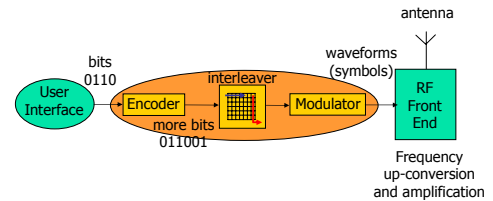


Introduction to Digital Modulation

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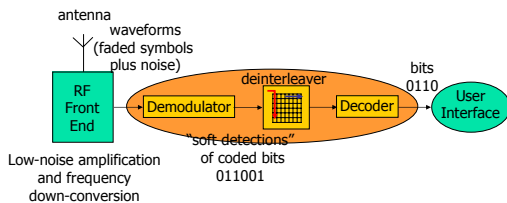
Digital Communication Transmitter

- A simplified block diagram:



Digital Communications Receiver

- A simplified block diagram:



Symbols

- In each symbol period, T_s , a digital modulator maps N coded bits word to a transmitted waveform from a set of $M=2^N$ possible waveforms
- Each waveform corresponds to an information symbol, x_n
- For Binary symbols, $N=1$

Detection

- The job of the receiver is to determine which symbols were sent and to reconstruct the bit stream that created them

Definitions

- Bit Rate (bits per sec or bps)

$$R = N / T_s$$

- Bandwidth Efficiency (bps/Hz)

$$\eta_B = R / B$$

where B is the bandwidth occupied by the signal

Shannon Theorem

- In a non-fading channel, the maximum bandwidth efficiency, or Shannon Capacity is

$$\eta_{B_{MAX}} = \log_2(1 + SNR)$$

SNR = signal-to-noise ratio

Pulses

- A symbol period, T_s , suggests a localization in time
- Localization in frequency is also necessary to enable frequency division multiplexing
- Regulatory agencies provide spectral masks to limit the distribution of power in the frequency domain

Example: Linear Modulation

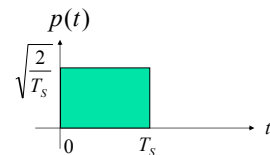
- $s(t)$ is the output of the modulator
- $g(t)$ is the complex envelope
- $p(t)$ is the basic pulse
- x_n is the n th symbol
- A is the amplification in the transmitter

$$s(t) = \text{Re}\{g(t)e^{j2\pi f_c t}\}$$

$$g(t) = A \sum_n x_n p(t - nT_s)$$

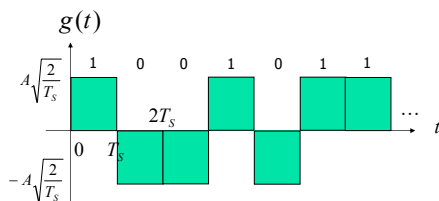
Rectangular Pulses

- Suppose $p(t)$ is a rectangular pulse
- This pulse is not used in practice, but is OK for illustration



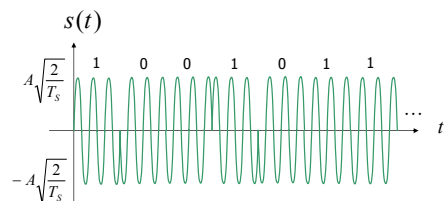
Binary Phase Shift Keying (BPSK)

- For BPSK, each symbol carries one bit of information $x_n \in \{-1, 1\}$



BPSK-Modulated Carrier

- The information is in the phase of the carrier



Power Spectral Density (PSD) of Linear Modulation

- Assume that the symbol sequence $\{x_n\}$ is iid and zero mean. Then, the PSD of $s(t)$ is

$$S_s(f) = \frac{1}{2} [S_g(f - f_c) + S_g(-f - f_c)]$$

where

$$S_g(f) = \frac{A^2 E \{ |x_n|^2 \}}{2T_s} |P(f)|^2$$

and $P(f)$ is the Fourier Transform of $p(t)$

Bandwidth Properties

- The RF bandwidth of the modulated carrier is two times the baseband bandwidth of

$$S_g(f) = \frac{A^2 E \{ |x_n|^2 \}}{2T_s} |P(f)|^2$$

which is clearly seen to depend on the bandwidth of the pulse

Fourier Transform of the Rectangular Pulse

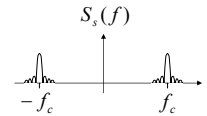
$$\begin{aligned} P(f) &= \int_{-\infty}^{+\infty} p(t) \exp(-j2\pi ft) dt = \int_0^{T_s} \sqrt{\frac{2}{T_s}} \exp(-j2\pi ft) dt \\ &= \sqrt{\frac{2}{T_s}} \frac{\exp(-j2\pi f T_s) - 1}{-j2\pi f} \Big|_0^{T_s} = \sqrt{\frac{2}{T_s}} \frac{\exp(-j2\pi f T_s) - 1}{-j2\pi f} \\ &= \frac{\exp(-j\pi f T_s)}{\pi f} \sqrt{\frac{2}{T_s}} \sin(\pi f T_s) = \exp(-j\pi f T_s) \sqrt{2T_s} \text{sinc}(fT_s) \end{aligned}$$

