Numerical Evaluation of the Energy for Upstream Opportunistic Large Array-based Transmissions

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Abstract— **The opportunistic large array (OLA) is a particularly simple form of cooperative transmission. Recently, the OLA concentric routing algorithm with limited flooding and transmission threshold (OLACRA-FT) was proposed for energy efficient upstream routing in wireless sensor networks that use an OLA-based physical layer. Because of the irregular shapes of the OLAs in OLACRA-FT, the OLAs were previously computed using time-consuming Monte Carlo techniques. In contrast, this paper applies numerical integration to determine the OLA boundaries in the OLACRA-FT protocol, under the continuum and orthogonal transmission assumptions. The energy saved relative to a simple OLA broadcast and the probability of successful message delivery to the sink are evaluated for variations on relay power, transmission threshold, and source OLA angular width.**

I. Introduction

Ad Hoc Wireless Sensor Networks (WSNs) are those in which simple, inexpensive devices are used to measure some quantity of interest, and report to a central computer. There is no central control or preestablished configuration in such networks. The network is established by a message being relayed through adjacent nodes, which are assumed to be in close proximity so as to require only low radio transmit power by a given node. A construct known as an Opportunistic Large Array (OLA) is a way to achieve diversity and reliability without the overhead of conventional multi-hop routing protocols [1]. Even though the network is *Ad Hoc* in nature, one can optimize energy usage by judicious choice of the way in which the network is initialized. The OLA Concentric Routing Algorithm with limited Flooding and a Threshold (OLACRA-FT) was introduced in [3] as a way to establish an Ad Hoc network so that the upstream transmissions are energy efficient. Our contribution is to use numerical integration to examine the upstream behavior of OLACRA-FT, in terms of the energy usage. This upstream routing back to the sink has been previously investigated using Monte Carlo simulations [2], [3].

We note that the state of the art in OLA upstream transmission is simple flooding of the entire network, unless location information, such as Global Positioning System for Navigation (GPS), is used to limit the flood [4], [6], [7]. In contrast, OLACRA-FT requires no location information to limit the flood.

II. Review of OLACRA-FT

WSNs consist of large number of nodes with sensing and communication capabilities conveying information in a networked manner to a central sink node. The Initialization Phase (InP) for the OLACRA-FT approach is achieved during a broadcast transmission from the central sink to the sensors, explained as follows. Let the sink transmit on a frequency f_1 [1], and selected nodes that can decode the sink transmission, forward the message on a new frequency f_2. For our analysis, we assume, as in [5] that a node successfully decodes if the received SNR exceeds a decoding threshold τ_d. The edge representing the nodes that barely are able to decode at f_1 will nominally form a circle. The edge nodes that decode on f_2, form a circle concentric with the first. The nodes between consecutive concentric rings form a Decoding Level, or just "Level". The disk defined by the smallest circle and which contains the central sink nodes is Level 1. In this manner, an ad hoc network is established, as the nodes in Level n transmit at a frequency f_{n+1}, forming a series of concentric circles. Nodes select themselves to relay if their received SNR is less than a "transmission threshold", τ_b [9]. In other words, the τ_b constraint prevents some of the decoding nodes from relaying because their contributions will not significantly impact the formation of the next level.

After the initialization phase, the upstream communication with OLACRA-FT works as follows. A source node in Level n transmits using f_{n+1}. Any node in Level n that can decode and meet the transmission threshold will repeat at f_n. Nodes in Level $n-1$ that can decode the message at f_n and meet the transmission threshold will relay at f_{n-1}. The transmission will proceed this way until the message is delivered to the sink.

It is known that in the InP, the radii growth is polynomial and the levels keep growing thicker [9]. This means that at a point far in the downstream, the levels could be so thick that the upstream approach described might fail if the source node is located far away from sink (and is located at outer the edge of a level). To remedy this, a few "broadcast" rings can be formed for the upstream transmission – just enough to reach the upstream border of the level. Suppose a source, indicated by the black dot in Fig. 1(a), is located near the downstream boundary of *Level n*. In this figure, three OLAs are required to fill the space between the upstream and downstream boundaries. The union of the three OLAs (all three shaded areas in Fig. 1(a) in Level n, is then considered an *extended source*. Next, the extended source behaves as if it were a single source node in an OLACRA-FT upstream transmission; this means that all the nodes in the extended source repeat the message together, and this collective transmission uses the same waveform as the original source. The formation of an extended source is a technique to ensure that the source transmission reaches the sink. In this paper, we shall approximate the extended source with an arc shape with straight sides as shown in Fig. 1(b).

