

## Other Linear Techniques

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## Recall Linear Modulation

- $s(t)$  is the output of the modulator
- $g(t)$  is the complex envelope
- $p(t)$  is the basic pulse
- $x_n$  is the  $n$ th symbol

$$s(t) = \text{Re}\{g(t)e^{j2\pi f_c t}\}$$

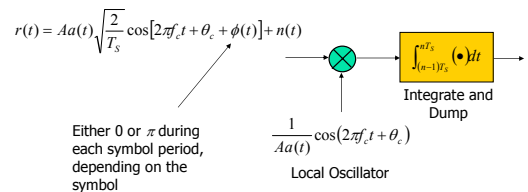
$$g(t) = A \sum_n x_n p(t - nT_s)$$

## Overview of Techniques

- Differential PSK (DPSK)
  - Inexpensive
- Quadrature Phase Shift Keying (QPSK)
  - Twice as spectrally efficient than BPSK
- Offset QPSK (OQPSK)
  - reduce envelope variation
- $\pi/4$  QPSK
  - reduces envelope variations to a lesser extent than OQPSK; can be noncoherently detected

## Recall the BPSK Receiver

- The local oscillator must match the phase of the incoming carrier

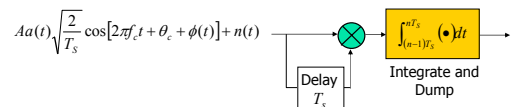


## DPSK

- Differential PSK does not require the Local Oscillator phase to match the incoming carrier phase
- The information is carried in the *difference* between the phases of the present and previous symbol waveforms

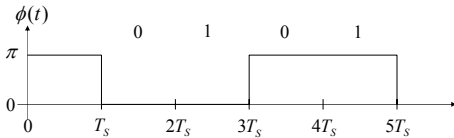
## DPSK Receiver

- Received signal in last symbol period is used as the Local Oscillator for the present symbol period



## Differential Encoding

- 1 = no change in phase
- 0 = change in phase



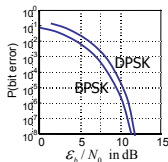
## DPSK Performance

- Because the "Local Oscillator" has some additive noise, the BER is not as low as for coherent BPSK

$$P_{DPSK}(\text{bit error}) = \frac{1}{2} \exp\left(-\frac{\mathcal{E}_b}{N_0}\right)$$

## SNR Penalty

- DPSK has an SNR penalty relative to BPSK of less than 2 dB for BER < 1E-2
- In other words,  $\mathcal{E}_b / N_0$  must be as much as 2 dB larger for DPSK to achieve the same BER as BPSK



## QPSK

- QPSK is like BPSK, except the phase can take four values instead of just two

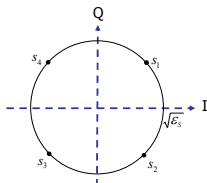
$$s(t) = \text{Re}\{g(t)e^{j2\pi f_c t}\}$$

$$g(t) = A \sum_n x_n p(t - nT_s)$$

$$x_n \in \{e^{j\pi/4}, e^{j3\pi/4}, e^{-j\pi/4}, e^{-j3\pi/4}\}$$

## QPSK Signal Space Diagram

- The bases functions are cosine and sine
  - The cosine component is called the "In Phase" component
  - The sine component is called the "Quadrature" component



## Square Pulse Case

- A QPSK waveform:

$$s_{QPSK}(t) = \sqrt{\mathcal{E}_s} \cos \phi_n \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t) - \sqrt{\mathcal{E}_s} \sin \phi_n \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t)$$

$$\text{where } \phi_n \in \{\pi/4, 3\pi/4, -\pi/4, -3\pi/4\}$$

$$\text{and } nT_s < t \leq (n+1)T_s$$

- $\mathcal{E}_s$  is the symbol energy
- $\mathcal{E}_b = \frac{1}{2} \mathcal{E}_s$  is the bit energy

## QPSK=2 BPSKs

$$s_{QPSK}(t) = \underbrace{\sqrt{\mathcal{E}_s} \cos \phi_n \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t)}_{\text{One BPSK Signal}} - \underbrace{\sqrt{\mathcal{E}_s} \sin \phi_n \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t)}_{\text{Another BPSK Signal}}$$

