CDMA

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ECE 4823

Motivation

- BER depends on bit energy—not on the bandwidth
- Large bandwidth signals are
  - less sensitive to multipath fading
  - less vulnerable to jamming
  - can be concealed
  - can share a common bandwidth without interfering with each other

Code-Division Multiple Access

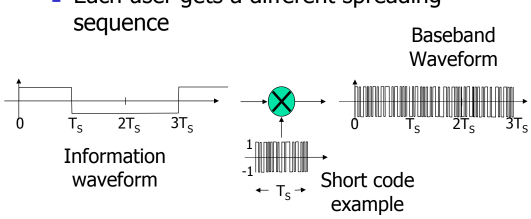
- Allows multiple users to share same bandwidth at the same time
- Each user’s waveform is like an independent noise random process
- Interference appears as white noise
- Matched filter pulls out desired user’s waveform, suppresses interference

Direct Sequence Spread Spectrum (DS-SS)

- DS-SS is one popular way to make the noise-like waveforms for CDMA
- Maximal-length shift registers make binary sequences that have noise-like properties
  - m-stage shift register produces a sequence with a period of length 2^m-1

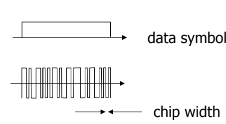
DS-SS Baseband Waveform

- Binary noise sequence is mapped to a chip spreading sequence of +/- 1’s
- Each user gets a different spreading sequence



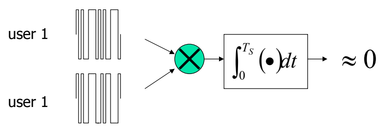
Chips

- The spreading sequence comprises chips (very short pulses) with width T_C
- There are an integer number of chips for each data symbol



Codes for Different Users

- Their cross-correlation is nearly zero:

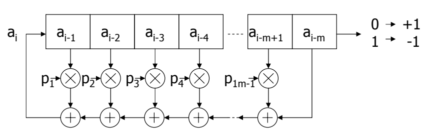


M-Sequences

- "Maximal-length" or m-sequences are a well-known class of spreading sequences
- Generated with a linear feedback shift register
- A register of length m generates a code N long, where $N=2^m-1$

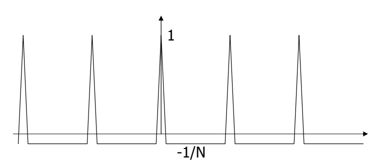
Generating M-Sequences

- The p_i's are the coefficients of a primitive polynomial



Autocorrelation of the M-Sequence

- Very much like an impulse



Processing Gain

- The number of chips per symbol is the processing gain (PG)
- This is also

$$PG=\frac{B_{SS}}{B}$$

where B_{SS} and B are the bandwidths of the chips and the data symbols, respectively. Usually, $B_{SS} >> B$

Signal Model for k-th User

$$s_k(t)=\sqrt{\frac{2E_S}{T_S}}m_k(t)p_k(t)\cos\left(2\pi f_c t+\theta_k\right)$$

- E_S = symbol energy
- $m_k(t)$ = information waveform for k-th user
- $p_k(t)$ = spreading sequence for k-th user
- B = bandwidth of $m_k(t)$
- B_{SS} = bandwidth of $p_k(t)$

Received Signal Model—No Multipath

$$r(t) = \sum_{k=1}^{K} s_k(t - \tau_k) + n(t)$$

- Assume K users and that the k-th user's signal is delayed by τ_k

Correlator Receiver

- Assume user 1's delay, τ_1, is known
- To receive the signal of user 1, correlate the received signal with user 1's spreading sequence delayed by τ_1

$$Z_i^{(1)} = \int_{(i-1)T_S+\tau_1}^{iT_S+\tau_1} r(t) p_1(t-\tau_1)\cos\left(2\pi f_c(t-\tau_1)+\theta_1\right)dt$$

$$= \int_{(i-1)T_S+\tau_1}^{iT_S+\tau_1} \left[\sum_{k=1}^{K} s_k(t-\tau_k)+n(t)\right] p_1(t-\tau_1)\cos\left(2\pi f_c(t-\tau_1)+\theta_1\right)dt$$

Multi-user Interference

- The receiver correlates to the code of the desired user
- Every undesired user's code has a small amount of residue because of imperfect orthogonality
- The multi-user interference from each user is approximated as a Gaussian RV based on a Central Limit Theorem argument
  - Contribution of lots of chips

Simplified Correlator Output

$$Z_i^{(1)} = I_i^{(1)} + \xi$$
$$= I_i^{(1)} + Y + \eta$$

- Y is the multiple access interference part
  - Y is $N(0, \sigma_\xi^2)$, based on Central Limit Theorem
- η is the thermal noise part

Random Sequence Model

- The analysis of the BER for DS-SS assumes that the K-1 interfering spreading sequences are random and N chips long
- The BER is obtained by averaging over all possible spreading sequences, including the desired sequence
- Therefore, $\sigma_\xi^2 > 0$, σ_ξ^2 is proportional to K-1, and σ_ξ^2 is inversely proportional to N

BER for BPSK Assuming AWGN

$$P_e = Q\left(\frac{1}{\sqrt{\frac{K-1}{3N} + \frac{N_o}{2E_b}}}\right)$$

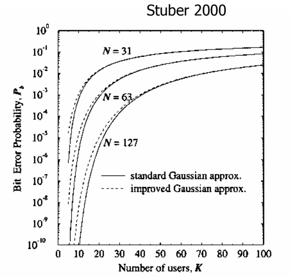
the "3" comes from an assumption of chip and phase asynchrony; this is dropped if synchronous