III. Aggregate Power from an Arbitrary OLA shape

We assume that the nodes are densely populated over some area under consideration, such that they can be approximated as a continuum of nodes as in [5]. Also, we assume the simple radio propagation model, which assumes that the power decreases with the square of the distance from the transmitting node. Further, the model assumes that node transmissions are orthogonal [5], however, non-orthogonal transmissions are expected to produce qualitatively similar results. We note that OLACRA has also been shown to work in Rayleigh fading channels in [10] . Also in [10],

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[1]Instead of a certain frequency, a certain spreading code or a certain preamble can be used.

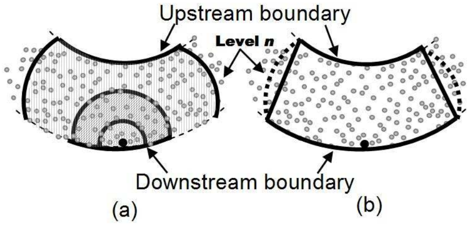

Fig. 1. (a) OLA extended source formation in OLACRA-FT; (b) Approximation of the extended source.

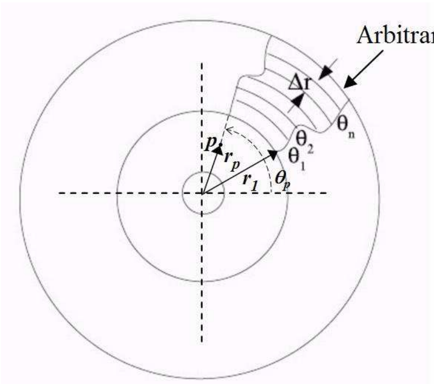

Fig. 2. Approximating an OLA of arbitrary shape in Polar Coordinates.

limited orthogonality was created by having the relays intentionally delay their spread spectrum transmissions. Thus to get the power power P_p received at a point p from this continuum of nodes in a given area, we integrate all the contributions from every differential element of area to give the integral

$$P_p = \int_A \frac{\bar{P}_r}{d^2} dA, \tag{1}$$

where $\bar{P}_r$ is the power transmitted by the relay nodes per unit area. In rectangular coordinates, d is given by

$$d(x, y) = \sqrt{(x_p - x)^2 + (y_p - y)^2}. \tag{2}$$

The polar coordinate system lends itself well to representing the concentric OLA approach. In polar coordinates, (1) becomes the double integral

$$P_p = \int_r \int_\theta \frac{\bar{P}_r}{d(r,\theta)^2} r dr d\theta, \tag{3}$$

where

$$d(r, \theta) = \sqrt{(r_p \cos\theta_p - r\cos\theta)^2 + (r_p \sin\theta_p - r\sin\theta)^2}. \tag{4}$$

The integral over the angle is available in closed form [8], so we will approximate the integral over radius as a Riemann Sum, for a sufficiently small radial thickness, Δr as

$$P_p \approx \sum_{i=1}^{N} \left\{ \frac{2\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan(\phi/2)\right]}{\sqrt{a^2-b^2}} \right\}_{\theta_1-\theta_p}^{\theta_2-\theta_p} r_i \Delta r, \tag{5}$$

with $a = r_p^2 + r^2$, and $b = -2rr_p$, $\Delta r = (r_2 - r_1)/N$, and $r_i = r_1 + i\Delta r$. We trade the accuracy of the approximation with computation time by controlling N. Thus, we can evaluate the integral over nearly arbitrary shapes by filling the shape in question with these arcs. This numerical approach dramatically shortens the computational time relative to the Monte Carlo approach, thereby enabling optimization of parameters.

IV. Comparison of OLACRA-FT to Flooding

We now examine the total power used by the network as compared to OLA flooding. Assuming the continuum approach, the Fractional Energy Savings (FES) is one minus the sum of the areas of all the trasmitting OLAs at each level divided by the total area of the network. This is obtained by summing the integrals over the various OLA shapes, given by

$$FES = 1 - \frac{\sum_{n=1}^{L} \int_{r_n} \int_{\theta_n} r dr d\theta}{\pi R_{net}^2}, \tag{6}$$

where r_n and θ_n define the OLA shapes in the nth level out of a total of L levels, and R_{net} is the radius of the entire network.

