

Log-normal Shadowing

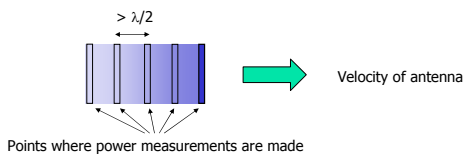
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Shadowing

- Also called *slow-fading*
- Accounts for random variations in received power observed over distances comparable to the widths of buildings
- Extra transmit power (*a fading margin*) must be provided to compensate for these fades

Local Average Power Measurements

- Take power measurements in Watts as the antenna is moved in a on the order of a few wavelengths
- Average these measurements to give a local average power measurement



Scanning in 3D

- After the cart is stabilized, the XYZ actuators move an antenna in 3 D to make a set of local power measurements
- One local average power is computed for each cart location

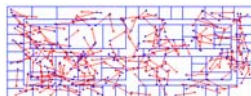
Smart Antenna
Research Laboratory



Same-Distance Measurements

- Local averages are made for many different locations, keeping the same transmitter-receiver distance
- These local averages will vary randomly with location

Example Tx-Rx locations within a floor of a building



Repeat for Multiple Distances

- Similar collections of average powers are made for other Tx-Rx distances

Likelihood of Coverage

- At a certain distance, d , what is the probability that the local average received power is below a certain threshold γ ?

$$P(P_r(d) < \gamma)$$

$$P_r(d) = \frac{P_t G_t G_r}{L_t L(d) L_r}$$

distance dependence shown explicitly

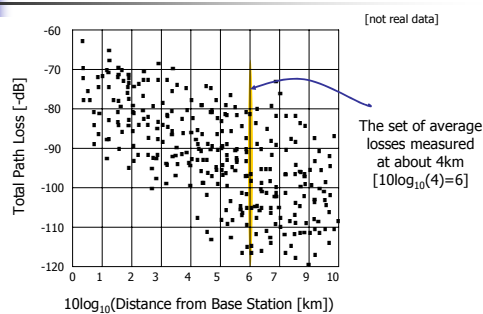
Likelihood of Coverage, cont'd

- Since only the path loss $L(d)$ is random, the probability can be expressed as a probability involving path loss:

$$P(P_r(d) < \gamma) = P(L(d) > \beta)$$

maximum tolerable path loss

Typical Macrocell Characteristics



Path Loss Assumptions

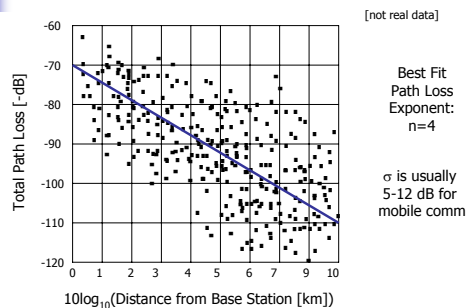
- The mean loss in dB follows the *power law* :

$$\bar{L}(d) = \bar{L}(d_o) + 10n \log_{10}\left(\frac{d}{d_o}\right)$$

- The measured loss in dB varies about this mean according to a zero-mean Gaussian RV, X_{σ} with standard deviation σ

$$L(d) = \bar{L}(d_o) + 10n \log_{10}\left(\frac{d}{d_o}\right) + X_{\sigma}$$

Typical Data Characteristics



Probability Calculation

- Since $L(d)$ is Gaussian, we need to know how to calculate probability involving Gaussian RVs

$$P(L(d) > \beta)$$

Q Function

- If X is a Gaussian RV with mean α and standard deviation σ , then

$$P(X > b) = Q\left(\frac{b - \alpha}{\sigma}\right)$$

where Q is a function defined as

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^{+\infty} \exp\left(-\frac{x^2}{2}\right) dx$$

The Problem with Q

- The integrand of Q has no antiderivative
- Q is found tabulated in books
- Q can be calculated using numerical integration

What is Log-normal Shadowing?