- The two BPSK signals are separated in the receiver using two LOs, one a cosine, the other a sine

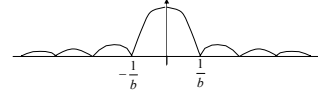
## QPSK Performance

- BER same as BPSK for the same  $\mathcal{E}_b$
- For the same data rate, QPSK has a bandwidth half of that of BPSK

$$S_s(f) = \frac{\mathcal{E}_s}{2} [\text{sinc}^2((f - f_c)T_s) + \text{sinc}^2((-f - f_c)T_s)]$$

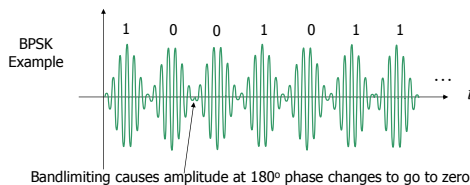
$$= \mathcal{E}_b [\text{sinc}^2((f - f_c)2T_B) + \text{sinc}^2((-f - f_c)2T_B)]$$

$$\text{sinc}^2(bf)$$



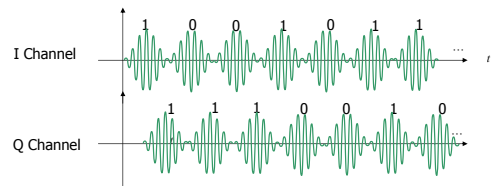
## Pulse-shaping Effects

- Because of realistic pulse-shaping, the envelope of BPSK or QPSK is not constant
- Undesirable because linear amplifiers, which are not as power efficient and more expensive, are required



## Offset QPSK

- Since the QPSK waveform is just a superposition of two independent BPSK waveforms, why not shift one relative to the other by half a symbol period to make the envelope more constant?



## Offset QPSK Performance

- By switching the phase twice as often as QPSK, the max phase change becomes  $\pm 90$  degrees instead of 180 degrees, so amplitude not forced to zero by bandlimiting
- Same spectrum as QPSK
- Same BER as QPSK

## $\pi/4$ QPSK

- A differential encoding technique that allows a maximum of a  $135^\circ$  phase shift at symbol transitions
- Used in mobile/personal radio standards IS-54 and IS-136, and in the European TETRA standard for private business radio [Burr, 2001]

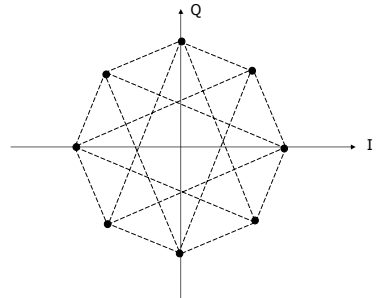
## Differential Mapping

- $\theta_{k-1}$  is the phase of the last symbol
- $\theta_k$  is the phase of the present symbol
- $\phi_k$  depends on the present two bits

$$\theta_k = \theta_{k-1} + \phi_k$$

Info bits	$\phi_k$
11	$\pi/4$
01	$3\pi/4$
00	$-3\pi/4$
10	$-\pi/4$

## Constellation



## BER Performance

- $\pi/4$  QPSK can be detected coherently or noncoherently (differentially)
- If coherently, the performance is the same as QPSK
- If noncoherent, the SNR penalty relative to QPSK is about 2.3 dB at high SNR [Proakis, 2000]

## Summary

- Many variations of linear modulation
- DPSK eliminates need for coherent receiver (no phased-lock loop), therefore cheaper; only 2 dB penalty
- QPSK is just two BPSKs, with carrier shifted by  $90^\circ$
- OQPSK makes maximum phase shift in carrier  $90^\circ$
- $\pi/4$  QPSK makes maximum phase shift in carrier  $135^\circ$ ; can be differentially detected

## References

- [Rapp, '02] T.S. Rappaport, *Wireless Communications*, Prentice Hall, 2002
- [Burr, 2001] Alister Burr, *Modulation and Coding for Wireless Communications*, Prentice Hall, 2001
- [Proakis, 2000] John G. Proakis, *Digital Communications* 4<sup>th</sup> ed., McGraw Hill, 2000