V. Simulation and Results

In this section, an example illustrating the downstream levels and upstream transmission to the sink is presented for a range of values of the upstream relay power density, $\bar{P}_r$. All figures share the same 10 downstream level boundaries, which were computed using the formulas in [2], assuming τ_d, τ_b, and $\bar{P}_r$ to be 1.0, 2.0, and 1.11 respectively.

Next, connectivity to the sink on the upstream was tested. Matlab simulations have been used to evaluate (5) and demonstrate how OLAs form, and their shapes. The simulations assume an approximate extended source, as described in Section II, and shown in Figs. 4–6 as a transparent thick arc in Level 10 outlined with blue solid line. Eq. (5) was used to compute the power received in each differential area in Level 9, and those areas that pass both τ_d and τ_b thresholds are indicated with a blue star. Therefore, the blue stars in Level 9 in each figure indicate the next upstream OLA after the extended source.

In Fig. 3, the upstream $\bar{P}_r$ is 0.9. In this case, the upstream OLAs are achieved all the way up to the sink. The reason upstream OLA formations are not contiguous is because of the transmission threshold τ_b.

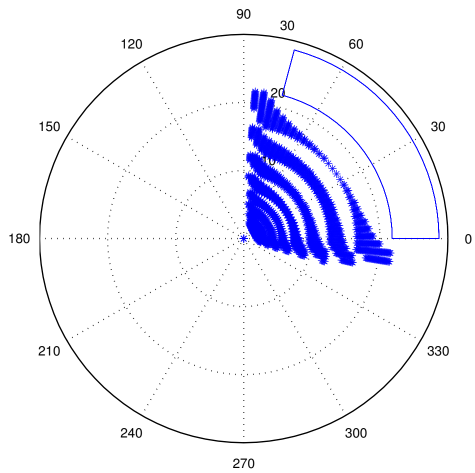

Fig. 3. Upstream OLA shapes for normalized relay power $\bar{P}_r = 0.9$ for $\tau_d = 1.0$ and $\tau_b = 2.0$.

Figs. 4–6 show results for other values of $\bar{P}_r$. In Fig. 4, the relay power of 0.75 is insufficient to propagate OLAs all the way to the sink. Fig. 5 shows a different phenomenon that results from the higher power level of $\bar{P}_r$ = 1.1. A hole is created in a couple of levels as the received power exceeds the τ_b threshold, and the nodes are therefore not allowed to transmit. If the relay power is reduced, or if τ_b is increased, the middle section in those levels will fill in. Finally, and of particular interest is the result shown in Fig. 6. Here, we see that if the power is *too high*, the message does not reach the sink, as the nodes near the center are seen to violate the τ_b threshold, and no nodes in the vicinity of the sink are allowed to transmit.

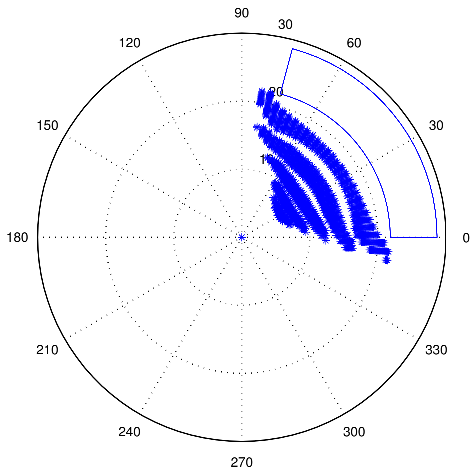

Fig. 4. Upstream OLA shapes for normalized relay power $\bar{P}_r = 0.75$ for $\tau_d = 1.0$ and $\tau_b = 2.0$.

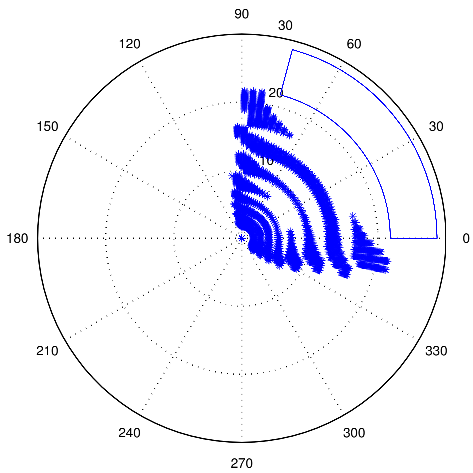

Fig. 5. Upstream OLA shapes for normalized relay power $\bar{P}_r = 1.1$ for $\tau_d = 1.0$ and $\tau_b = 2.0$.