- If Y is a Gaussian RV and Z is defined such that $Y = \log Z$, then Z is a log-normal RV
- Shadowing is log-normal shadowing when the path loss in dB is Gaussian; this means that the path loss expressed as a ratio is log-normal

Inverse Q Problems

- Sometimes, the probability is specified and we must find one of the parameters in the argument of Q

$$P(X > b) = Q\left(\frac{b - \alpha}{\sigma}\right)$$

- Suppose the value of $P(X > b)$ is given, along with values of b and α . Solve for σ
- Must look up the argument of Q that gives the specified value.

Example Inverse Q Problem

- Suppose the mean of the local average received powers at a certain distance is -30dBm, that the standard deviation of shadow fading is 9 dB, and that the observed received power is above the threshold 95% of the time. What is the threshold power?

$$P(P_r > b) = Q\left(\frac{b - (-30)}{9}\right) = 0.95$$

- Q is usually tabulated for arguments of 0.5 and less, so we must use the fact that

$$Q(z) = 1 - Q(-z)$$

- The argument of Q that yields 0.05 is about 1.65

$$-\frac{b + 30}{9} = 1.65, \text{ and } b = -44.85 \text{ dBm}$$

Boundary Coverage

- Suppose that a cell has radius R and γ is the minimum acceptable received power level
- Then $P(P_r(R) > \gamma)$ is the "likelihood of coverage" at the boundary of the cell
- $P(P_r(R) > \gamma)$ is also the "fraction of time" that a mobile's signal is acceptable at a distance R from the transmitter, assuming the car moves around that circle

Percentage of Useful Service Area

- By integrating these probabilities over all the circles within a disk, one can compute the fraction of the area within the cell that will have acceptable power levels

$$U(\gamma) = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R P(P_r(r) > \gamma) r dr d\theta$$

Integral Evaluated

- Assuming log-normal shadowing and the power path loss model, the fraction of useful service area is

$$U(\gamma) = \frac{1}{2} \left(1 - \operatorname{erf}(a) + \exp\left(\frac{1-2ab}{b^2}\right) \left[1 - \operatorname{erf}\left(\frac{1-ab}{b}\right) \right] \right)$$

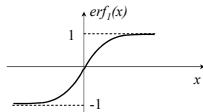
where

$$a = \frac{\gamma - \overline{P_r(R)}}{\sigma\sqrt{2}} \quad \text{and} \quad b = \frac{10n \log_{10} e}{\sigma\sqrt{2}}$$

The Error Function

- $\operatorname{erf}(x)$ is another form of the Gaussian integral (like $Q(x)$)
- $\operatorname{erf}(x)$ has odd symmetry, with extreme values ± 1 .

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$



- Note that some authors may define erf differently

erf and Q

- The erf function and Q are related:

$$\operatorname{erf}(z) = 1 - 2Q(\sqrt{2}z)$$

When the Average Boundary Power is Acceptable

- Suppose $P_r(R) = \gamma$. Then we may use this graph from [Rappaport '96] to figure the percent useful service area

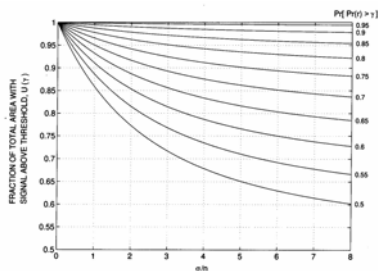


Figure 3.18
Family of curves relating fraction of total area with signal above threshold, $U(\gamma)$ as a function of probability of signal above threshold on the cell boundary.

Summary

- Local averages in dB of received power (or path loss) tend to be Gaussian when the ensemble is all Tx-Rx locations with the same distance in the same type of environment
- The mean local average path loss follows the standard power model (proportional to $10 \log d^{\alpha}$)
- Can use Q or erf to calculate the likelihood of boundary coverage or the percent of useful service area



References

- [Rapp, '96] T.S. Rappaport, *Wireless Communications*, Prentice Hall, 1996
- [Saunders, '99] Simon R. Saunders, *Antennas and Propagation for Wireless Communication Systems*, John Wiley and Sons, LTD, 1999.