Nyquist Pulses

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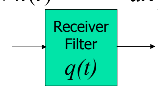
ECE4823

Receiver Filter

- Baseband output is a superposition of filtered pulses plus filtered noise

$$aA\sum_n x_n p(t-nT_S)+n(t) \qquad aA\sum_n x_n \tilde{p}(t-nT_S)+v(t)$$



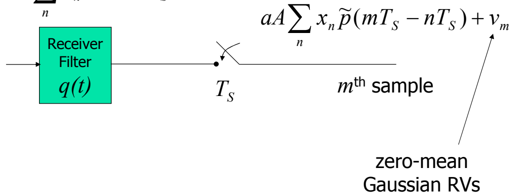
$$\tilde{p}(t)=\frac{1}{2}\int_{-\infty}^{+\infty} p(t-\tau)q(\tau)d\tau$$

Sampling in the Receiver

- The baseband representation of the receiver in additive Gaussian white noise (AWGN) :

$$aA\sum_n x_n \tilde{p}(t-nT_S)+v(t)$$

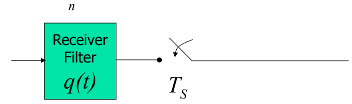
$$aA\sum_n x_n \tilde{p}(mT_S-nT_S)+v_m$$



Ideal Situation

- Ideally, the m^{th} sample depends on only the m^{th} symbol and the noise

$$aA\sum_n x_n \tilde{p}(mT_S-nT_S)+v_m = aAx_m + v_m$$



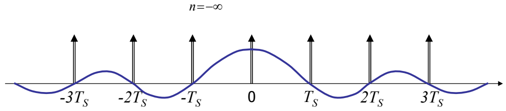
Necessary Condition

- To have this ideal situation, we must have

$$\tilde{p}(mT_S-nT_S)=\begin{cases}1 & m=n\\ 0 & m\neq n\end{cases}$$

- or, alternatively,

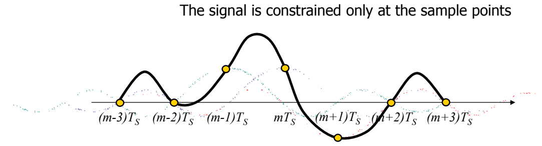
$$\tilde{p}(t)\sum_{n=-\infty}^{+\infty}\delta(t-nT_S)=\delta(t)$$



Received Signal Example

- Suppose the m-1$^{\text{st}}$, m^{th}, and m+1$^{\text{st}}$ symbols were +1, +1, -1, respectively

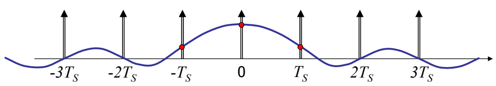
The signal is constrained only at the sample points



Non-Ideal Situation

- Suppose the received pulse did not satisfy the condition, but did this instead:

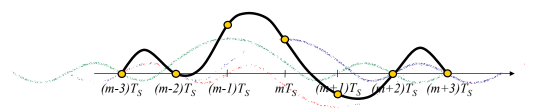
$$\tilde{p}(t)\sum_{n=-\infty}^{+\infty}\delta(t-nT_S)=\delta(t)+0.3\delta(t-T_S)+0.3\delta(t+T_S)$$



Intersymbol Interference (ISI)

- The m-1st, mth, and m+1st received samples become

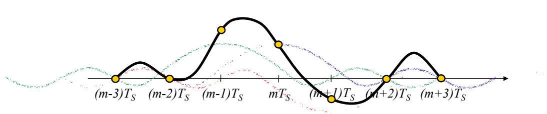
$$1.3+v_{m-1},\ 1+v_m,\ -0.7+v_{m+1}$$



The Negative Effect of ISI

- The worst case dominates the probability if bit error

$$1.3+v_{m-1},\ 1+v_m,\ 0.7+v_{m+1}$$



Nyquist Pulses

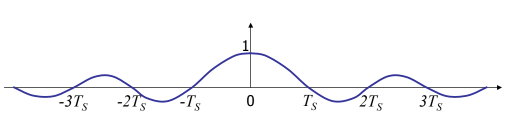
- Pulses that satisfy the condition for no ISI are called Nyquist Pulses

$$\tilde{p}(t)\sum_{n=-\infty}^{+\infty}\delta(t-nT_S)=\delta(t)$$

Sinc Pulse

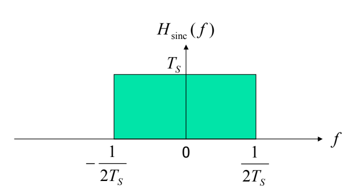
- The sinc pulse is a Nyquist pulse

$$\mathrm{sinc}(t/T_S)=\frac{\sin(\pi t/T_S)}{\pi t/T_S}$$



Fourier Transform of Sinc Pulse

- The F.T. of the sinc pulse is the "brick wall" characteristic

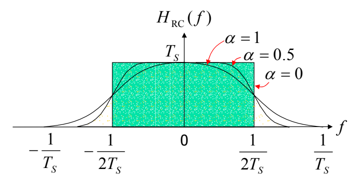


Pros and Cons of Sinc

- The F.T. has the narrowest possible bandwidth of all Nyquist pulses
- Noncausal
- Roll-off too gradual

Popular Alternative: Raised Cosine

- Single parameter α



Raised Cosine Formulas

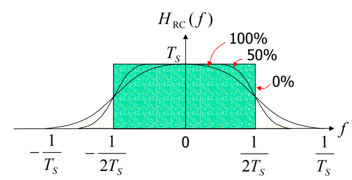
[Pahlavan `95]

$$H_{RC}(f) = \begin{cases} T_S & 0 \le |f| \le (1-\alpha)/2T_S \\ \frac{T_S}{2}\left[1 - \sin \frac{\pi T_S}{\alpha}\left(|f| - \frac{1}{2T_S}\right)\right] & (1-\alpha)/2T_S \le |f| \le (1+\alpha)/2T_S \\ 0 & (1+\alpha)/2T_S \le |f| \end{cases}$$

$$p_{RC}(t) = F^{-1}\{H_{RC}(f)\} = \frac{\sin \pi t / T_S}{\pi t / T_S} \frac{\cos \alpha \pi t / T_S}{1 - 4\alpha^2 t^2 / T_S^2}$$

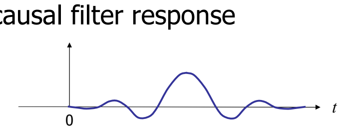
Excess Bandwidth

- The bandwidth of any pulse will be X% greater than that of the sinc Nyquist pulse
- This percentage is the excess bandwidth



Pros of Raised Cosine

- Smooth F.T.
  - Easy to build filters that approximate it
- Falls off as $1/t^3$
- Can approximate a delayed pulse with a causal filter response



Summary

- Intersymbol interference (ISI) can dominate BER
- Nyquist Pulses are pulses that do not cause ISI in an AWGN channel
- Sinc is the Nyquist Pulse with the least bandwidth, but it is impractical
- Raised Cosine is a popular Nyquist Pulse
- Excess Bandwidth indicates how much broader is the bandwidth than that of the sinc Nyquist Pulse

References

- [Rapp, '02] T.S. Rappaport, *Wireless Communications*, Prentice Hall, 2002
- [Stuber, '01] Gordon Stuber, *Principles of Mobile Communication, 2nd ed*, Kluwer Academic Publishers, 2001
- [Pahlavan, '95] Pahlavan and Levesque, *Wireless Information Networks*, Wiley Interscience, 1995