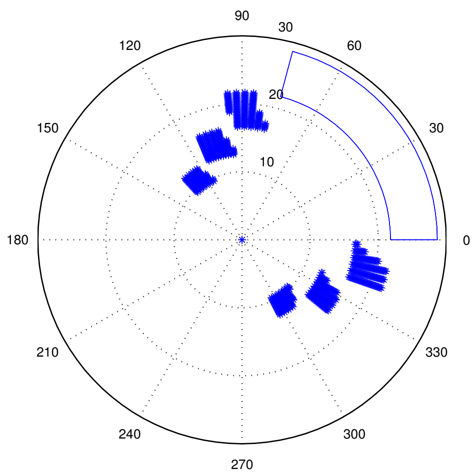

Fig. 6. Upstream OLA shapes for normalized relay power $\bar{P}_r = 1.5$ for $\tau_d = 1.0$ and $\tau_b = 2.0$.

In figure(7), Fractional Energy Savings (FES) is plotted by

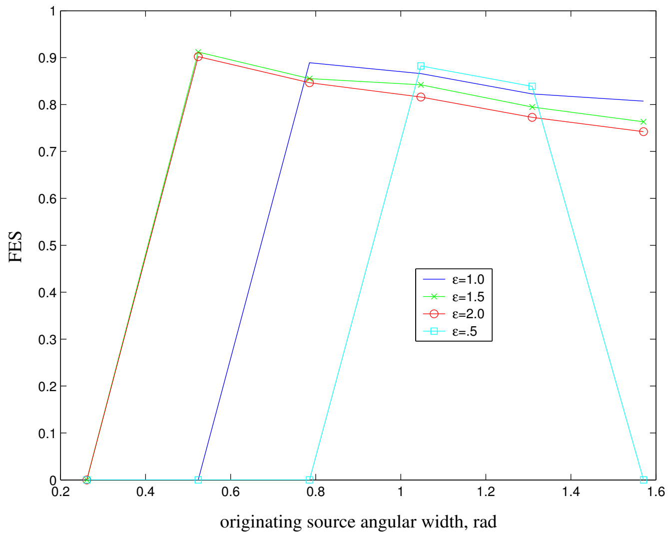

Fig. 7. Fractional Energy Savings vs Source Angular Width for different values of ϵ for $\bar{P}_r = 1.1$, net radius of 28.

evaluating (6) as a function of extended source angular width and ϵ, which is defined as $\epsilon = \tau_b - \tau_d$. Fig. 7 assumes $\bar{P}_r$ = 1.1, network radius of 28, and τ_d=1.0. When the network fails to deliver the message to the Sink, the FES is set to zero, which is the case for excessively small values of angular width. It can be seen that the value of $\epsilon = 0.5$ yields the highest FES, and is also the one that has the narrowest range of operation in terms of angular width. Operation over the widest range of angular widths is achieved with ϵ values of 1.5 and 2.0, respectively. For the value of ϵ=2.0, many more nodes are allowed to participate in the transmission at each level, reducing the overall FES. A value $\epsilon = 1.0$ is compromise between large required angular width and FES. In general, as ϵ is decreased, a larger extended (originating) source angular width is required for messages to reach the sink.

VI. Conclusions

We investigated upstream OLA behavior using a numerical technique, assuming the OLACRA-FT protocol. Our numerical technique for evaluating the closed form expression for the received power significantly reduced computation time when compared to running Monte Carlo simulations. By plotting the shapes of the successive OLAs at each level in the upstream propagation, we gain an intuitive feel for the effect of using the dual threshold approach of OLACRA-FT. We examined the FES behavior and found that for smaller values of ϵ, less nodes are allowed to transmit at each level, allowing for the greatest FES. However, these lower values of ϵ were found to require a lager source angular width in order to ensure the messages reach the sink. In all cases, considerable energy savings were observed employing OLACRA-FT compared to simple OLA flooding.

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