PSD for BPSK and the Rectangular Pulse

- $|P(f)|^2 = 2T_s \text{sinc}^2(fT_s)$
- $|x_n|^2 = 1$

$$\begin{aligned} S_g(f) &= \frac{A^2 E \{ |x_n|^2 \}}{2T_s} |P(f)|^2 \\ &= A^2 \text{sinc}^2(fT_s) \end{aligned}$$

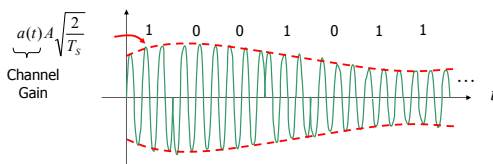
$$S_s(f) = \frac{A^2}{2} [\text{sinc}^2((f - f_c)T_s) + \text{sinc}^2((-f - f_c)T_s)]$$



Received Signal

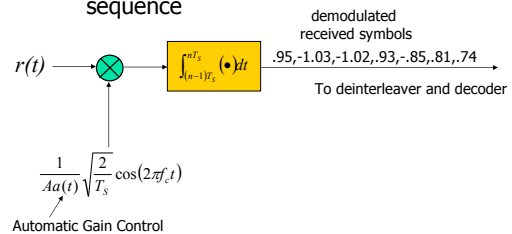
- The received signal has been attenuated by path loss and multipath fading and has added noise

$$r(t) = a(t) A \sum_n x_n p(t - nT_s) \cos(2\pi f_c t + \theta_c) + n(t)$$



BPSK Demodulator

- The output of the correlator is a sequence of noisy versions of the transmitted symbol sequence



Integrate and Dump Output

- Consider one symbol period and the rectangular pulse:

$$R = \int_0^{T_s} r(t) \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t) dt = \int_0^{T_s} \left(x_n \sqrt{\frac{2\mathcal{E}_b}{T_s}} \cos(2\pi f_c t) + n(t) \right) \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t) dt$$

channel gains and TX power lumped into \mathcal{E}_b

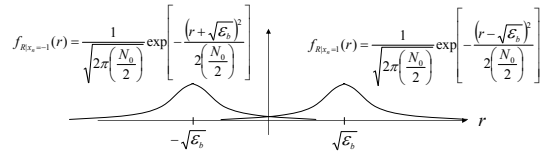
$$= \frac{x_n}{T_s} \sqrt{\mathcal{E}_b} \int_0^{T_s} 1 dt + \sqrt{\frac{2}{T_s}} \int_0^{T_s} n(t) \cos(2\pi f_c t) dt = x_n \sqrt{\mathcal{E}_b} + v_n$$

- v_n is a zero-mean Gaussian RV with variance

$$\sigma_v^2 = \frac{N_0}{2}$$

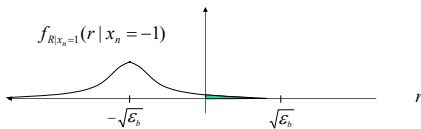
Conditional PDFs

- Recall $R = x_n \sqrt{\mathcal{E}_b} + v_n$



Conditional Probability of Bit Error

- If x_n is -1, then an error happens if $R > 0$



$$P(\text{error} | x_n = -1) = P(R > 0 | x_n = -1)$$

$$= \int_0^{+\infty} f_{R|x_n=-1}(r | x_n = -1) dr$$

Conditional Probability of Error Expression

$$\int_0^{+\infty} f_{R|x_n=-1}(r | x_n = -1) dr = Q\left(\frac{0 - (-\sqrt{\mathcal{E}_b})}{\sqrt{\frac{N_0}{2}}}\right) = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$$

- Same for other kind of error, by symmetry

Unconditional Probability of Bit Error

- Assume the two possible values of x_n are equally likely

$$P(\text{error}) = \frac{1}{2} P(\text{error} | x_n = -1) + \frac{1}{2} P(\text{error} | x_n = 1)$$

$$= Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$$

How to Improve System Performance

- Increase symbol energy \mathcal{E}_b
- Decrease average noise power $N_0/2$

$$P(\text{error}) = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$$



References

- [Rapp, '02] T.S. Rappaport, *Wireless Communications*, Prentice Hall, 2002
- [Stuber, '01] Gordon Stuber, *Principles of Mobile Communication, 2nd ed*, Kluwer Academic Publishers, 2001.