Frequency Shift Keying (FSK)

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Why Study FSK?

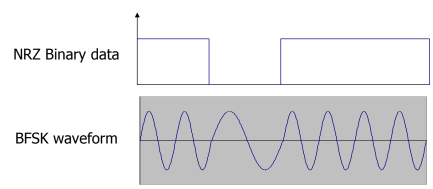
- Constant envelope
  - More efficient, less costly power amplifiers
- Gaussian minimum shift keying (GMSK), a special type of FSK, is used in the European digital cellular communications system (GSM)

Angle Modulation

- General form:
  - $s(t) = A \cos\{2\pi f_c t + \phi(t)\}$
- Phase modulation (PM)
  - $\phi(t) = k_p\, m(t)$, where k_p = constant, $m(t)$=modulating signal
  - Examples: BPSK, QPSK, 8PSK, etc.
- Frequency modulation (FM)
  - $\phi(t) = 2\pi \int_0^t k_f m(\tau)\, d\tau$, where k_f = constant, $m(\tau)$= modulating signal
  - Examples: FSK

Continuous Phase BFSK Illustration

- Phase continuity is important to reduce bandwidth



M-ary Continuous Phase FSK

- Recall FM phase: $\phi(t) = 2\pi \int_0^t k_f m(\tau) d\tau$
- The modulating waveform is pulse-amplitude modulated (PAM)

$$m(\tau) = \sum_n I_n g(\tau - nT_S)$$

where

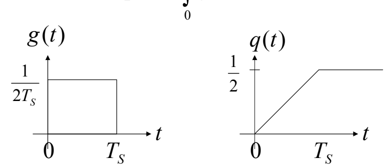
$$I_n \in \{\pm 1, \pm 3, \ldots, \pm(M-1)\}$$

and $g(\tau)$ is a pulse with area 1/2

Integrated Pulse

- Let $q(t)$ be the integrated pulse

$$q(t) = \int_0^t g(\tau) d\tau$$



Modulation Index

- The FSK phase can be written

$$\phi(t) = 2\pi k_f \sum_n I_n q(t - nT_S)$$

- k_f is the modulation index $k_f = 2 f_d T_S$ with units of cycles/symbol period
- f_d is the peak frequency deviation
- The sum increments in multiples of 1/2

Example

- Suppose $g(t)$ is the rectangular pulse, k_f =10, and I_n=1, then

Orthogonal Waveforms

- BFSK has the following waveforms:

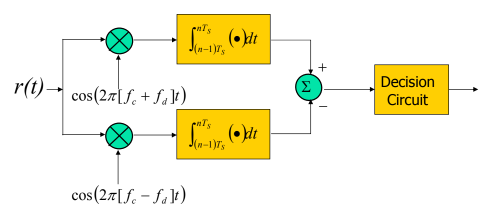
$$s_1(t) = \sqrt{\frac{2\mathcal{E}_b}{T_S}}\cos(2\pi[f_c + f_d]t) \qquad 0 < t < T_S$$

$$s_2(t) = \sqrt{\frac{2\mathcal{E}_b}{T_S}}\cos(2\pi[f_c - f_d]t) \qquad 0 < t < T_S$$

- If $f_d = n/4T_s$, for n a positive integer, these waveforms will be orthogonal

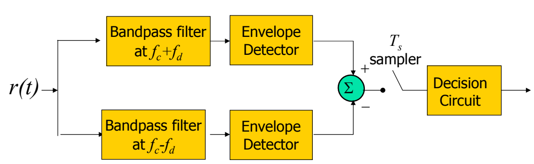
BFSK Coherent Detection

$$P_{CBFSK}(error) = Q\left(\sqrt{\frac{\mathcal{E}_b}{N_0}}\right)$$



BFSK Noncoherent Detection

$$P_{NBFSK}(error) = \frac{1}{2}\exp\left(-\frac{\mathcal{E}_b}{2N_0}\right)$$



Comparison

- In terms of BER,
  - CBFSK is to NBFSK as
  - BPSK is to DPSK
- The SNR for FSK is half as large as it is for PSK

$$P_{BPSK}(error) = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right) \qquad P_{CBFSK}(error) = Q\left(\sqrt{\frac{\mathcal{E}_b}{N_0}}\right)$$

$$P_{DPSK}(error) = \frac{1}{2}\exp\left(-\frac{\mathcal{E}_b}{N_0}\right) \qquad P_{NBFSK}(error) = \frac{1}{2}\exp\left(-\frac{\mathcal{E}_b}{2N_0}\right)$$

MSK

- For minimum shift keying (MSK), the modulation index is as small as it can be and still yield orthogonal waveforms
  - This minimum is k_f=1/2
  - For rectangular $g(t)$, the peak frequency deviation is ¼ the data rate

GMSK

- Gaussian MSK (GMSK) uses k_f=1/2 and pulses related to the Gaussian shape
- GMSK pulses are distinguished by their time-bandwidth product BT_S
- B is the 3-dB bandwidth of the pulse
- BT_S=0.3 is used in GSM

Summary

- Frequency shift keying is a constant-envelope modulation technique
- Phase continuity reduces bandwidth
- FSK can be detected noncoherently
- GMSK, treated in detail in another module, is used in GSM

References

- [Rapp, '02] T.S. Rappaport, *Wireless Communications*, Prentice Hall, 2002
- [Proakis, 2000] John G. *Proakis, Digital Communications* 4th ed., McGraw Hill, 2000