^2$, yields $\tilde{\Psi} = 1 - \frac{\alpha(\tau_b)}{\alpha(\tau_d)}$. We observe that the ratio of areas is invariant to the network size L. When $\tilde{\Psi}$ is evaluated at $\overline{P_r}^{(\mathrm{A})} = \overline{P_r}_{\min}^{(\mathrm{A})}$ for the A-OLA-T with two alternating sets [7], it is found that $\tilde{\Psi} \approx 0.5$.

B. $\mathcal{D}_{max}^{(A)}$ for m-set A-OLA-T

Still focussing on Broadcast 1, which is an OLA-T broadcast, set $\tilde{\Psi} = \frac{1}{m}$. Substituting the definitions of $\alpha(\cdot)$ into the expression for $\tilde{\Psi}$ yields

$$\mathcal{R} = \left(\frac{\pi}{\mathcal{D}}\right) \ln \left[\frac{m \exp\left(\frac{\mathcal{D}}{\pi}\right) - 1}{m - 1} \right].$$

This expression tells us that for a given $\mathcal{D}$ (i.e. a given data rate and relay power density) there will be exactly one value of transmission threshold that will yield a ratio of areas of $1/m$. There is no guarantee, however, that the transmission threshold is sufficiently high to ensure sustained OLA propagation (i.e. that the step sizes do not approach zero). That guarantee is provided by the following bound for OLA-T [2]. From [2], the condition for a successful OLA-T broadcast takes the form of a lower bound on $\mathcal{R}$ given by

$$\mathcal{R}_{\text{lower bound}} = (-1) \left\{ \frac{\pi \ln \left[2 - \exp\left(\frac{\mathcal{D}}{\pi}\right) \right]}{\mathcal{D}} \right\}.$$

Here is where we make our conjecture. In [7], we found for the $m = 2$ case that the upper and lower bounds for $\mathcal{R}$ converged at the maximum possible value of $\mathcal{D}$, denoted $\mathcal{D}_{\max}^{(\mathrm{A})}$. Therefore, the value of $\mathcal{D}$ that we get when we set $\mathcal{R}$ equal to $\mathcal{R}_{\text{lower bound}}$, is assumed to be the maximum $\mathcal{D}$ (corresponding to the lowest $\overline{P_r}$ and consequently the lowest energy, since eventually every node transmits in A-OLA-T).

$$\mathcal{R} = \mathcal{R}_{\text{lower bound}},$$
$$\left[\frac{m \exp\left(\frac{\mathcal{D}}{\pi}\right) - 1}{m - 1} \right] \cdot \left[2 - \exp\left(\frac{\mathcal{D}}{\pi}\right) \right] = 1.$$

Replacing $\exp\left(\frac{\mathcal{D}}{\pi}\right)$ with q, we can re-write the above as a quadratic equation in q as follows:

$$mq^2 - (2m+1)q + (m+1) = 0,$$

the roots of which are $q = \frac{m+1}{m}$. So, $\mathcal{D}_{\max}^{(\mathrm{A})} = \pi \ln \left(\frac{m+1}{m}\right)$.

REFERENCES

[1] Y. W. Hong and A. Scaglione, "Energy-Efficient Broadcasting with Cooperative Transmissions in wireless Sensor Networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 10, pp. 2844–55, Oct. 2006.

[2] A. Kailas, L. Thanayankizil, and M. A. Ingram, "A Simple Cooperative Transmission Protocol for Energy-Efficient Broadcasting Over Multi-Hop Wireless Networks," *accepted for publication in Journal of Communications and Networks (Special Issue on Wireless Cooperative Transmission and Its Applications)*, vol. 10, no. 2, pp. 1–8, Jun. 2008.

[3] A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation – Part I: System Description, Part II: Implmentation Aspects and Performance Analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–48, Nov. 2003.

[4] J. N. Laneman, D. Tse, and G. W. Wornell, "Cooperative Diversitry in Wireless Networks: Efficient Protocols and Outage Behaviour, "*IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3063–3080, Dec. 2004.

[5] B. Sirkeci-Mergen and A. Scaglione, "On the Power Efficiency of Cooperative Broadcast in Dense Wireless Networks," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 497–507, Feb. 2007.

[6] B. Sirkeci-Mergen, A. Scaglione, G. Mergen, "Asymptotic Analysis of Multi-Stage Cooperative Broadcast in Wireless Networks," *Joint special issue of the IEEE Trans. Inf. Theory and IEEE/ACM Trans, On Networking*, vol. 52, no. 6, pp. 2531–50, Jun. 2006.

[7] A. Kailas and M. A. Ingram, "Alternating Opportunistic Large Arrays in Broadcasting for Network Lifetime Extension," *accepted subject to revision in the IEEE Trans. Wireless Commun.*, Feb. 2009.

[8] B. Sirkeci-Mergen, A. Scaglione, G. Mergen, "Asymptotic Analysis of Multi-Stage Cooperative Broadcast in Wireless Networks," *Joint special issue of the IEEE Trans. Inf. Theory and IEEE/ACM Trans, On Networking*, vol. 52, no. 6, pp. 2531–50, Jun. 2006.

[9] L. Thanayankizil and M. A. Ingram, "Opportunistic Large Array Concentric Routing Algorithm (OLACRA) over Wireless Fading Channels," *Proc. of Wireless Sensors Workshop, IEEE GLOBECOM*, Nov. 2007.

[10] R. Mudumbai, G. Barriac, and U. Madhow, "Spread-Spectrum Techniques for Distributed Space-Time Communication in Sensor Networks," *Proc. of Thirty-Eighth Asilomar Conference Signals, Systems and Computers*, Nov. 2004, pp. 908–912.

[11] A. Y. Wang and C. G. Sodini, "On the Energy Efficiency of Wireless Transceivers," *Proc. ICC*, vol. 8, Jun. 2006, pp. 3783-88.

[12] L. Thanayankizil, A. Kailas, and M. A. Ingram, "Energy-Efficient Strategies for Cooperative Communications in Wireless Sensor Networks,"*Proc. IEEE SENSORCOMM*, Sep. 2007, pp. 541–546 .