- Reduces to standard BPSK BER expression when K=1

Graceful Degradation

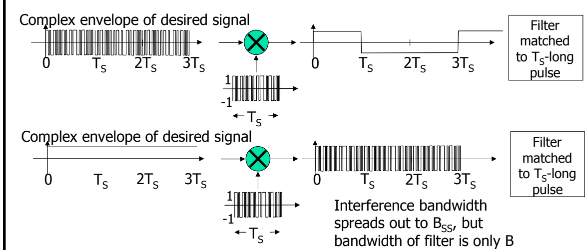
- Unlike TDMA, CDMA BER increases gradually as more users are added

Stuber 2000

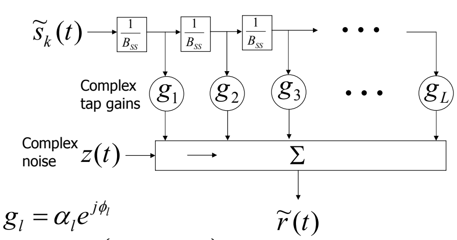


Narrowband Interference

- Interference signal is spread and then filtered



Baseband Tapped-Delay Line Model of Received Complex Envelope



$$g_l = \alpha_l e^{j\phi_l}$$

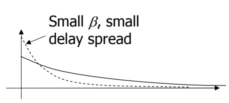
$$r(t) = \mathrm{Re}\left\{\tilde{r}(t) e^{j2\pi f_c t}\right\}$$

Statistical Models of Tap Gains

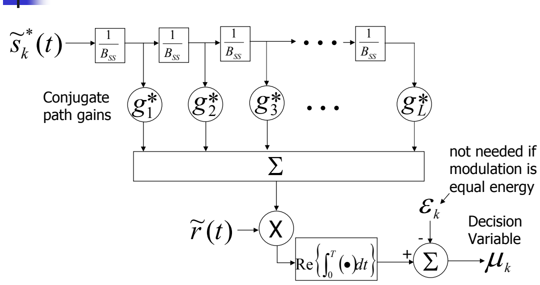
- Under the wide-sense stationary uncorrelated scattering (WSSUS) assumption, the tap gains are uncorrelated complex RVs
- A reasonable model for the tap gain magnitudes, α_l, is Rayleigh with exponentially decreasing mean square values

$$E\left\{\alpha_l^2\right\} = Ce^{-l/\beta}$$

$$s.t. \quad \sum_{l=1}^{L} Ce^{-l/\beta} = 1$$



Correlator (RAKE) Receiver



The Decision Variable

- The RAKE receiver output is

autocorrelation of spreading sequence

$$\mu_k = 2\varepsilon \sum_{l=1}^{L} \alpha_l^2 + 4\varepsilon \sum_{k=1}^{L-1} \sum_{i=1}^{L-1-k} Y_{i,i+k} \phi_p(k) + \tilde{n}_k$$

where the self-interference is

$$Y_{m,l} = \mathrm{Re}\left\{g_m g_l^*\right\}$$

and generally non-Gaussian and correlated

BER

- For DS-SS-BPSK, and assuming ideal speading sequence (impulse autocorrelation), then

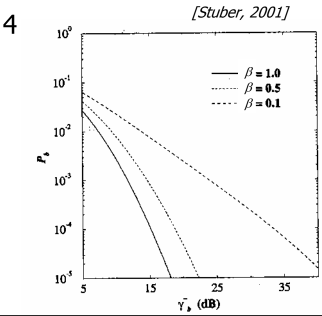
$$P_b = \frac{1}{2}\sum_{l=1}^{L} A_m \left[1 - \sqrt{\frac{\bar{\gamma}_m}{1+\bar{\gamma}_m}}\right]$$

$$A_m = \prod_{l=1, l\neq m}^{L} \frac{\bar{\gamma}_m}{\bar{\gamma}_m - \bar{\gamma}_l}$$

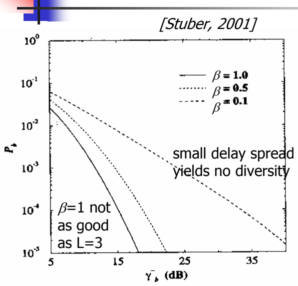
$$\bar{\gamma}_m = \frac{\left(1-e^{-1/\beta}\right)e^{-m/\beta}}{e^{-1/\beta} - e^{-(L+1)/\beta}}\bar{\gamma}_b$$

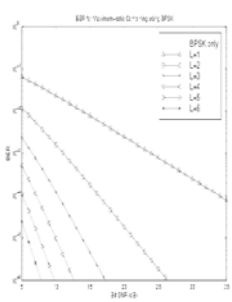
BPSK DS-SS BER Curves

- Channel has 4 taps
- RAKE has 4 taps



Compare to 4-th Order Spatial Diversity

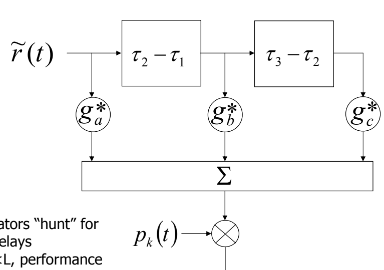




Sliding Correlator RAKE Receiver

- The received signal is split into M branches (M could be less than L)
- Each branch signal is weighted with the conjugate of the path gain (like MRC)
- Each weighted branch signal is correlated with a differently delayed version of the spreading sequence

Sliding Correlator RAKE Receiver; Example: M=3



3 RAKE "fingers"

- Correlators "hunt" for best delays
- for M<L, performance won't be as good

Summary

- CDMA allows efficient use of spectrum by putting all users on top of each other in time and frequency
- Graceful degradation as traffic increases
- Robust against interference
- RAKE receiver provides some fading mitigation that depends on delay spread