Small-Scale Fading Distributions

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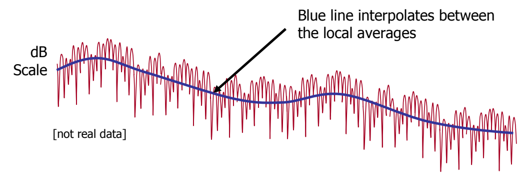
ECE4823

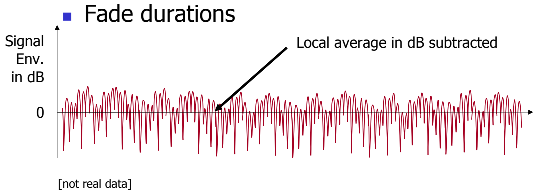
Small-Scale Fading

- Describes the fluctuations in the received signal envelope relative to the local average used for path loss



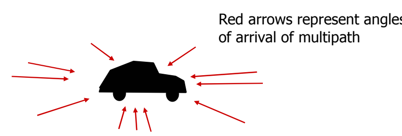
Types of Statistics

- PDF or CDF describing the values ⇦ In this Module
- Level crossings
- Fade durations



Physical Motivation

- In a typical non-line-of-sight (NLOS) channel, many paths of comparable strength combine at the receiver
- The Central Limit Theorem predicts that both the real and imaginary parts of the resulting waveform have Gaussian statistics



The Envelope

- Suppose $\alpha = X + jY$, where X and Y are independent Gaussian RVs, both zero mean and variance σ^2
- Let $R = |\alpha|$, or

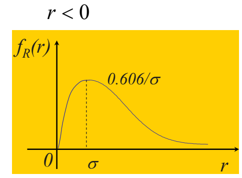
$$R = \sqrt{X^2 + Y^2}$$

- Then, R has a Rayleigh Distribution

Rayleigh Distribution

- The PDF for the Rayleigh RV is

$$f_R(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left\{ -\frac{r^2}{2\sigma^2} \right\} & r \geq 0 \\ 0 & r < 0 \end{cases}$$



[Rappaport, 1996]

Rayleigh Moments

- The general formula for moments is [Papoulis, 1984]

$$E\{R^n\} = \begin{cases} 1\cdot 3 \cdots n\sigma^n \sqrt{\pi/2} & n = 2k+1 \\ 2^k k! \sigma^{nk} & n = 2k \end{cases}$$

- In particular,

$$E\{R\} = \sigma\sqrt{\pi/2}$$
$$E\{R^2\} = 2\sigma^2 \qquad Var\{R\} = 2\sigma^2 - \sigma^2\pi/2$$

LOS Multipath Channel

- Now, suppose that in addition to the many non-LOS paths, there is a LOS path with peak amplitude A
- Let $K = |A|^2 / 2\sigma^2$
  be the ratio of deterministic signal power $|A|^2/2$ to the average power of the rest of the signal, σ^2
- This is the "K factor" or "Ricean Factor"
- We don't expect such deep fades if K>1

The Envelope

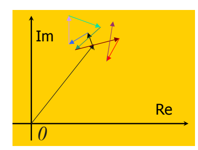
- Let $\alpha = X + jY + A$, where X and Y are as before, and A is non-random
- Again, let $R = |\alpha|$, or

$$R = \sqrt{(X + \mathrm{Re}(A))^2 + (Y + \mathrm{Im}(A))^2}$$

- Then R has a Ricean Distribution

Ricean Distribution

- This PDF describes the magnitude of the sum of one deterministic vector with a lot of iid random vectors

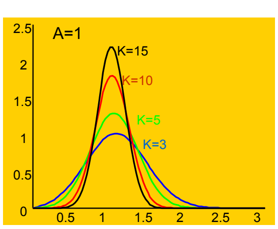


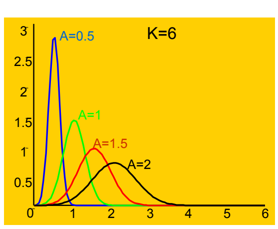
$$f_R(r) = \begin{cases} \dfrac{r}{\sigma^2} \exp\left\{-\dfrac{r^2 + A^2}{2\sigma^2}\right\} \cdot \mathrm{I}_0\left(\dfrac{\mathrm{Ar}}{\sigma^2}\right) & A \ge 0, r \ge 0 \\ 0 & r < 0 \end{cases}$$

Modified Bessel function of the first kind and zero order

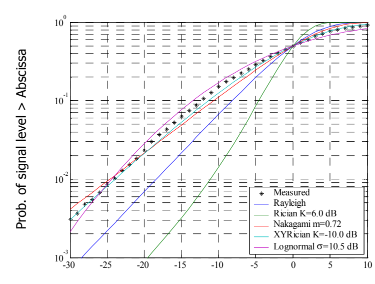
Ricean Examples

- Left: the PDF for fixed A and various K
- Right: the PDF for fixed K and various A





Example CDFs



Back To Rayleigh

- The Rayleigh Distribution is a special case of the Ricean Distribution when K=0

Summary

- Two popular distributions for describing small-scale fading are the Rayleigh and Ricean distributions
- Rayleigh for non-LOS channels
- Ricean for LOS channels
- The K factor is the key parameter

References

- [Rapp, '02] T.S. Rappaport, *Wireless Communications*, Prentice Hall, 2002
- [Papoulis, '85] Athanasios Papoulis, *Probability, Random Variables, and Stochastic Processes*, McGraw-Hill, 1985