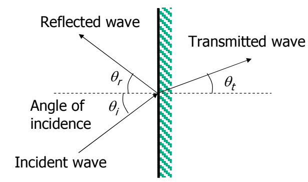
Diffraction

ECE 4823
Instructor: M.A. Ingram

Recall Snell's Laws

- Snell's Laws account for reflection, refraction and transmission at an infinite interface

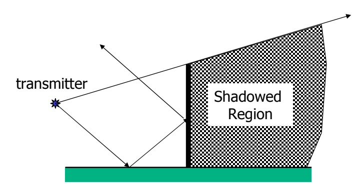


Note: Geometrical Optics

- The geometrical optics approach allows finite, curved interfaces to be modeled as infinite, planar interfaces
- This approach represents a simple way to approximate the field at a receiver

The Shortcoming of Geometrical Optics

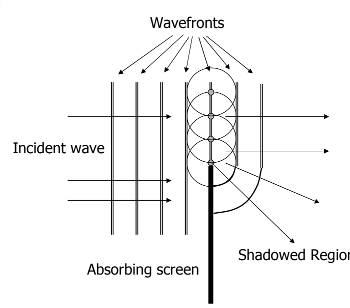
- Geometrical optics predicts no field in the shadow of an obstruction



Huygen's Principle

- Huygen's principle predicts the diffraction of a wave around obstructions
  - Each element of a wavefront at a point in time may be regarded as the center of a secondary disturbance, creating spherical wavelets
  - The wave at any later time is the superposition of all such wavelets

Knife-Edge Diffraction



[Saunders '99]

Diffraction of Water Waves

[Saunders '99]

Use of Knife-Edge Diffraction

- Gives insight into the order of diffraction loss over buildings or hills

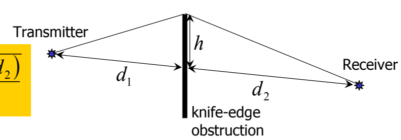
Knife-Edge Diffraction Loss

- The reduction in field strength in dB relative to free-space is

$$L_{ke} = -20\log_{10}\left(\left|\frac{E_d}{E_{fs}}\right|\right) = -20\log_{10}|F(v)|$$

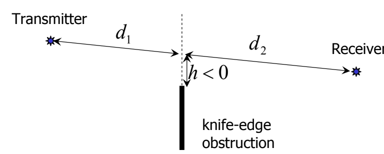
where $F(v)$ is the Fresnel Integral and v is a normalized parameter

$$v = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}}$$



h Can Be Negative

- When the knife edge is below the LOS, h is negative



Diffraction Gain=-Loss

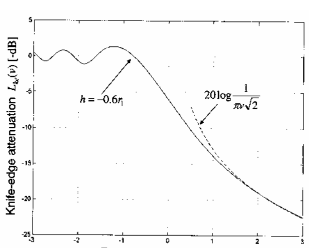
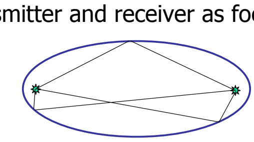


Figure 3.15: Knife-edge diffraction attenuation: (—) exact (– –) large v approximation

[Saunders '99]

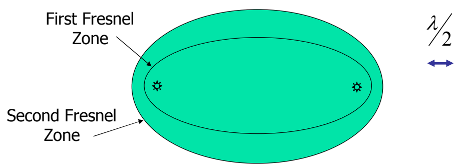
Contours of Constant Path Length

- Consider the lengths of paths of propagation that have exactly one bounce or bend
- The bounce/bend points for paths of the same length form an ellipsoid with the transmitter and receiver as foci



Fresnel Zones

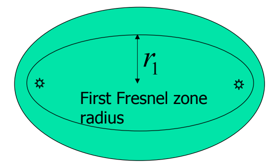
- The ellipsoid that corresponds to path lengths $n\lambda/2$ longer than the LOS path encloses the nth Fresnel Zone



Fresnel Zone Radii

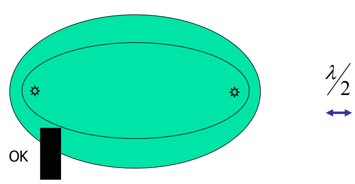
- The radius of the nth Fresnel zone is half the length of its minor axis

$$r_n = \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}}$$



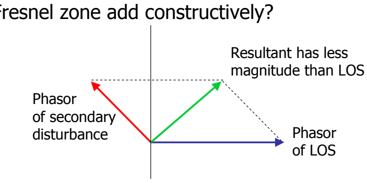
Fresnel Zones & Obstructions

- All paths within the first Fresnel Zone combine constructively
- An object that does not obstruct the first Fresnel Zone causes insignificant diffraction loss



A Seeming Paradox

- Paths within the first Fresnel zone that are more than λ/4 longer than the LOS path have phases that differ by more than 90 degrees from the LOS path phase
- Then why is it claimed that all paths in the first Fresnel zone add constructively?



[Born & Wolf, *Principles of Optics*, 6th ed, 1989]

The Answer

- The overall contribution of all paths within the Fresnel zone is based on an integral over a continuous domain

$$\int_a^b \exp\left(i\frac{2\pi}{\lambda}s\right)ds = \frac{\lambda}{2\pi i}\exp\left(i\frac{2\pi}{\lambda}s\right)\Big|_a^b$$

$$= \frac{\lambda}{2\pi i}\left[\exp\left(i\frac{2\pi}{\lambda}b\right) - \exp\left(i\frac{2\pi}{\lambda}a\right)\right]$$

$$= \frac{\lambda}{2\pi i}\exp\left(i\frac{2\pi}{\lambda}\frac{(b+a)}{2}\right)\left[\exp\left(i\frac{2\pi}{\lambda}\frac{(b-a)}{2}\right) - \exp\left(-i\frac{2\pi}{\lambda}\frac{(b-a)}{2}\right)\right]$$

$$= \frac{\lambda}{\pi}\exp\left(i\frac{2\pi}{\lambda}\frac{(b+a)}{2}\right)\sin\left(\frac{2\pi}{\lambda}\frac{(b-a)}{2}\right)$$

The Answer, Concluded

- Sine is maximized when $\frac{2\pi}{\lambda}\frac{(b-a)}{2} = \frac{\pi}{2}$ or when b-a=λ/2
- Therefore, the integral limits 0 and λ/2 are optimum even if selected values of the integrand do not add constructively

$$\int_0^{\frac{\lambda}{2}} \exp\left(i\frac{2\pi}{\lambda}s\right)ds$$

Knife-Edge Diffraction Again

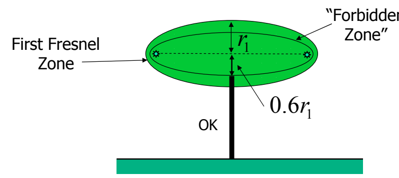
- Can express the Fresnel diffraction parameter in terms of the first Fresnel radius

$$v = h\sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}} = \frac{h\sqrt{2}}{r_1}$$

"Forbidden Zone"

[Saunders '99]

- When $h = -0.6r_1$, $v = -0.8$, and the knife-edge diffraction loss is 0dB



Summary

- Diffraction enables radio reception behind obstructions
- Huygen's Principle explains diffraction
- Knife-edge diffraction loss gives insight into diffraction from buildings and hills
- As long as obstructions stay outside of most of the first Fresnel zone, losses are insignificant
  - knife edge can cut to -0.6r_1 without significant loss