Coherence Time

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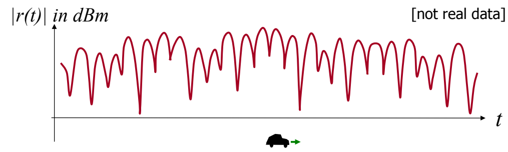
ECE4823

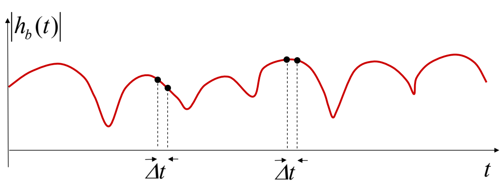
Recall Flat-fading Channels

- The response to a CW (or long pulse) probe signal will fluctuate in time as the vehicle moves



Correlation in Time

- We can view the time fluctuation of a channel as a Random Process as a function of t
- We can ask, "What is the correlation between responses at different times?"



Wide Sense Stationary Uncorrelated Scattering (WSSUS)

- Assumes correlations between flat fades at different times depends only on the time difference Δt

$$\rho_{\Delta t} = \frac{E\left\{h_b(t)h_b(t+\Delta t)^*\right\}}{E\left\{|h_b(t)|^2\right\}}$$

Coherence Time

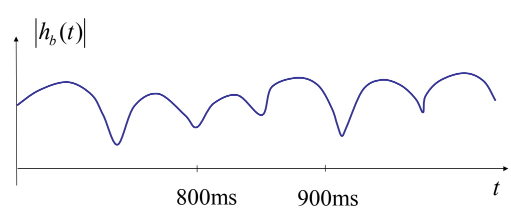
- The X% coherence time is that value of $\Delta\tau$ such that

$$\rho_{\Delta t} = \frac{X}{100}$$

- If the 90% coherence time is 3ms, then the response to a pulse of length 100μs will have a nearly constant envelope

Illustration

- Do you think the 90% coherence time is > or < 100ms?



Doppler Effect

- Suppose a mobile transmitting at carrier frequency f_o approaches a stationary receiver directly at a speed of v

- The carrier frequency of the received signal will be

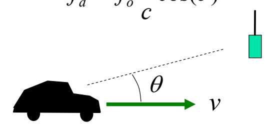
$$f_{rec} = f_o + f_d, \quad \text{where} \quad f_d = f_o \frac{v}{c}$$

Maximum Doppler Shift

Non-direct Approach

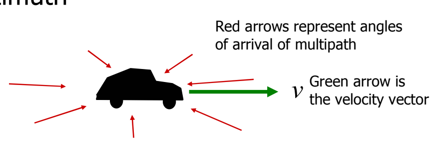
- If the velocity of the vehicle is at an angle θ to the receiver, then the Doppler shift is

$$f_d = f_o \frac{v}{c} \cos(\theta)$$



Clarke's Model

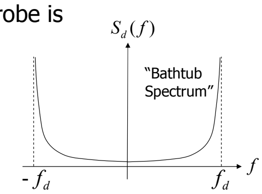
- Assumes that paths of equal average amplitude and random phases arrive at the mobile from random directions in azimuth



Gans' Power Spectrum of Clarke's Model

- Let f_d be the maximum Doppler frequency for the given speed
- Then the baseband power spectrum of the response to a CW probe is

$$S_d(f) = \begin{cases} \dfrac{\sigma}{\pi f_d \sqrt{1 - \left(\dfrac{f}{f_d}\right)^2}} & |f| \le f_d \\ 0 & |f| > f_d \end{cases}$$



Relation to Maximum Doppler

- Assuming Clarke's model, the 50% coherence time can be defined [Shankar, 2002][Rapp,2002]

$$T_C = \frac{9}{16\pi f_d}$$

where f_d is the maximum Doppler frequency

- A popular rule of thumb is [Rappaport,2002]

$$T_C = \frac{1}{f_d}\sqrt{\frac{9}{16\pi}} = \frac{0.423}{f_d}$$

Fast-fading

- A channel is ***fast-fading*** if the symbol period is longer than the coherence time

$$T_S > T_C$$

- Alternatively, the condition can be expressed in terms of the signal bandwidth and the ***Doppler spread***, which is inversely related to the coherence time

$$B_S < B_D = \frac{k}{T_C}$$

Slow-fading

- Slow-fading is the (conservative) opposite of fast-fading
- Safe to assume $h_b(t)$ is constant during at least one symbol period

Doppler-related Relationships

Fast Fading	Slow Fading
$B_S < B_D$	$B_S >> B_D$
$T_S > T_C$	$T_S << T_C$

Combinations of Descriptions

- Channels can be
  - Slow and Flat
  - Fast and Flat
  - Slow and Frequency Selective
  - Fast and Frequency Selective

Summary

- Coherence time and Doppler Bandwidth are inversely related and quantify the time-variation of the channel
- Clarke's model yields the "bathtub spectrum" and depends only on the maximum Doppler frequency

References

- [Rapp, '02] T.S. Rappaport, *Wireless Communications*, Prentice Hall, 2002
- [Shankar, '02] P. Mohana Shankar, *Introduction to Wireless Systems*, John Wiley & Sons, 2002