Coherence Bandwidth

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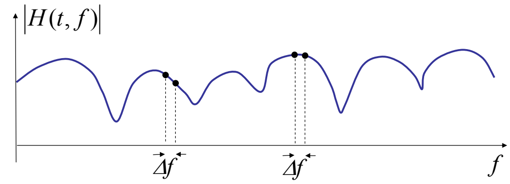
ECE4823

Correlation in Frequency

- We can view the frequency response of a channel as a Random Process as a function of f
- We can ask, "What is the correlation between responses at different frequencies?"



Review: Correlation Coefficient

- Suppose X and Y are two complex RVs
- Their correlation coefficient is defined

$$\rho_{XY} = \frac{E\left\{(X - m_X)(Y - m_Y)^*\right\}}{\sigma_X \sigma_Y}$$

- This is a normalized covariance; it varies between +1 and -1

Correlation Coefficient for Random Processes

- Now suppose $H(f)$ is a RP wrt f
- That means that for any fixed f, $H(f)$ is a RV
- Consider two frequencies f_1 and f_2. The correlation coefficient becomes

$$\rho_{H(f_1)H(f_2)} = \frac{E\left\{\left(H(f_1) - m_{H(f_1)}\right)\left(H(f_2) - m_{H(f_2)}\right)^*\right\}}{\sigma_{H(f_1)}\sigma_{H(f_2)}}$$

Wide Sense Stationary Uncorrelated Scattering (WSSUS)

- Assumes that path gains at different delays are uncorrelated
- Assumes correlations between frequency responses depends only on the frequency difference Δf

$$\rho_{\Delta f} = \frac{E\left\{(H(f) - m_H)(H(f+\Delta f) - m_H)^*\right\}}{\sigma_H^2}$$

Mean is Zero

- Because the phase is uniformly distributed over $[0,2\pi]$, $m_H=0$, so the correlation coefficient becomes

$$\rho_{\Delta f} = \frac{E\left\{H(f)H(f+\Delta f)^*\right\}}{E\left\{|H(f)|^2\right\}}$$

Coherence Bandwidth

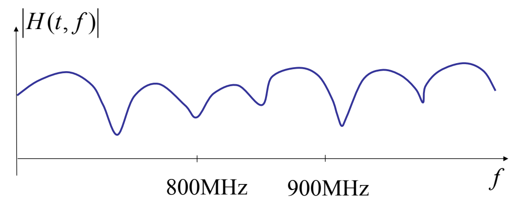
- The X% coherence bandwidth is that value of Δf such that

$$\rho_{\Delta f} = \frac{X}{100}$$

- If the 90% coherence bandwidth is 30KHz, then responses for frequencies separated by 30KHz or less will be nearly equal

Illustration

- Do you think the 90% coherence bandwidth is > or < 100MHz?



Relation to RMS Delay Spread

[Rappaport, 2002]

- The 90% coherence bandwidth is approximately

$$B_{C,90} = \frac{1}{50\sigma_\tau}$$

- The 50% coherence bandwidth is approximately

$$B_{C,50} = \frac{1}{5\sigma_\tau}$$

Need for Equalization

- If a transmitted signal's bandwidth is greater than the 50% coherence bandwidth, then the channel is frequency selective
- An equalizer (adaptive tapped delay filter) will be needed in the receiver
- Flat-fading channels do not require equalization

Time Dispersion Relationships

Flat Fading	Frequency Selective Fading
$B_{C,50} > B_S$	$B_{C,50} < B_S$
$\sigma_\tau < 0.2T_S$	$\sigma_\tau > 0.2T_S$

Summary

- Delay spread and coherence bandwidth are inversely related and quantify the effects of multipath delays
- They can be used to estimate the maximum data rate that can be supported without the use of an equalizer

References

- [Rapp, '02] T.S. Rappaport, *Wireless Communications*, Prentice Hall, 2002