More on MSK

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ECE4823

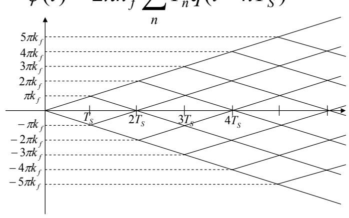
MSK is a Linear Modulation

- MSK is an unusual type of FSK because it can be represented as a linear modulation
- It looks like OQPSK with pulses that are twice as wide
- The "symbols" weighting the pulses are not the original symbol sequence

Recall BFSK

- Recall the phase trajectories for binary FSK when rectangular pulses are used for *m(t)*:

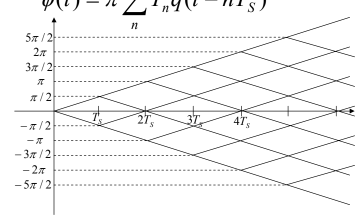
$$\phi(t) = 2\pi k_f \sum_n I_n q(t - nT_S)$$



The Phase for MSK

- In MSK, k_f=0.5

$$\phi(t) = \pi \sum_n I_n q(t - nT_S)$$



Y-intercept Form

[Simon et al., '94]

- The phase can be expressed

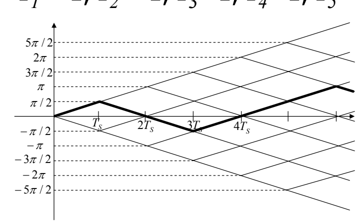
$$\phi(t) = I_n \left(\frac{\pi t}{2T_S} \right) + x_n$$

where x_n is the Y-intercept for the nth symbol interval

$$[x_n \bmod(2\pi)] \in \{0, \pi\}$$

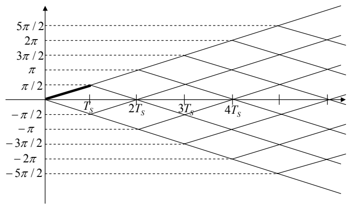
Example

- Consider the symbol sequence I_o=1, I_1=-1, I_2=-1, I_3=1, I_4=1, I_5=1, I_6=-1



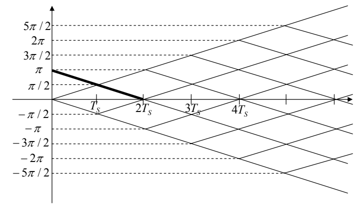
For I_0=1 and $0<t<T_S$

$$\phi(t) = I_0\left(\frac{\pi t}{2T_S}\right) + x_0 = 1\cdot\left(\frac{\pi t}{2T_S}\right) + 0 \quad \Rightarrow \quad x_0 = 0$$



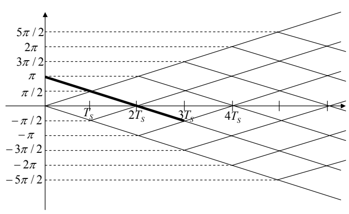
For I_1=-1 and $T_S<t<2T_S$

$$\phi(t) = I_1\left(\frac{\pi t}{2T_S}\right) + x_1 = -1\cdot\left(\frac{\pi t}{2T_S}\right) + \pi \quad \Rightarrow \quad x_1 = \pi$$



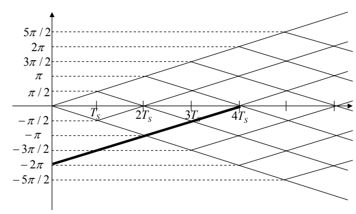
For I_2=-1 and $2T_S<t<3T_S$

$$\phi(t) = I_2\left(\frac{\pi t}{2T_S}\right) + x_2 = -1\cdot\left(\frac{\pi t}{2T_S}\right) + \pi \quad \Rightarrow \quad x_2 = \pi$$



For I_3=1 and $3T_S<t<4T_S$

$$\phi(t) = I_3\left(\frac{\pi t}{2T_S}\right) + x_3 = 1\cdot\left(\frac{\pi t}{2T_S}\right) - 2\pi \quad \Rightarrow \quad x_3 = -2\pi = 0$$



I/Q Formulation of MSK

- It can be shown that

$$s(t) = \sqrt{\frac{2\mathcal{E}_b}{T_S}}\left[\sum_n c_n p(t-(2n-1)T_S)\cos(2\pi f_c t) - \sum_n d_n p(t-2nT_S)\sin(2\pi f_c t)\right]$$

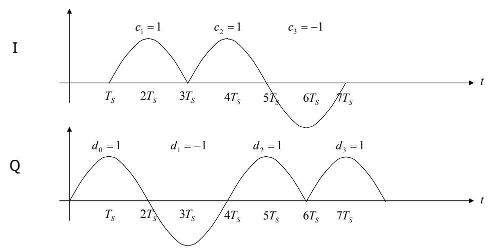
where

$$p(t) = \begin{cases} \sin\left(\dfrac{\pi t}{2T_S}\right) & 0 \le t \le 2T_S \\ 0 & \text{otherwise} \end{cases} \qquad c_n = (-1)^n \cos x_{2n-1} \qquad d_n = (-1)^n I_{2n} \cos x_{2n}$$

Example, cont'd

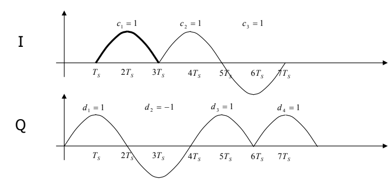
n	I_n	$x_n mod 2\pi$	$\cos x_n$	c_n	d_n
0	1	0	1		d_0=1
1	-1	π	-1	c_1=1	
2	-1	π	-1		d_1=-1
3	1	0	1	c_2=1	
4	1	0	1		d_2=1
5	1	0	1	c_3=-1	
6	-1	0	1		d_3=1

Baseband Waveform for the Example



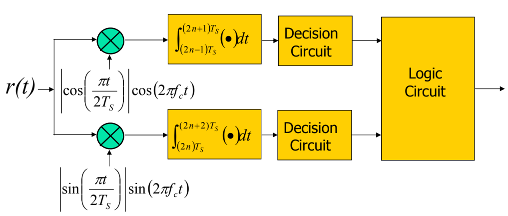
MSK Resembles OQPSK

- The basic pulse shape is half a sinusoid that is $2T_S$ wide



Quadrature Detection

- The MSK receiver can be implemented coherently like an OQPSK receiver



Summary

- MSK is actually a linear modulation method
- The basic pulse shape is half a sinusoid that is two bit periods wide
- Resembles OQPSK with this basic pulse shape

References

- [Rapp, '02] T.S. Rappaport, *Wireless Communications*, Prentice Hall, 2002
- [Simon et al., '94] M.K. Simon, S.M. Hinedi and W.C. Lindsey, *Digital Communication Techniques, Signal Design and Detection*, PTR Prentice Hall, 1994.